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SIMULATION OF GUST RESPONSE OF COMMERCIAL AIRCRAFT USING THE PEELE METHOD

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Abstract. *This work consists of a comparative study of two methodologies for the analysis of aircraft dynamic responses, due to vertical and lateral continuous gusts. The objective of such calculations is to provide the load spectra, which are needed for aircraft structural analysis and fatigue life estimation.*
The first methodology uses an in-house, Matlab-based code (PGUST), based on the Peele method to estimate the gust response parameters. The airplane is modeled as a rigid body with two degrees of freedom. This method is very straightforward and requires a minimum amount of input data, which is extremely useful for the early stages of an aircraft design cycle.
The second methodology is based on more detailed, finite-element-based models using MSC/NASTRAN. With this approach, it is possible to estimate response parameters for vertical and lateral gust taking into account the influence of the aircraft main flexible modes. Both methods use the Von Kármán turbulence spectrum as input.
The present article compares the results obtained from the application of the two methodologies for the simulation of gust responses of several commercial aircraft. The simplified methodology is validated and its advantages and limitations are discussed.

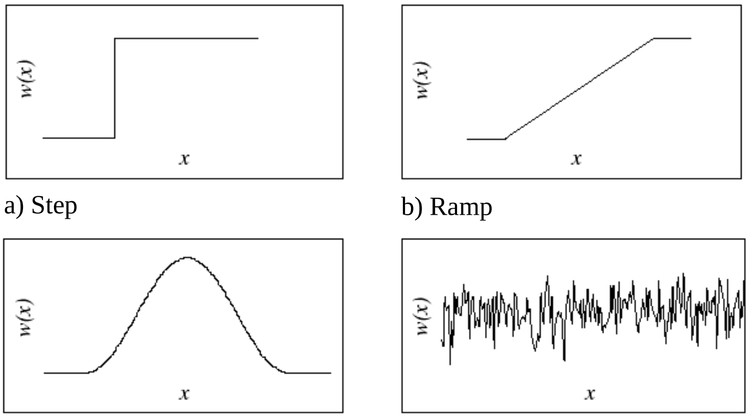
Keywords*: Continuous gust, Peele, aircraft dynamic response, gust loads*

1. Introduction

Around 1930, it was recognized that gust effects had a strong influence on the determination of aircraft design loads. The first approach to model the effects of a gust on the airplane dynamics was a simple description of the localized wind velocity profile, known as the step model (Fig.1a). Later, in the 1940's, a better model was required and a ramp model was developed (Fig.1b). However, those models were too conservative, and would not permit the fulfillment of the weight reduction and performance requirements that emerged with the advances in commercial aviation. Therefore, a more realistic model, known as the "(1-cos)" gust model (Fig.1c) was adopted in 1955 (Lomax, 1996).

All of the previously mentioned methods are based on a deterministic description of the gust loads. In the 1960's the development of continuous gust methodologies started (Pratt et al., 1960), but the first certification requirement was published only in 1980. Continuous gust models are stochastic models (Fig.1d), and therefore are a much better description of the atmospheric phenomena, which are essentially random.

The main objective of this paper is the comparison between the dynamic responses of commercial aircraft, obtained using two different methodologies. The first one is based on the Peele method, which models the aircraft as a rigid body. This method is very straightforward and can provide quick answers, as required during the early stages of the design of a new airplane. The second approach is based on a more detailed, finite element model (FEM), created with MSC/NASTRAN, which includes the effects of the airplane flexible modes.



a) Step
b) Ramp
c) (1-cos)
d) Continuous gust

Figure 1. Gust classified by geometry.

2. Formulation of the Peele method

This work discusses a method that uses Peele parameters to estimate the aircraft CG acceleration response, or more specifically the parameters $\overline{A}$ and N_0, respectively the gain and exceedance frequency of the gust response, defined as:

$$\overline{A} = \frac{\sigma_y}{\sigma_w} \tag{1}$$

and

$$N_0 = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{\int_0^\infty \omega^2 \cdot \phi(\omega) d\omega}{\int_0^\infty \phi(\omega) d\omega}} \tag{2}$$

Where:

σ_y : Output RMS signal.

σ_w : Input RMS signal.

$\phi(\ )$: Power spectra density (PSD) of the signal.

ω : Radial frequency.

The Peele method is one of the so-called “short-cut methods”, which models the aircraft as a rigid body. Short-cut methods are based on simplified calculations and are used in a preliminary design phase, where accurate data are difficult to obtain, and a fast load response has to be obtained. In order to simplify the $\overline{A}$ and N_0 calculation, Peele developed the formulas below:

$$\overline{A}_{\Delta n} = \frac{V_T}{g \cdot \delta} \cdot \sqrt{R_4 + B_1 \cdot R_2} \tag{3}$$

$$N_{0\Delta n} = f_0 \cdot \sqrt{\frac{R_6 + B_1 \cdot R_4}{R_4 + B_1 \cdot R_2}} \tag{4}$$

$$\overline{A}_{\ddot{\theta},\ddot{\psi}} = \frac{(2\cdot\pi\cdot f_0)^2}{V_T}\cdot\sqrt{R_4} \tag{5}$$

$$N_{0_{\ddot{\theta},\ddot{\psi}}} = f_0\cdot\sqrt{\frac{R_6}{R_4}} \tag{6}$$

Where:

V_T : True airspeed.

f_0 : Undamped natural frequency.

δ : Translational response distance constant.

B_1 : Gust intensity measure.

R_j : Peele response integral.

g : Gravity acceleration.

In the Peele method, both input and output can be described as statistical data. The most usual mathematical description of the continuous gust is the Von Kármán profile (Hoblit, 1988).

Vertical and lateral gust formulations are very similar, having as difference, basically, the stability derivatives used. The inputs for this method are few and easy to obtain in a preliminary design phase, consisting of basic geometric data, aircraft mass and inertia, atmospheric data, preliminary aerodynamic parameters and some stability derivatives. A more detailed formulation of the method can be found in Hoblit, 1988.

3. PGUST development

PGUST is a piece of software developed in Matlab and based on Peele method, which estimates the aircraft response due to both vertical and lateral gust.

The main challenge found for the aircraft response calculation due to gusts using Peele method was to obtain intermediate response integrals, the R-parameters (R_0, R_2, R_4 e R_6). Usually, these variables are available in graphical form, and one has to graphically interpolate in order to obtain the actual values for each particular aircraft (Hoblit, 1988). In order to avoid the errors intrinsic to the interpolation process, the PGUST routine includes a numerical calculation of the R integrals (Peele, 1971). The general form of the response integrals is given by:

$$R_j = \frac{sk_0}{\pi}\cdot\int_0^{\overline{\beta}^*}\frac{\overline{\beta}^j\cdot e^{-\tilde{a}k_0\cdot\overline{\beta}}\cdot\left[1+\frac{8}{3}\cdot\left(1.339\cdot sk_0\cdot\overline{\beta}\right)^2\right]}{\left[\left(1-\overline{\beta}^2\right)^2+4\cdot\zeta^2\cdot\overline{\beta}^2\right]\cdot\left[1+\left(1.339\cdot sk_0\cdot\overline{\beta}\right)^2\right]^{\frac{11}{6}}}d\overline{\beta} \tag{7}$$

Where:

R_j : Peele response integral.

s : Relative gust scale, $\frac{2\cdot L}{c}$

L : Scale of Turbulence

c : Mean aerodynamic chord.

ζ : Damping ratio.

$\overline{\beta}$: Ratio between circular frequency and natural frequency.

$\tilde{a}$: Küssner parameter.

k_0 : Reduced undamped natural frequency.

As Peele already noted in his original work (Peele, 1971), the numerical integrations for R_2 and R_4 converge very fast, while the integration of R_6 can lead to physically unacceptable results. In order to solve this problem, Peele suggests that the same $\overline{\beta}^*$ found in R_4 convergence in the calculation of $\overline{A}_{\Delta n}$ should to be used as an upper integral limit for the R_6 calculation.

During the research, no information about the R_4 convergence precision was found, and in order to solve this problem, two examples were solved, from Hoblit, 1988 and Peele, 1971, finding 1% as convergence precision.

The PGUST output parameters are:

$\overline{A}_{\Delta n}$, for vertical and lateral gust.

$N_{0\,\Delta n}$, for vertical and lateral gust.

$\overline{A}_{\ddot{\theta}}$, for pitch acceleration.

$\overline{A}_{\ddot{\psi}}$, for yaw acceleration.

$N_{0\ddot{\theta}}$, for pitch acceleration.

$N_{0\ddot{\psi}}$, for yaw acceleration.

4. Results

In order to validate the quality of the results, the first step is to analyze the influence of input data in the final results. For that, sensibility analyses of the response due to variations in *R* values and stability derivatives were performed, since these are the inputs that are the most likely to include imprecision and errors in a preliminary design phase.

4.1. Sensibility analysis for R_2, R_4 and R_6 values

Firstly, the influence of R_2 and R_4 on $\overline{A}$ results was analyzed. Figure 2 and Fig. 3 show, respectively, the influence of R_2 and R_4 on $\overline{A}$.

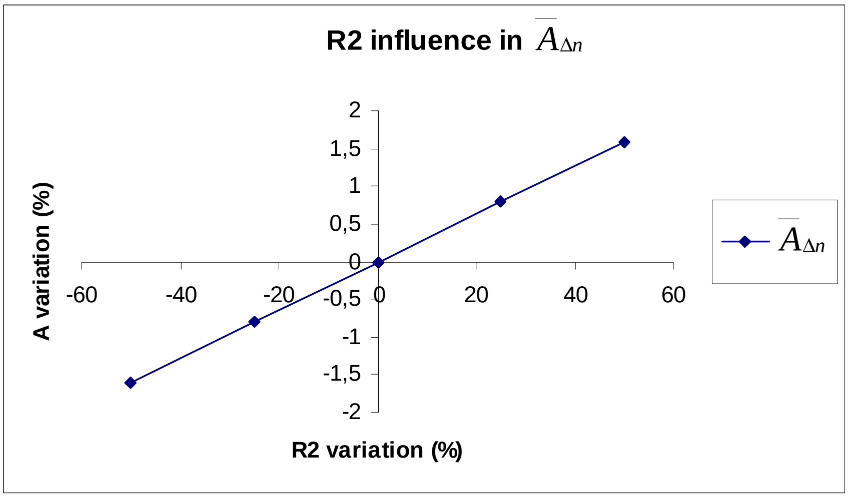


Figure 2. Influence of R_2 in $\overline{A}_{\Delta n}$ value.

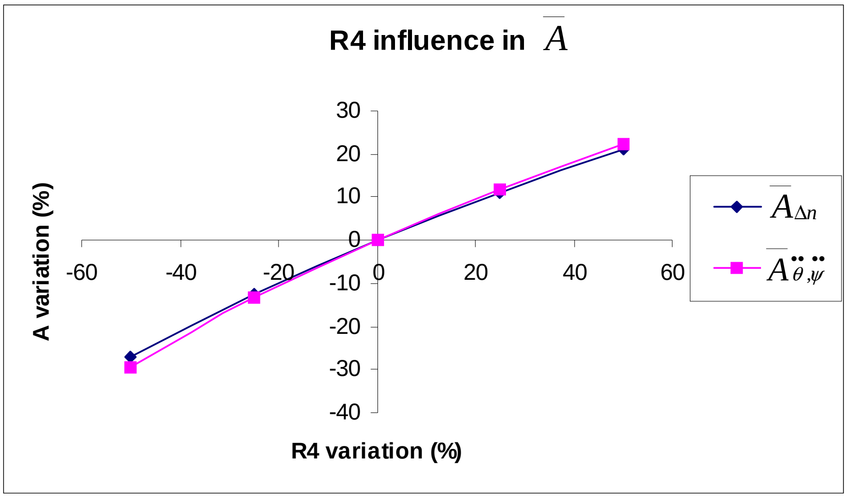


Figure 3. Influence of R_4 in $\overline{A}_{\Delta n}$ and $\overline{A}_{\ddot{\theta},\ddot{\psi}}$ values.

It can be observed from Fig. 2 and Fig. 3 that the parameter R_4 has to be determined very carefully, since its value can strongly influence the results of $\overline{A}_{\Delta n}$ and $\overline{A}_{\ddot{\theta},\ddot{\psi}}$.

Figure 4 and Fig. 5 show the influence of *R* values in $N_{0_{\Delta n}}$ and $N_{0_{\ddot{\theta},\ddot{\psi}}}$.

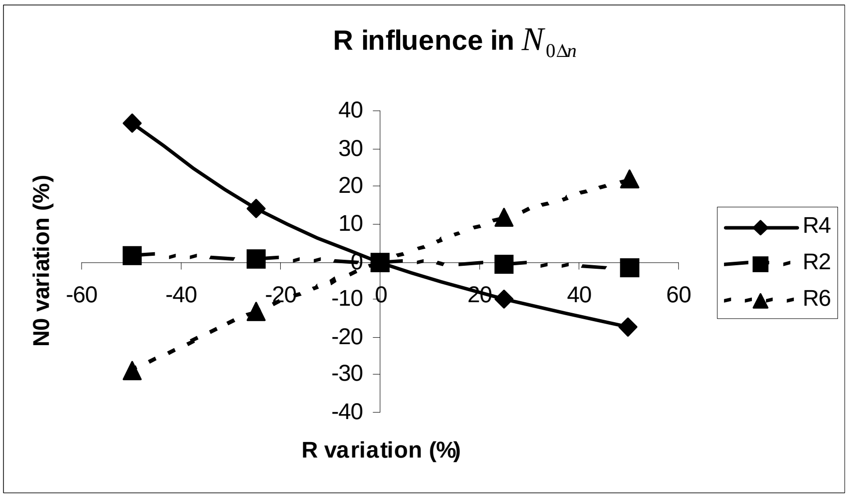


Figure 4. Influence of R values in $N_{0_{\Delta n}}$.

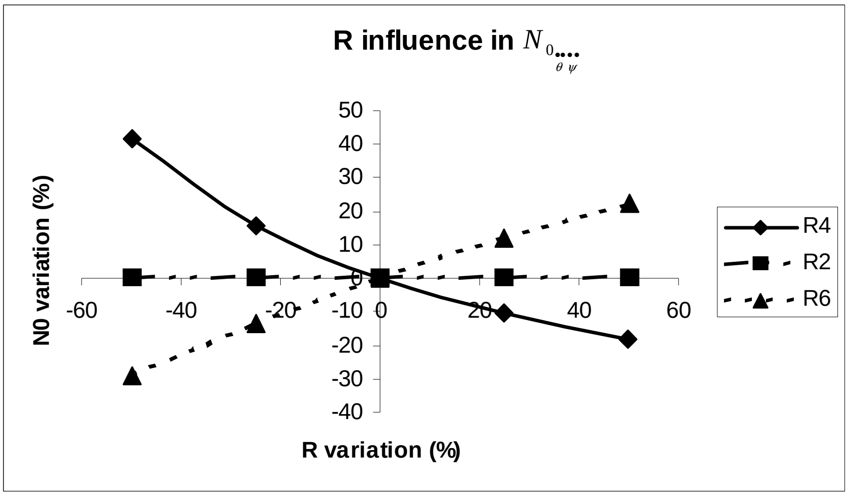


Figure 5. Influence of R values in $N_{0_{\ddot{\theta},\ddot{\psi}}}$.

Observing Fig. 4 and Fig. 5, one can notice that R_4 is a very sensible variable in the system again. However, in order to obtain reliable values of N_0, R_6 is also very important, since its influence on the results is similar to those caused by variations in R_4.

The results above confirm the necessity of including a numerical integration of R_j in PGUST, since the errors involved even in an automated graphical interpolation procedure could lead to unacceptable errors in the response integrals.

4.2. Sensibility analysis for stabilities derivatives

In order to evaluate the importance of the stability derivatives in the calculation process for vertical gust response in the Peele method, a sensibility study was done. Figure 6 and Fig. 7, show the influence of the variations of the stability derivatives on $\overline{A}_{\Delta n}$ and $\overline{A}_{\ddot{\theta}}$. It can be noticed that the most important derivative for $\overline{A}_{\Delta n}$ calculation is $C_{l\alpha}$ (aircraft lift curve slope), while for $\overline{A}_{\ddot{\theta}}$, $C_{m\alpha}$ (longitudinal static stability derivative) is the most important one.

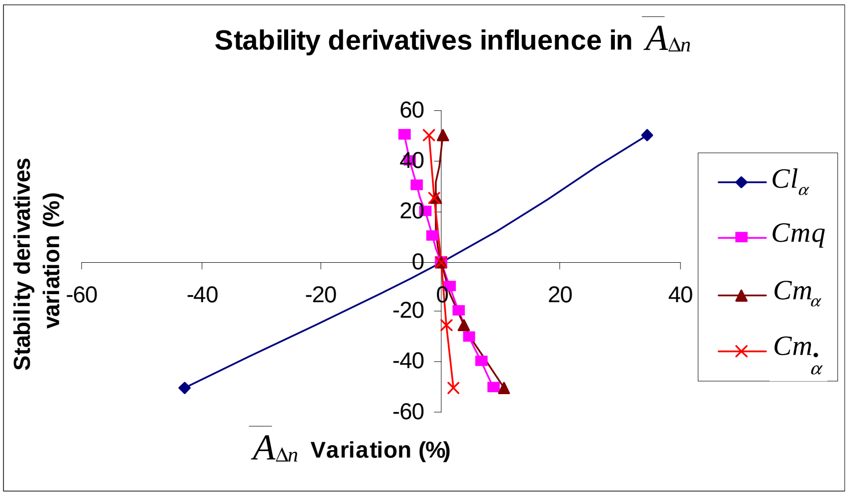


Figure 6. Effect of variation of stability derivatives on $\overline{A}_{\Delta n}$.

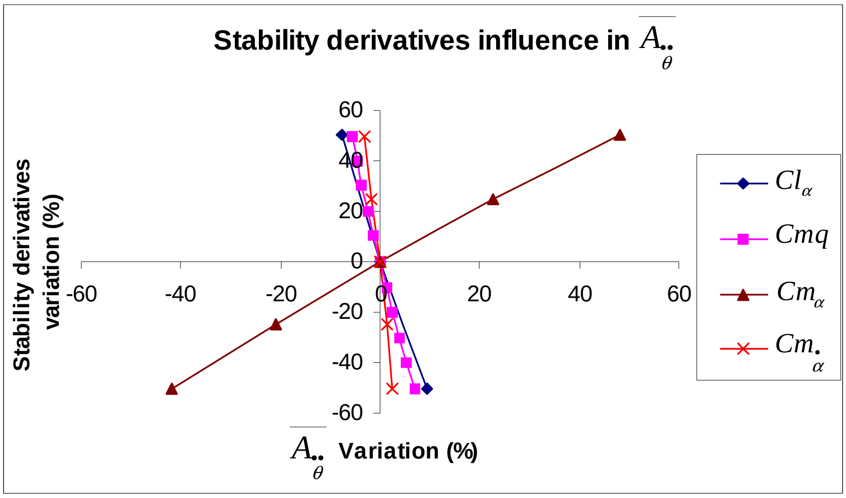


Figure 7. Effect of variation of stability derivatives on $\overline{A}_{\ddot{\theta}}$.

The sensibility study was primordial to asses the reliability of PGUST response outputs, before the results could be compared with those obtained by the more detailed FEM model.

4.3.Comparison between results by PGUST and MSC/NASTRAN

The MSC/NASTRAN model includes aircraft flexibility, and the number of flexible modes varies according to the simulated aircraft. In this study, the simulated aircraft natural frequency upper bound frequency varied from 20Hz to 70 Hz, depending on the aircraft. For aircraft A1 and A2, the dynamic model consists of beam elements, while for aircraft A3, A4 and A5, a more detailed FE model, based on plate and shell elements, was used. The results for gust response parameters obtained by both methods are shown in Tab. 1 and Tab. 2.

Table 1: Comparison between NASTRAN [N] (flexible modes included) and PGUST [G] for vertical gust.

	AIRPLANE				
	A1	**A2**	**A3**	**A4**	**A5**
Weight [Kg]	16245	18084	28807	29788	41222
$\overline{A}_{\ddot{z}}$ [s^{-1}] [N]	0,590	0,607	0,530	0,522	0,519
$\overline{A}_{\ddot{z}}$ [s^{-1}] [G]	0,465	0,443	0,447	0,439	0,395
%	-21,2	-26,9	-15,6	-15,9	-23,9
$N_{0\ddot{z}}$ [Hz] [N]	4,495	4,760	4,035	3,301	3,456
$N_{0\ddot{z}}$ [Hz] [G]	0,995	0,899	1,146	1,024	0,873
%	-77,8	-81,11	-71,5	-68,9	-74,7
$\overline{A}_{\ddot{\theta}}$ [rad/ms] [N]	0,0621	0,0783	0,068	0,0350	0,0427
$\overline{A}_{\ddot{\theta}}$ [rad/ms] [G]	0,0170	0,0153	0,023	0,0165	0,0103
%	-72,6	-80,4	-66,2	-52,8	-75,8
$N_{0\ddot{\theta}}$ [Hz] [N]	23,801	24,069	29,438	13,962	15,564
$N_{0\ddot{\theta}}$ [Hz] [G]	1,033	0,925	1,194	1,076	0,932
%	-95,6	-96,1	-95,9	-92,2	-94,0

Table 2: Comparison between NASTRAN [N] (flexible modes included) and PGUST [G] for lateral gust.

	AIRPLANE				
	A1	**A2**	**A3**	**A4**	**A5**
Weight [Kg]	16245	18084	28807	29788	41222
$\bar{A}_{\ddot{y}}$ [s^{-1}] [N]	0,1033	0,0910	0,181	0,172	0,138
$\bar{A}_{\ddot{y}}$ [s^{-1}] [G]	0,1238	0,1137	0,169	0,169	0,136
%	19,8	24,8	-6,4	-1,7	-1,2
$N_{0\ddot{y}}$ [Hz] [N]	10,041	6,730	5,247	4,509	3,946
$N_{0\ddot{y}}$ [Hz] [G]	0,355	0,316	0,727	0,697	0,550
%	-96,4	-95,3	-86,1	-84,5	-86,1
$\bar{A}_{\ddot{\psi}}$ [rad/ms] [N]	0,0229	0,0181	0,0311	0,0259	0,0229
$\bar{A}_{\ddot{\psi}}$ [rad/ms] [G]	0,0097	0,0067	0,0338	0,0295	0,0193
%	-57,6	-63,1	8,5	13,9	-15,5
$N_{0\ddot{\psi}}$ [Hz] [N]	22,445	19,039	12,547	6,564	7,526
$N_{0\ddot{\psi}}$ [Hz] [G]	0,356	0,318	0,732	0,702	0,550
%	-98,4	-98,3	-94,1	-89,3	-92,6

Analyzing the results shown in Tab. 1 and Tab. 2, large differences between N_0's values calculated by NASTRAN and PGUST are verified. This can be explained mainly by the differences in the stiffness representation of the aircraft (rigid body for PGUST and flexible model for NASTRAN). However, N_0 is less important than $\bar{A}$ for load calculations, as described in the following equations.

$$n_z = U_\sigma \cdot \bar{A}_{\Delta n} + 1 \tag{8}$$

$$n_y = U_\sigma \cdot \bar{A}_{\Delta n} \tag{9}$$

$$\ddot{\theta} = U_\sigma \cdot \bar{A}_{\ddot{\theta}} \tag{10}$$

$$\ddot{\psi} = U_\sigma \cdot \bar{A}_{\ddot{\psi}} \tag{11}$$

$$N(y) = \sum_i t_i \cdot N_0 \cdot \left(P_1 \cdot e^{-\frac{y}{b_1 \cdot \bar{A}}} + P_2 \cdot e^{-\frac{y}{b_2 \cdot \bar{A}}} \right) \tag{12}$$

Where:

$\ddot{\psi}$: Yaw acceleration.

U_σ : Gust velocity.

t : Time.

$\ddot{\theta}$: Pitch acceleration.

P_1 : fraction of time in nonstorm turbulence.

P_2 : fraction of time in storm turbulence.

b_1 : rms value of σ_w corresponding to P_1

b_2 : rms value of σ_w corresponding to P_2

As can be observed from Eq. (12), $\bar{A}$ is more important than N_0 for the fatigue spectra N(y) calculation due to its influence in the exponent of the exceedance expression. Figure 8 and Fig. 9 illustrate this difference.

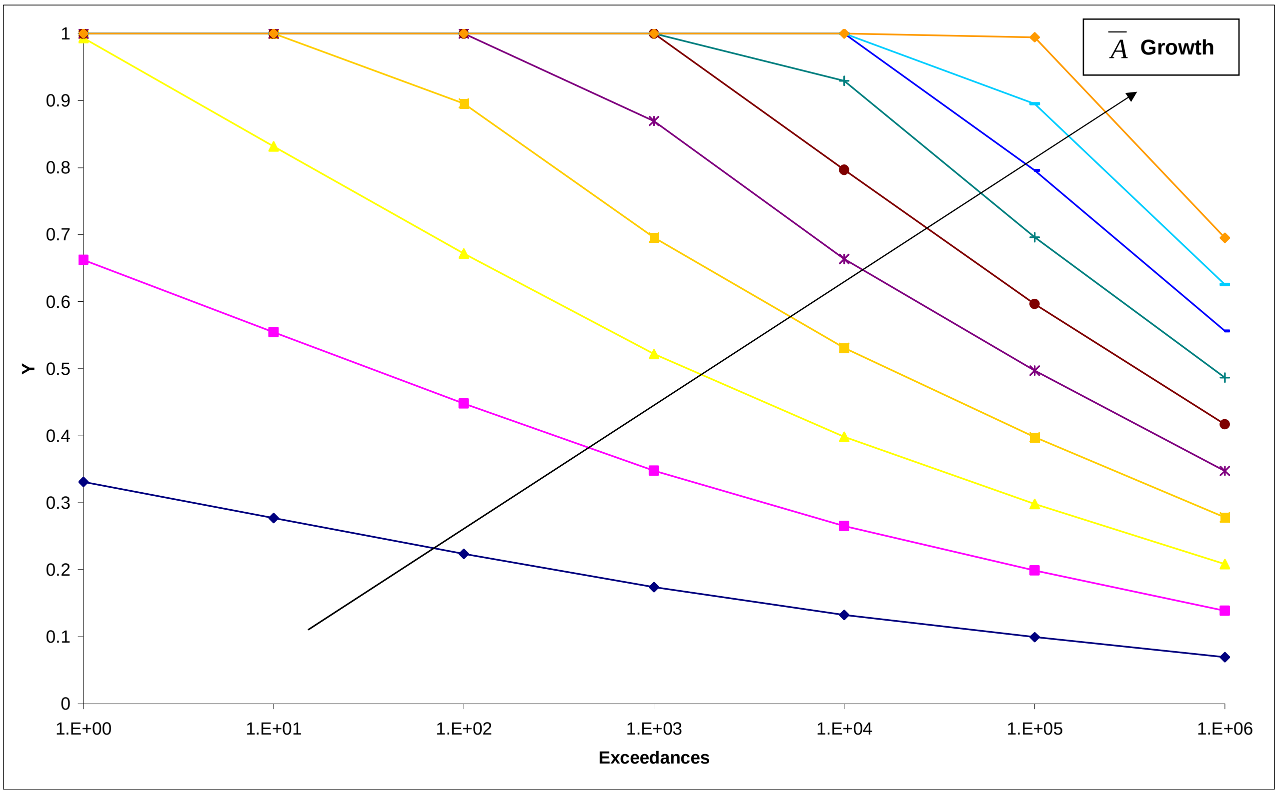


Figure 8. $\overline{A}$ influence in load factor spectra.

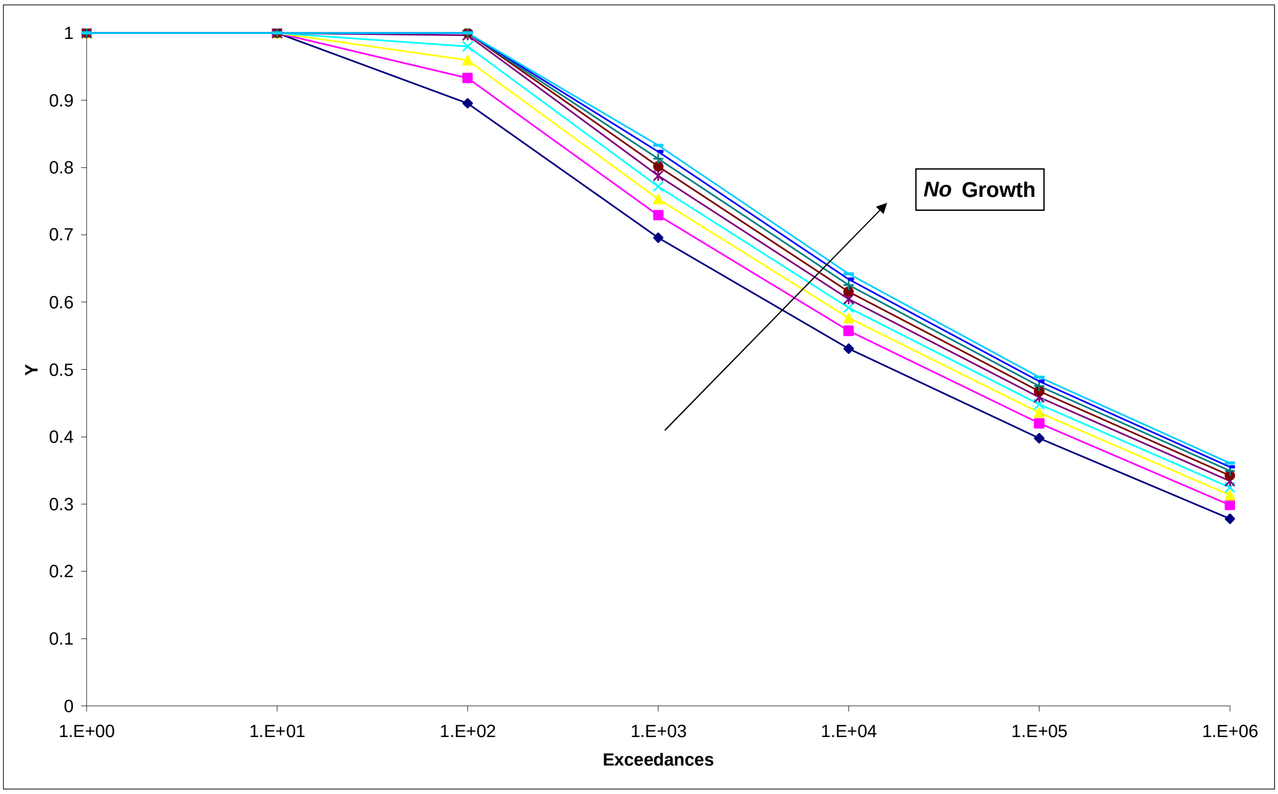


Figure 9. N_0 influence in load factor spectra.

5. Conclusion

A comparison of aircraft gust response results obtained by two different methodologies was presented. The first approach is based on the Peele method, which considers the aircraft as a rigid body and is suitable for preliminary design phases. The second approach consists of a detailed finite element-based dynamic response of the aircraft, including its flexibility.

Sensitivity investigations were performed in order to evaluate the influence of the most relevant input parameters on the gust responses obtained by the Peele method, providing qualitative guidelines for the required precision of the input data.

From the response results obtained by both approaches for several commercial aircraft, it can be noticed that Peele method can provide good results, with acceptable errors for the early stages of the design of an aircraft. The largest differences are found in the calculation of N_0. However, it was shown that the influence of N_0 on the fatigue load spectra is small when compared to the effects of variations on $\bar{A}$.

Further studies are recommended, in order to quantify the impact of the response results obtained with the simplified approach on the prediction of the fatigue life of the airplane, by analyzing the different flight phases in more detail.

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