

EXPERIMENTAL IDENTIFICATION OF IN-PLANE LOADS IN RECTANGULAR PLATES USING LIFECYCLE MODEL

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Abstract. *The influence of normal and shear membrane stresses on the flexural dynamic behavior of thin rectangular plates is well known, and referred to as stress-stiffening effect. The authors have demonstrated that, by exploring such effect, it is possible to obtain information about stress distributions or alternatively, external loads, through an inverse problem approach. In this paper, the underlying theory is first reviewed, including the development of a mathematical model for the flexural vibrations of plates subjected to in-plane loads, based on Kirchhoff's theory and assumed-modes method. In previous publications the authors verified that the solution of inverse problems by using the traditional gradient-based Sequential Quadratic Programming (SQP) may encounter some difficulties due to the existence of local minima in the design space and the necessity of initial configuration. These aspects have motivated the authors to explore an alternative inverse problem approach for the determination of external loading in structures, based on a heuristic named LifeCycle (LC) Model together with the SQP. Within the LifeCycle Model, Genetic Algorithms and Particle Swarm Optimization are used as search methods. The LifeCycle Model enables to combine the advantageous characteristics of both methods. SQP is used to refine the solution obtained by using LC. A cost function is defined to represent the differences between the experimental values of the natural frequencies of the loaded plate and their model-predicted counterparts. A set of design parameters is defined in relation with the stress state. The method is applied to a plate tested in laboratory. The results obtained are found to be in good agreement with experimental data, demonstrating the feasibility of identification methodology and the efficiency of optimization procedure.*

Keywords: *Stress-stiffening, Inverse Problems, Assumed Modes, LifeCycle Model*

1. Introduction

In structural systems for which operation, integrity and safety rely on the dynamic characteristics, the effect of external loading on the dynamic behavior of the system is to be carefully analyzed. Thus, it is very important to determine the external loading under real service conditions. However, the determination of external loading in most real-life structures is not feasible from the experimental point of view because, in general, transducers can not be easily introduced in the structure during its mounting.

By taking into account the influence exerted by the external loading on the dynamical response of the system through the so-called stress-stiffening effect, it is possible to obtain information about the loading distribution, given the dynamic responses, through an inverse problem approach. The *stress-stiffening* effect is characterized by the fact that when subjected to a given stress field, structural components such as strings, beam and plates exhibit variations in their bending stiffness and, as a result, modifications in their static and dynamic behavior. Such phenomenon is observed in the presence of stress fields generated either by external loading or by self-equilibrating residual stresses that can be produced by various thermo-mechanical manufacturing processes such as welding and cold-forming.

A number of studies have been reported in the literature focusing the stress-stiffening effect in connection with the static or dynamic structural behavior of structural components. Simons and Leissa (1971) used the Rayleigh-Ritz approach to investigate the influence of in-plane acceleration loads on the natural frequencies of rectangular cantilever plates. The same method, combined with the use of beam functions in the deflection series was used by Kaldas and Dickinson (1981). Lieven and Greening (2001) demonstrated experimentally the variability in the dynamic responses of nominally identical structures, due to residual stresses introduced through manufacturing processes. The same authors developed a strategy for including the stress-stiffening effect in a model updating procedure intended for the identification of the axial loads acting on the member of two-dimensional frames (Greening and Lieven, 2003). In a particular application to the determination of welding induced residual stresses, (Vieira Jr. and Rade, 2003) and (Vieira Jr., 2003) proposed a methodology for the identification of membrane stresses in rectangular thin plates from the transverse vibration responses. Rojas (2004) carried-out a comprehensive study about the stress-stiffening effect on the dynamic behavior of two-dimensional frames and rectangular plates, considering both direct and inverse problems. In a similar approach, Rojas *et al.* (2004) used heuristic optimization methods for the identification of external loads in frames.

More recently, Rade *et al.* (2005) analyzed the influence of normal and shear membrane stresses on the dynamic behavior of thin rectangular plates in direct and inverse problems using classical optimization. These studies demonstrated the strong influence the stress-state can have upon the dynamic characteristics of vibrating systems, leading to conclude that in many circumstances, the stress-stiffening effect must be taken in account in the modeling procedures so as to guarantee the accuracy of model predictions.

This paper is devoted to a feasibility study of a methodology for the experimental identification of external in-plane loads applied to rectangular plates, given a set of natural frequencies corresponding to bending modes of the loaded plate and a mathematical model relating such frequencies with the loading. An inverse problem is formulated in such a way that the unknown loading parameters are identified by solving a nonlinear constrained optimization problem. In an attempt to circumvent some difficulties in dealing with local minima, experienced in previous works when using gradient-based optimization algorithms, in this paper, optimization is performed by combining both classical and heuristic approaches. The authors explore an inverse problem procedure based on the LifeCycle Model (LC). The results obtained by LC are used in a cascade scheme with the Lagrange-Newton Sequential Quadratic Programming (SQP) method to refine the results in the neighborhood of the optimum. In the remainder, various aspects related to the identification method are discussed, including the underlying theory and the results obtained from laboratory tests.

2. Modeling of flexural vibrations of rectangular plates subjected to in-plane loads

Figure 1 illustrates a uniform rectangular plate of dimensions $a \times b \times h$, being also depicted an element which is acted upon by normal and shear stress components σ_x , σ_y and τ_{xy} .

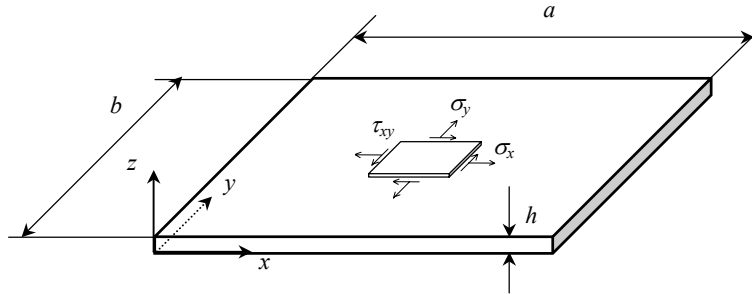


Figure 1. Stress components applied to an element of the plate

According to Kirchhoff's plate theory (Craig Jr., 1981), neglecting dissipation effects, the Assumed Modes Method (AMM) is used to derive a discrete model for the flexural vibrations of the plate considering the membrane stresses. With this aim, the kinetic and strain energies of the plate are first written as follows (Géradin and Rixen, 1997):

$$T = m \int_0^b \int_0^a \left(\frac{\partial w}{\partial t} \right)^2 dx dy \quad (1)$$

$$U = \frac{1}{2} \int_0^b \int_0^a \left\{ D \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2D(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] + h \left[\sigma_x \left(\frac{\partial w}{\partial x} \right)^2 + \sigma_y \left(\frac{\partial w}{\partial y} \right)^2 + 2\tau_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] \right\} dx dy \quad (2)$$

where $w = w(x,y,t)$ represents transverse displacement field; m is the mass density of the plate (mass per unit area); $D = Eh^3/[12(1-\nu^2)]$ is the plate flexural stiffness; ν and E are the Poisson's ratio and Young modulus, respectively.

The AMM expresses the plate transverse displacement field as a truncated linear combination of arbitrarily selected admissible functions. Following the approach adopted by Kaldas and Dickinson (1981), these functions are chosen to be the eigenfunctions of vibrating beams satisfying the geometrical boundary conditions of the plate in directions x and y . Thus, one writes:

$$w(x,y,t) = \sum_{i=1}^p \sum_{j=1}^q q_{ij}(t) \phi_i(x) \psi_j(y) \quad (3)$$

where p , q are the numbers of eigenfunctions considered in the series in x and y directions, respectively; $q_{ij}(t)$ are unknown generalized coordinates and $\phi_i(x)$ and $\psi_j(y)$ designate the beam eigenfunctions which are expressed as combinations of trigonometric and hyperbolic functions as follows (Young, 1950):

$$\phi_i(x) = A_i \sin\left(\eta_i \frac{x}{a}\right) + B_i \cos\left(\eta_i \frac{x}{a}\right) + C_i \sinh\left(\eta_i \frac{x}{a}\right) + D_i \cosh\left(\eta_i \frac{x}{a}\right) \quad (4)$$

In Eq. (4), A_b , B_b , C_b , D_b , η_i are constant coefficients that depend on the boundary conditions. In the same way, $\psi_j(y)$ with respect y direction. Young (1950) provides the numerical values of these coefficients for three combinations of boundary conditions: *clamped-clamped*, *clamped-free* and *free-free*.

Rewriting the expansion expressed by Eq. (3), introducing matrix notation and manipulating this equations, the following expressions for the kinetic and strain energies are obtained:

$$T(t) = \frac{I}{2} \{\dot{C}(t)\}^T [M] \{\dot{C}(t)\}; \quad U(t) = \frac{I}{2} \{C(t)\}^T [K] \{C(t)\} \quad (5)$$

where:

$$[M] = \rho h \int_0^b \int_0^a \{\eta(x, y)\} \{\eta(x, y)\}^T dx dy; \quad [K(\{\sigma\})] = [K_l] + [K_2(\{\sigma\})] \quad (6)$$

with:

$$[K_l] = \int_0^a \int_0^b D \left[\{\eta_{xx}\} \{\eta_{xx}\}^T + \{\eta_{yy}\} \{\eta_{yy}\}^T + 2\nu \{\eta_{xx}\} \{\eta_{yy}\}^T + 2(1-\nu) \{\eta_{xy}\} \{\eta_{xy}\}^T \right] dx dy \quad (7)$$

$$[K_2(\{\sigma\})] = \int_0^a \int_0^b \left[\sigma_x \{\eta_x\} \{\eta_x\}^T + \sigma_y \{\eta_y\} \{\eta_y\}^T + 2\tau_{xy} \{\eta_x\} \{\eta_y\}^T \right] dx dy \quad (8)$$

In the equations above: $\{\eta(x, y)\} = [\eta_1(x, y) \ \eta_2(x, y) \ \dots \ \eta_N(x, y)]^T$; $\{C(t)\} = [C_1(t) \ C_2(t) \ \dots \ C_N(t)]^T$; $\eta_k(x, y) = \phi_i(x) \psi_j(y)$; $C_k(t) = q_{ij}(t)$. The indices indicate partial derivatives of the products of beam functions with respect to the corresponding space variables and $\{\sigma\} = [\sigma_x \ \sigma_y \ \tau_{xy}]^T$ indicates the vector of the stress components.

It should be pointed out that the influence of the membrane stresses on the plate dynamics appear in matrix $[K_2(\{\sigma\})]$ which is referred to as *initial-stress stiffness* or *geometric stiffness matrix*. As it can be seen in Eq. (8), this matrix is a linear function of the stress components.

From Lagrange equations ($L = T - U$), it is possible to obtain the following eigenvalue problem:

$$([K(\{\sigma\})] - \lambda_r [M]) \{C_r\} = \{0\} \quad (9)$$

Once solved this problem, the eigenvalues λ_r provide the natural frequencies and the eigenvectors $\{C_r\}$, after back transformation into physical coordinates, provide the corresponding vibration mode shapes.

The harmonic responses in terms of physical coordinates observed in a set of points of the plate identified by their coordinates can be obtained by using the coordinate transformation (Rade *et al.*, 2005).

In the context of the identification procedure focused in this paper, it is convenient to express the stress field over the plate in terms of a small number of parameters. Thus, the parameterization of the stress field was discussed by the authors using polynomial approximations for the Airy's stress function and a priori knowledge about stress distributions by means of a base-line finite element model (Rade *et al.*, 2005).

Considering the dependency of the dynamic responses on the stress state (or, alternatively, on the external load), one is lead to consider the possibility of assessing the magnitudes and/or distributions of the stress components (or the external loading), given a set of measured natural frequencies of a loaded plate. The feasibility of such an inverse approach is examined in the next sections.

3. Optimization strategy

The linear search algorithm used in this paper is based on the Lagrange-Newton SQP and is devoted to the minimization of a function of several variables $f(x)$, subjected to linear/non-linear equality and inequality constraints and side constraints. To obtain the optimal solution it is required an initial estimation of the optimization parameters (Vanderplaats, 1999). Obviously, in the case of the present contribution, the results obtained from SQP depend on the initial estimation of the external forces and the number of variables to be identified.

LifeCycle Model is inserted in the natural optimization context, in which Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) also are. In this sense, LifeCycle is inspired in the concept of life cycle (Krink and Løvberg, 2002). In biology, the term refers to the passage through the phases during the life of an individual. As life phases, can be cited the sexual maturity and the mating seasons, for example. As it happens in nature, the ability of an individual to actively change its own phase or stage in response to its success in the environment is the main inspiration for LifeCycle. In fact, the idea behind LifeCycle is to use the transitions to deal with the mechanism of self-adaptation to

the optimization problem. The fitness offers a criterion used by each individual to shift to another life stage. To close the definition, LifeCycle stages must be presented. In the present work, two heuristics are used as stages, namely the GA and the PSO. Other versions of the LifeCycle can be proposed by considering other heuristics and a mix of them, as shown in Krink and Løvberg (2002).

Since the algorithm is composed by various heuristics, it is necessary to set the parameters of every heuristic used in the LifeCycle model. Nevertheless, there is a parameter inherent to the LifeCycle model, namely the number of iterations that represents a stage of the LifeCycle, called as stage interval. At the end of each stage interval, the less well-succeeded individuals must change their stage in order to improve their fitness. This means that the optimization approach does not follow a rigid scheme as proposed in (Assis and Steffen, 2003), in which various techniques are used sequentially in a cascade-type structure. In other words, it is the mechanism of self-adaptation of the optimization problem in action.

It is important to notice that the algorithm can run in a parallel scheme, since the original population is divided in a subpopulation of PSO particles and another one of GA individuals. During the optimization procedure the agents of each subpopulation commute to the other in such a way to improve its own fitness.

Details about GA are provided by Michalewicz (1994) and Haupt and Haupt (1998), while PSO is comprehensively presented by Kennedy and Eberhart (1995) and Venter and Sobieski (2002).

The outline of a basic LifeCycle algorithm is presented in Fig. 2 and Fig. 3 shows a scheme of the hybrid optimization strategy consisting in the combination of LC and SQP algorithms. In Fig. 2, *mod* is the mathematical function that expresses the modulus after division, for example $mod(13,5) = 3$ (*mod* is already implemented in MATLAB). For more detailed information about LifeCycle Model and SQP the reader should refer to Rojas *et al.* (2004).

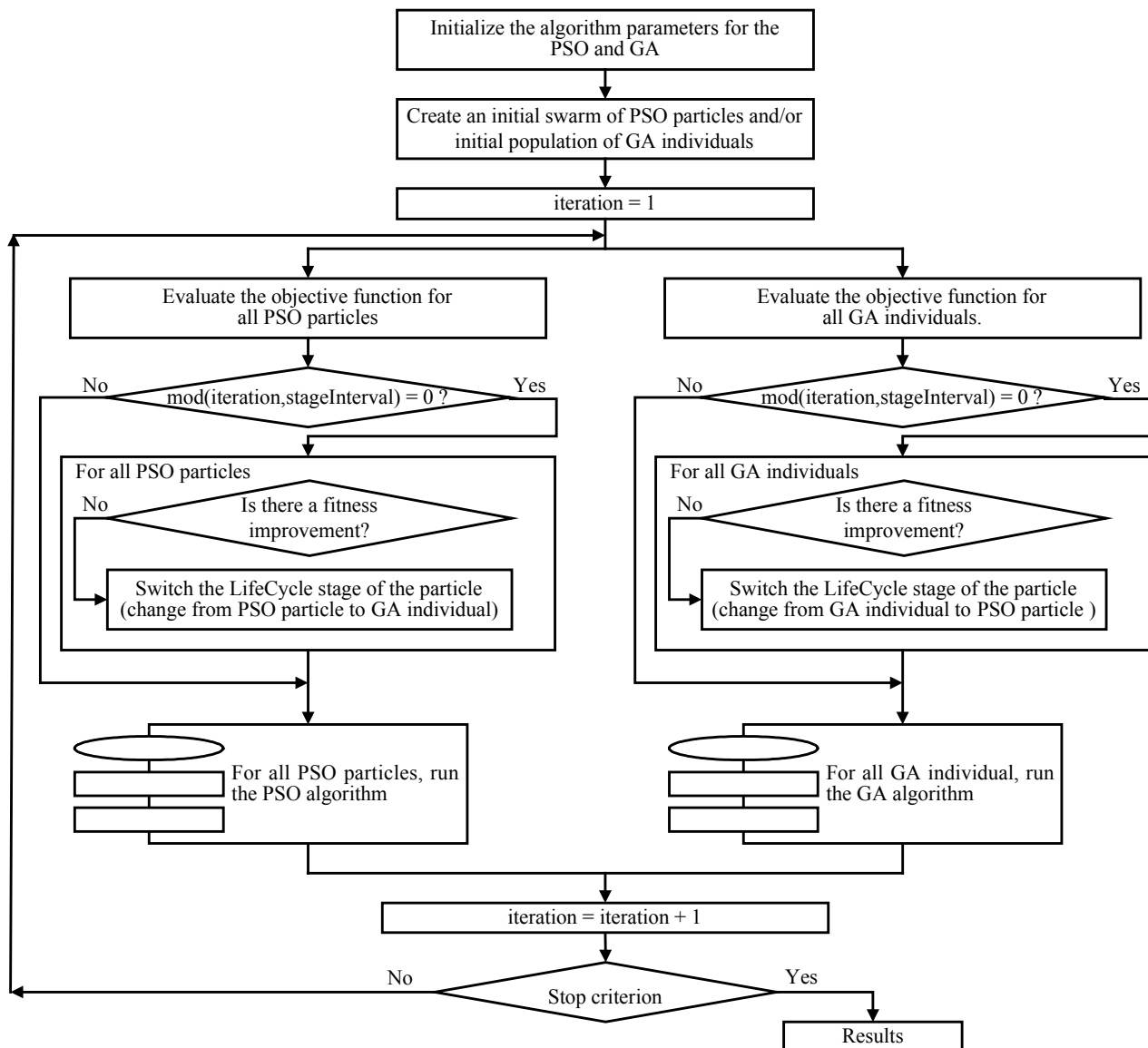


Figure 2. Outline of a basic LifeCycle algorithm.

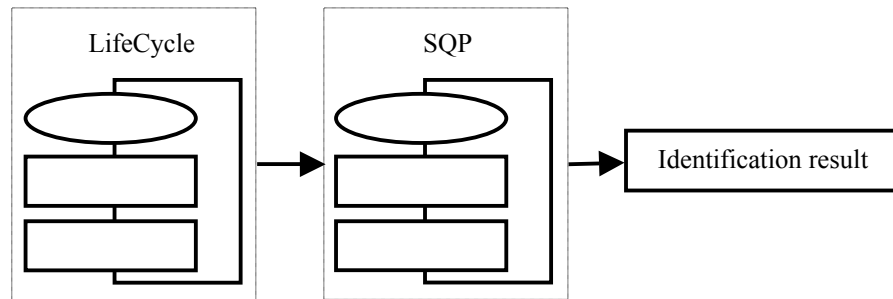
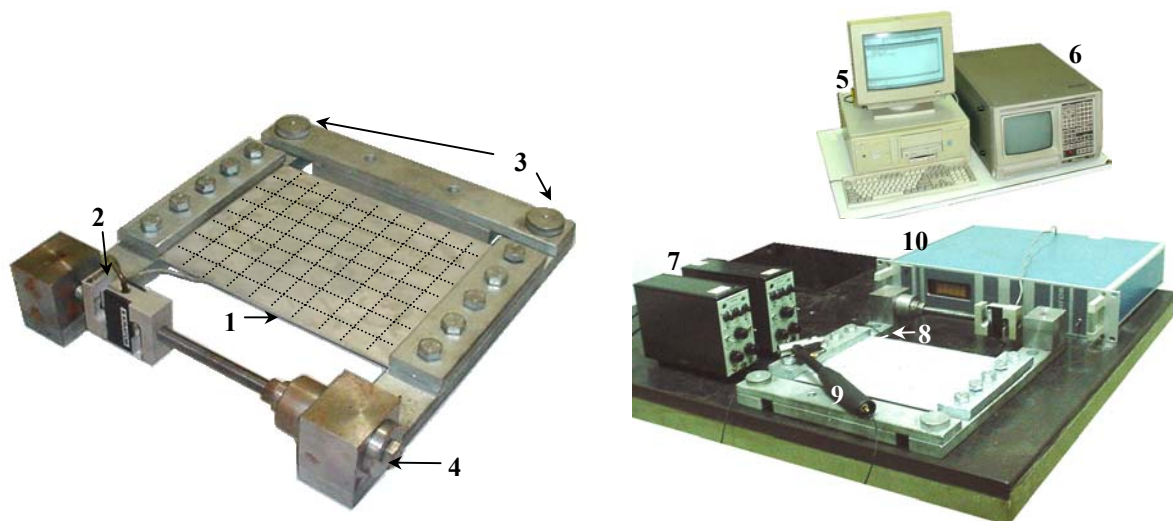


Figure 3. Scheme of the hybrid optimization strategy.

4. Application

The load identification procedure was evaluated by using experimental data obtained from laboratory tests performed on a rectangular plate made of aluminum, subjected to a specially designed fixture, which acts also as a loading device (Fig. 4). The plate was attached to the fixture in such a way to simulate clamped-clamped-free-free boundary conditions. A non-uniform in-plane traction (either tension or compression) could be applied to the plate by means of a bolt and the force could be measured through a load cell. The basic components of the experimental setup are illustrated in Fig. 4.

The vibration tests were performed for different values of the applied load, either in tension or compression. For each value of the load, a set of frequency responses were obtained by processing the Fourier-transformed input (impact forces) and output (transverse accelerations). The amplitudes of four of these, a driving-point FRF, corresponding to the different loads, are illustrated in Fig. 5, as compared to the amplitudes of the same FRF of the plate under the unloaded condition. In this figure, one can evaluate the influence of the in-plane loading on the natural frequencies of the plate. It can be noticed that, since the applied loads are tensile, the general trend observed is the increase of the values of the natural frequencies, meaning that the plate becomes stiffer in bending, as a result of the stress-stiffening effect.



1-Plate; 2-Load cell; 3-Pivot points; 4-Loading bolt; 5-Microcomputer; 6-Dual-channel frequency analyser; 7-Charge amplifiers; 8-Piezoelectric accelerometer; 9- Modal hammer instrumented with load cell; 10-Signal conditioner.

Figure 4. Scheme of the plate, fixture and loading device and experimental setup of the vibration tests

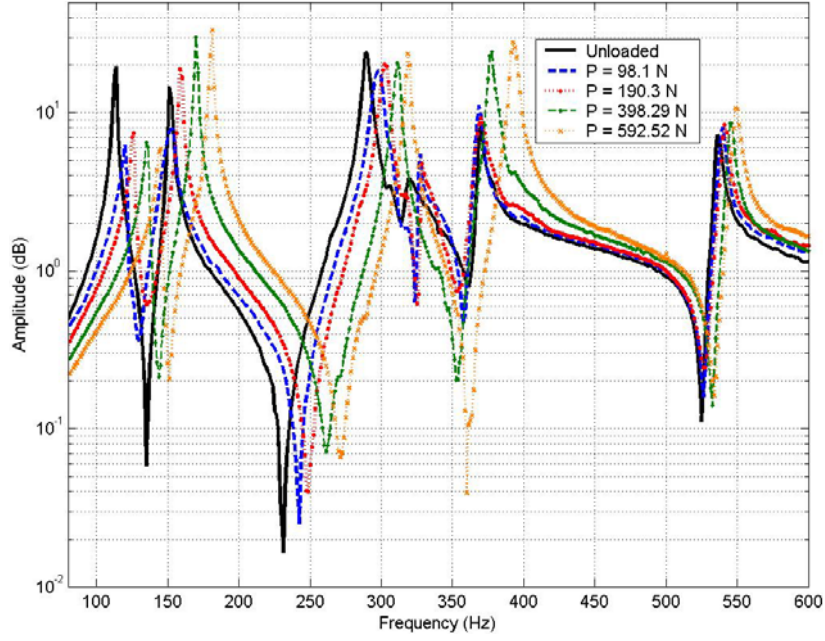


Figure 5. Amplitudes of a driving-point FRFs of the unloaded and loaded plate

5. Modeling of stress distribution

For the purpose of loading identification, it was adopted the following simplified model, also illustrated in Fig. 6(a), for the load distribution along the clamped borders of the plate, in which σ_1 and σ_2 are assumed to be unknown. According to this model, the stress distribution over the plate is the following:

$$\sigma_x(x, y) = \sigma_1 + \frac{\sigma_2 - \sigma_1}{b} y \quad \sigma_y(x, y) \equiv 0 \quad \tau_{xy}(x, y) \equiv 0 \quad (10)$$

By imposing the equilibrium of moments about the pivot joints of the fixture's arms, as illustrate in Fig. 6(b), the following relation between the applied force and parameters σ_1 and σ_2 is obtained:

$$P = \frac{\frac{h(\sigma_2 - \sigma_1)b}{2} \left(\frac{2b}{3} + d_1 \right) + h\sigma_1 b \left(\frac{b}{2} + d_1 \right)}{d_1 + d_2 + b} \quad (11)$$

It can thus be seen that two parameters (σ_1 , σ_2) are to be determined, enabling to fully characterize the stress-state of the plate, according to Eq. (10). Subsequently, the applied external load P can be estimated according to Eq. (11) and the estimated value can then be compared to the exact one provided by the load cell. Clearly, there exist multiples solutions (σ_1 , σ_2) leading to a same value of P .

An important question that arises concerns the accuracy of the model expressed by Eq. (11) in representing the actual stress-distribution of the plate. Rade *et al.* (2005) demonstrated that the simplified model is good enough by using a detailed finite element model of the system plate-fixture.

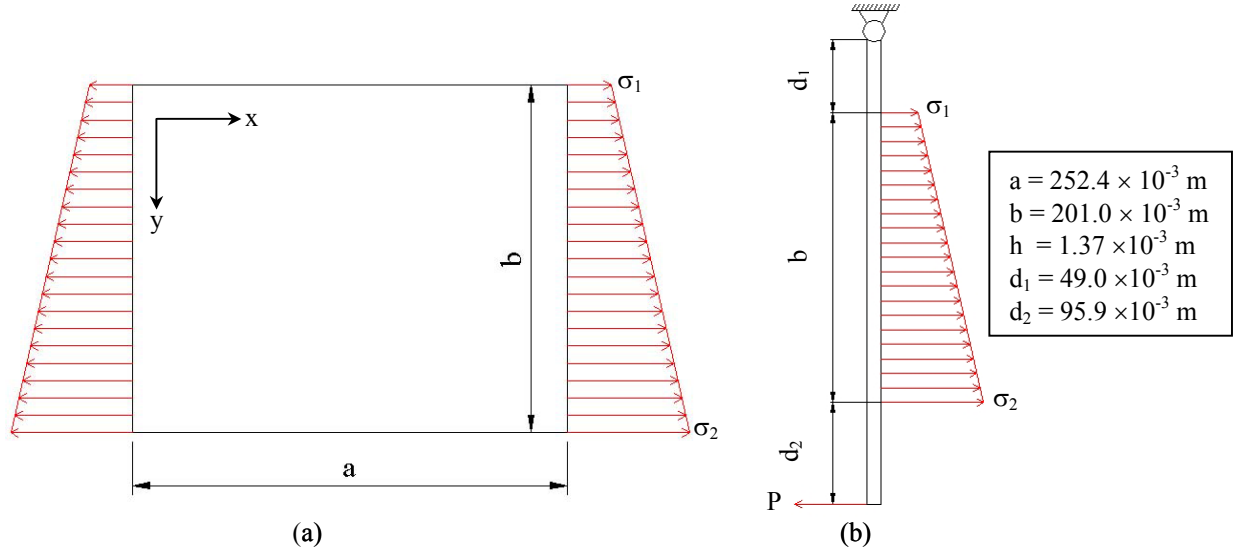


Figure 6. (a) model for the load distribution along the clamped edges; (b) free-body-diagram of the fixture arm

6. Formulation of the inverse problem

The identification procedure to determine the external loads consists in solving a constrained optimization problem in which the design variables are the parameters featuring in the model the stress distributions (either the coefficients of the defined in the previous section). In this paper, it was adopted a cost function representing the dimensionless difference between the values of the measured natural frequencies of the loaded plate and those predicted by the Assumed-Modes model, described in Section 2. Thus, the optimization problem is formulated as follows:

$$\min_{\{p\}} J = \frac{1}{\bar{\omega}} \sum_{p=1}^{nb_freq} W_p \left[\omega_p^{(m)} - \omega_p^{(c)}(\{p\}) \right] \quad (12)$$

$$\bar{\omega} = \frac{1}{nb_freq} \sum_{i=1}^{nb_freq} \omega_i^{(m)}$$

where $\{p\}$ designates, generically, the set of unknown stress or load parameters, nb_freq is the number of natural frequencies used for identification, W_p are user-defined weighting factors and $\omega_p^{(m)}$ and $\omega_p^{(c)}$ and designate the measured and calculated values of the natural frequencies, respectively.

The side constraints are introduced to limit the values of the design variables within a feasible design space, avoiding the possibility of buckling or structural collapse due to extreme external load levels.

7. Load identification

Initially, some tests were carried-out to verify the influence of the initial guess (starting point on the search space) on the identification results using SQP algorithm available in the MATLAB® *Optimization Toolbox*. The cost function was constructed according to Eq. (12) by considering the first six natural frequencies of the plate. Table 1 shows the results obtained for two values of the load: 190.31 N and 592.52 N. As it can be seen, for the first load, the identification results were satisfactory (error below 6 %) regardless of the initial guess. However, for the second load, the identification results change significantly according to the initial guess. Thus, it is confirmed that the SQP, like other unidirectional-search algorithms, requires a reasonable initial solution to provide satisfactory results.

Optimization computations were carried-out by using the LC method to obtain an initial guess which was then used in SQP, in an attempt to improve the results. Table 2 presents the identification results obtained for different values of the in-plane loads. Different weighting factors were adopted for each case, according to the variations observed in the values of the natural frequencies with respect to the unloaded counterparts, in such a way that “more sensitive” eigenfrequencies are more weighted. In the last two cases, the fourth natural frequency could not be properly identified in the experimental frequency responses functions. Therefore, it was not considered in the optimization procedure (this is indicated by zero weighting in Table 2).

Table 1. Results for the loaded plate identification tests (SQP).

Force (N)	Initial values		Identified values		Identified force	Force error (%)	Cost function
	σ_1 (N/m)	σ_2 (N/m)	σ_1 (N/m)	σ_2 (N/m)	P (N)		J
190.31	-700	4100	-698.13	4207.34	200.17	5.18	0.0821
	-600	2700	-541.28	4061.24	197.69	3.88	0.0132
	400	2000	419.78	3149.83	181.62	-4.57	0.0827
	500	3000	502.02	3071.66	180.24	-5.29	0.0827
	600	4400	568.41	3007.62	179.07	-5.91	0.0828
592.52	-3000	10000	3164.57	9252.88	598.64	1.03	0.5664
	-1000	7500	5354.64	6168.18	508.45	-14.19	0.5564
	500	2000	1527.66	12969.05	741.05	25.07	0.5820
	2000	6000	3262.24	9140.68	595.97	0.58	0.5638
	1500	8000	4269.17	7522.28	543.86	-8.21	0.5586

An interesting fact is that the identified value of the parameter σ_1 is negative in the first case, increasing gradually to a positive value. In the cases presented, the results obtained by using LC were used as an initial solution for the SQP optimization run. However, by comparing the results presented in Tables 1 and 2 for the values of the load 190.3 N and 592.52 N, it can be concluded that identification using LC was in general more accurate than that using SQP. Moreover, the use of SQP did not lead to significant improvement of the results obtained with LC in most cases.

Table 2. Identification results by using LC and SQP (hybrid method).

Case	Method	σ_1 (N/m)	σ_2 (N/m)	Load (N)		Error (%)	Cost Function	Weighting factors
				Identified	Experimental			
1	LC	-974.27	2535.87	101.99	98.10	3.97	0.0563	5 2 2 1 1 1
	SQP	-983.17	2537.48	101.78		3.75	0.0598	5 2 2 1 1 1
2	LC	-968.38	4370.94	199.76	190.30	4.96	0.0822	1 1 1 1 1 1
	SQP	-967.30	4456.82	204.36		7.39	0.0829	1 1 1 1 1 1
3	LC	185.17	8748.99	471.42	398.29	18.36	0.2060	4 3 2 0 1 1
	SQP	185.17	8747.87	471.36		18.35	0.2060	4 3 2 0 1 1
4	LC	2923.27	10009.27	630.72	592.52	6.45	0.5755	6 6 2 0 4 1
	SQP	3692.40	8400.66	571.12		-3.61	0.5611	6 6 2 0 4 1

Table 3 shows the values of the experimental natural frequencies used in the optimization procedure and the “identified” natural frequencies corresponding to the identified load parameters shown in Tab. 2. It is observed a good updating of the natural frequencies performed by the optimization algorithms

Table 3. Natural frequencies of the identified loads.

Case		Natural frequencies (Hz)					
		1	2	3	4	5	6
1	Experimental	120.00	152.50	297.50	327.50	368.75	540.00
	LC	120.04	152.23	293.63	326.15	373.76	536.75
	SQP	120.01	152.22	293.62	326.12	373.74	536.73
2	Experimental	126.25	158.75	302.50	328.75	370.00	541.25
	LC	123.83	156.79	295.58	331.38	380.91	541.05
	SQP	124.17	157.23	295.77	338.35	381.60	541.46
3	Experimental	135.00	170.00	311.25	-	377.50	546.25
	LC	135.00	169.32	301.36	347.34	401.04	553.74
	SQP	135.00	169.32	301.36	347.34	401.04	553.74
4	Experimental	145.00	181.25	318.75	-	392.50	548.75
	LC	145.34	175.74	305.33	362.95	411.66	562.29
	SQP	145.00	173.12	304.42	362.51	407.35	560.24

8. Conclusions

In this work, a methodology for in-plane loads identification in rectangular plates was presented. A hybrid optimization procedure was used to carry out this task, consisting in using a pseudo-heuristic model (LifeCycle) combined with a gradient-based method (Sequential Quadratic Programming).

Among the inherent difficulties of inverse problems, the choice of the initial guess to the solution is one of the most relevant for the success of the optimization procedure. Therefore, the LC model was used to provide an adequate initial initial solution.

The results obtained confirm the possibility of obtaining the external load or the stress-state within the structure by means of an inverse procedure. In some of the cases analyzed, the second optimization run using SPQ was important to improve the results obtained, since heuristic technique could be able to provide an adequate initial solution.

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11. Responsibility notice

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