ANALYSIS OF RADIATIVE PROPERTIES IDENTIFICATION WITH REFRACTIVE INDEX

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Abstract. *An analysis for the radiative properties identification at the anisotropic media with refractive index ratio has been performed. Discrete-ordinate method is used to solve the radiative transfer equation. The methodology for the radiative properties identification is analyzed function on the physical model, the experimental collection of data, and the detector noise.*

Keywords*: Radiation Heat Transfer, Ordinate Discrete Method, Refractive Index*

1. Introduction

In recent years, many works to semitransparent media (STM) had been carried considering the medium without interface, that is, the distance among the particles are important so that the porosity are extremely high. Consequently, the effects of reflection and refraction in interfaces can not be considered. However, many materials, such as ceramic, thin films, coatings and metals with low porosity, are far to be considered with refractive index≈1. Besides the direction change with the refractive index, in some problems are necessary to take in account the increase of local blackbody emission by a factor of refractive index square.

Recently, some works can be found referring the study of radiative transfer in materials with refractive index≠1. Special conditions in interfaces must be taken in account due to refraction (Wu *et al.*, 1994, Liou and Wu, 1996 and Hottel *et al.*, 1968). Moreover, it is necessary special quadrature for angular discretization to Discrete Ordinate Method (DOM). Numerical simulations allow us to understand the radiative heat transfer process and the influence of the parameters on this process (Moura, 2003). Wu *et al.* (1994) had analyzed an isotropic medium for a problem with azimuthal symmetry, considering a variable refractive index. They had analyzed the change direction of the beam as well as the critical angle. Liou *et al.* (1996), using this same formulation, had analyzed the influence of the refractive index in a multilayer problem. More recently, Abdallah and Le Dez (2000) had analyzed, using the method of "ray-tracing" and formal equations of Kernel (functions error), the emitted intensity of radiation of a non-diffusing semitransparent plate with a variable refractive index. Lemonnier and Le Dez (2002) had also analyzed the same problem using DOM with good agreement. The solution is approached because the radiative intensity directions vary strongly with the internal variation of the refractive index. Lacroix *et al.* (2002) had analyzed a similar problem to the previous ones, but in this work, they consider the conduction equation coupling.

Baillis and Sacadura (2000) had presented a review on radiative properties determination on STM. Two different techniques can be used to evaluate the radiative transfer on STM. The first one considers that the radiative transfer as a term that must be added in heat conduction equation (Tong and Tien, 1980). Although this method is simpler, it always demands the experimental parameters determination and these parameters are restricted to the range of experimental analyses. The second technique consists of using the RTE coupled to heat conduction equation and/or Navier Stokes equation. The solution of the RTE requires the knowledge of the radiative properties of the medium. These properties can be determined using two different techniques: i) using Maxwell equations (electromagnetic field), being necessary to know the morphologic parameters and the spectral optical properties of the medium; ii) measuring the radiative intensities field emitted, transmitted and/or reflected by a sample with an experimental device and, identifying the radiative properties by inversion techniques.

In this work it is analyzed the radiative properties identification of scatter STM consisted by a solid matrix with refractive index. The direct model is based on the method of the Discrete Ordinate Method (DOM) assuming the condition of symmetry on radiative intensity field. This model is used on parameter identification routine. Results are carried to experimental simulations obtained from the direct model and increases of random errors.

2. Radiative transfer equation - RTE

The RTE, which describes the variation of the spectral radiation intensity, I, (in a solid angle Ω, function of optical depth τ) in an absorbing-emitting-scattering medium, can be written as:

$$\mu\frac{\partial I(\tau,\mu)}{\partial\tau}+I(\tau,\mu)=(1-\omega)I_o(T)+\frac{\omega}{2}\int_{-1}^{1}I(\tau,\mu')p(\mu',\mu)d\mu' \qquad (1)$$

where ω is the albedo, p is the phase function, and I_o is Planck's blackbody function (in order to simplify the notations, the spectral subscript λ is omitted in the text). These properties are those of a pseudo-continuum medium equivalent, in terms of radiative transport, to the real dispersed material. The boundary conditions assumed a normally incident collimated beam onto the sample with bidirectional transmittance and reflectance measurements. The boundary conditions can be expressed like:

$$\begin{cases} I(\tau = 0, \mu) = I_C\,; & \mu_o < \mu < 1 \\ I(\tau = 0, \mu) = 0\,; & 0 < \mu < \mu_o \\ I(\tau = \tau_o, \mu) = 0\,; & -1 < \mu < 0 \end{cases} \tag{2}$$

where I_C is the radiative intensity of the incident beam with a divergence angle, θ_o ($\mu_o = cos\ \theta_o$).

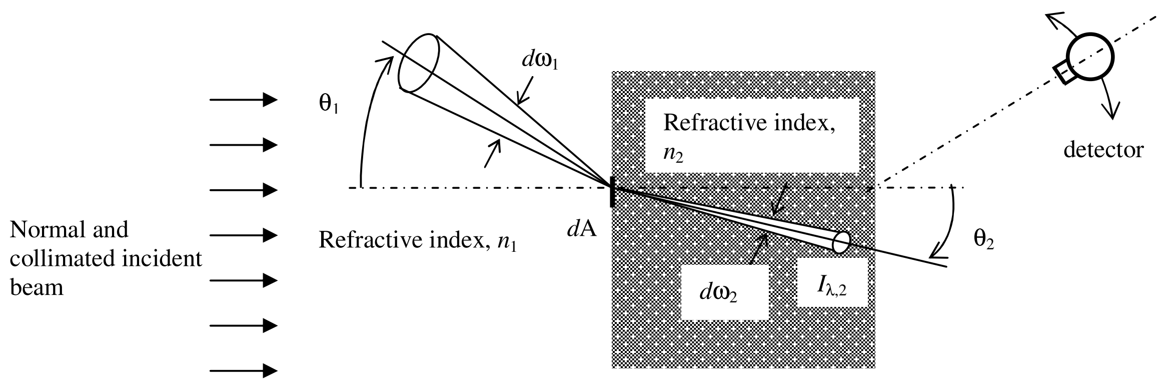


Figure 1. Boundaries conditions to the simulate experimental device

More information about the direct problem to a medium with a refractive index can be found on Moura (2003). Even when there is a normal incidence onto the sample the solid angle of the incident beam changes. This must be considered to the quadrature setup and the use of classic quadratures, like Gauss and Radau, are not possible. Moura (1999) shows that the small variations in solid angle of the incident beam can result larger errors on phase function identification. In this way, it is recommended either to use adaptive quadratures function of the solid angle in the incident beam, or, function of the refractive index medium and the directions $\mu=\pm 1$ are calculated with respective weights.

3. Radiative properties identification

The radiative properties identification, using RTE inversion, generally consider a sample with well-know boundaries conditions, measuring transmittances and reflectances around the sample on one-dimensional condition. However, the identification of some parameters together is not always possible. The success of identification method depends on the chosen of the physical model, the available experimental data (transmittance and/or reflectance bidirectional or hemispherical and the number of measurement points and angular direction choice) and detector noise level. In this work, it is used the Gauss linearization method to identify the radiative properties which was initially applied by Nicolau *et al*. (1994) for the radiative properties identification of glass wool. Nicolau *et al*. (1994) had identified one strong correlation of the optical thickness with optical thickness; they preferred to identify the optical thickness by direct method: a model type Beer's law. In this method, a second order polynomial is used to determine the diffuse transmittance in the directions closed on the incident beam and they extrapolate these values to the direction of incidence. However, Moura *et al*. (1999) had demonstrated that the accuracy of the estimation depends on the divergence angle of the incident beam and the anisotropy of the phase function. When the anisotropy factor increase, g_1=0.95, the optical thickness identification can present uncertainties superiors at 20% (especially when the divergence angle of the incidental beam is closed to unity). For this reason, the optical thickness should be identified by inverse method. But, the fact of increase one parameter in identification procedure, it will increase the ill-conditioned of the system. To assure the convergence, an underrelaxation coefficient is used. This parameter should increase the number of iterations. The radiative properties are calculated from measurements of transmittances ($0<\mu<1$) e reflectances ($-1<\mu<0$) calculate by:

$$T(\mu) = \frac{i(\mu)}{i_o d\omega_o} \tag{3}$$

where $d\omega_o$ is the solid angle in respect to the divergence angle, θ_o.

The radiative properties vector, $\hat{\chi}_{k=1,\ldots,6} = n, \omega, g, f_1, f_2, \tau_o$, is identified minimizing the quadratic difference between theoretical transmittances, T_t,, and experimental transmittances, T_e, , for Nd measurements:

$$\mathbf{F}\left(\hat{\chi}_{k=1,\ldots,K}\right) = \sum_{n=1}^{Nd}\left[T_{t,n} - T_{e,n}\right]^2 \tag{4}$$

The method adopted to achieve this minimization is the Gauss linearization method that minimizes $\mathrm{F}(\hat{\chi}_k)$ by setting to zero the derivatives with respect to each of unknown parameters, $\hat{\chi}_k$. As the system is non-linear, an iterative process is performed over m iterations (Nicolau, 1994):

$$\begin{bmatrix} \sum_{n=1}^{Nd}\left(\frac{\partial T_{tn}}{\partial \chi_1}\right)^2 & \sum_{n=1}^{Nd}\frac{\partial T_{tn}}{\partial \chi_1}\frac{\partial T_{tn}}{\partial \chi_2} \cdots & \sum_{n=1}^{Nd}\frac{\partial T_{tn}}{\partial \chi_1}\frac{\partial T_{tn}}{\partial \chi_K} \\ \sum_{n=1}^{Nd}\frac{\partial T_{tn}}{\partial \chi_1}\frac{\partial T_{tn}}{\partial \chi_2} & \sum_{n=1}^{N}\left(\frac{\partial T_{tn}}{\partial \chi_2}\right)^2 \cdots & \sum_{n=1}^{Nd}\frac{\partial T_{tn}}{\partial \chi_2}\frac{\partial T_{tn}}{\partial \chi_K} \\ -- & -- & -- \\ \sum_{n=1}^{Nd}\frac{\partial T_{tn}}{\partial \chi_K}\frac{\partial T_{tn}}{\partial \chi_1} & \sum_{n=1}^{Nd}\frac{\partial T_{tn}}{\partial \chi_K}\frac{\partial T_{tn}}{\partial \chi_2} \cdots & \sum_{n=1}^{Nd}\left(\frac{\partial T_{tn}}{\partial \chi_K}\right)^2 \end{bmatrix} \cdot \begin{bmatrix} \Delta\chi_1 \\ \Delta\chi_2 \\ -- \\ \Delta\chi_K \end{bmatrix} = - \begin{bmatrix} \sum_{n=1}^{Nd}\left(T_{tn} - T_{en}\right)\frac{\partial T_{tn}}{\partial \chi_1} \\ \sum_{n=1}^{Nd}\left(T_{tn} - T_{en}\right)\frac{\partial T_{tn}}{\partial \chi_2} \\ -- \\ \sum_{n=1}^{Nd}\left(T_{tn} - T_{en}\right)\frac{\partial T_{tn}}{\partial \chi_K} \end{bmatrix} \tag{5}$$

The solution of this system gives the increments $\Delta\chi_{k=1,\ldots,K}$ to be added to each parameter, χ_k, at each step of iterative process, as:

$$\chi_{k+1} = \chi_k + \lambda_c \Delta\chi_k \tag{6}$$

where λ_k is an underrelaxation coefficient for the parameter k to assure the convergence. The convergence is obtained than $\Delta\chi_k^m / \chi_k^m$ is less than a convergence tolerance.

The matrix on the left hand side is composed of the sensitivity coefficient products, calculated from the theoretical model; it does not directly depend on the experimental values. This matrix, **S**, can be used in the sensitivity analysis to verify possible linear dependences between the sensitivity coefficients calculated for each parameter. The calculation of a condition number CN of this matrix can be used to determine the degree of ill-posedness of the identification problem.

$$CN(\mathbf{S}) = \left\|\mathbf{S}^{-1}\right\| \cdot \left\|\mathbf{S}\right\| \tag{7}$$

where the norm, $\|\mathbf{S}\|$, is calculated from the elements $\mathrm{S}_{k',k}$. The condition number is greater than one. The larger the condition number is, the worse ill-conditioned the system is; small changes in the right hand side of equation (5), i.e. in the measurements, result in very large change in the solution vector, i.e. the increments, $\Delta\hat{\chi}_k$. It is then almost impossible to simultaneously determine all of the unknown parameters. Poor conditioning occurs when at least two of the sensitivity coefficients are quasi-linearly dependent or when at least one is very small or very large compared to the others.

Figure 2 shows the identification procedure used. A second order is used to obtain the first approximation of the optical thickness. After that, the iterative process is performed until convergence and the radiative parameters (n, ω, f_1, f_2, g, τ_o) are obtained. The underrelaxation is adaptable, that is, more the corrective values, $\Delta\chi_{k=1,\ldots,K}$, decrease, more the underrelaxation factor, λ_c, close to one.

Figures 3 to 5 show the condition number (CN) for two different refractive indexes function the optical thickness of the sample. Figure 3 shows the CN presented by Moura *et al.,* 1999, to unite refractive index medium. The parameter vector is $\hat{\chi}_{k=1,\ldots,6} = \tau_o, \omega, g_1, f_1, g_2, f_2$ to case (a) and $\hat{\chi}_{k=1,\ldots,6} = n, \omega, g, f_1, f_2, \tau_o$ to case (b). It can be shown that the optimal optical thickness is close to 6. The increase of NC in case (b) is more important that show a difficulty to identify the radiative properties. However, the increase of refractive index decreases the CN, making possible the radiative properties identification, Fig. 4. A CN very high in n=1, probably is due to the poor sensibility function because does not change in directions of radiative intensities.

Figure 5 shows the CN function of refractive index. Refractive index less than 1.0 had not been analyzed. It can be shown the difficulty on the procedure to identify materials with a refractive index close to unity. It is also shown that to refractive index to values close to 1.0 the CN decrease drastically, above all when the identification of the six parameters is considered, then after that it increase to values around 10^5, to refractive index greater than 1.2.

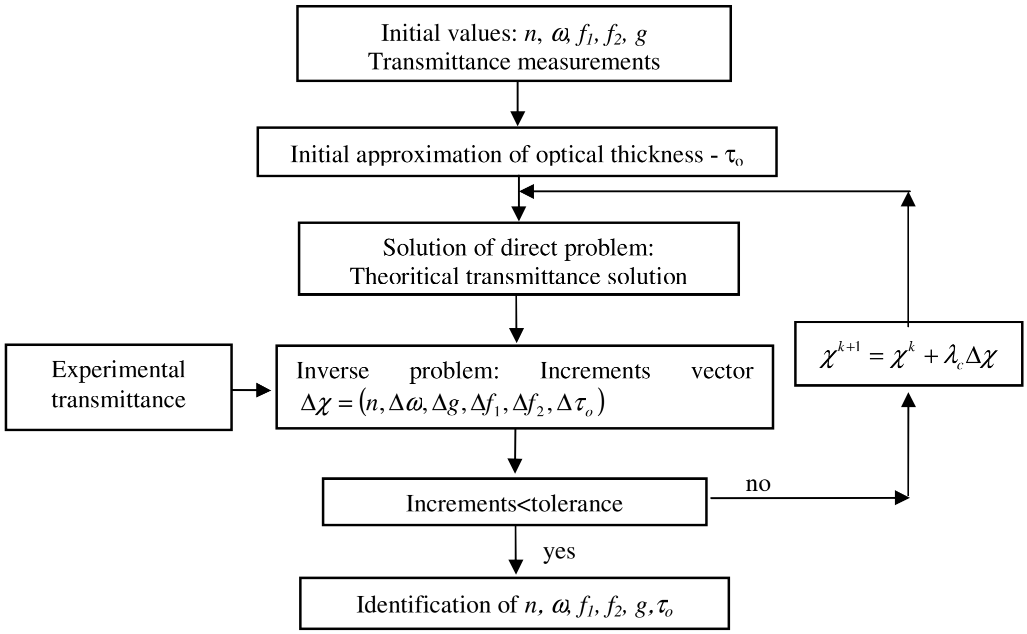
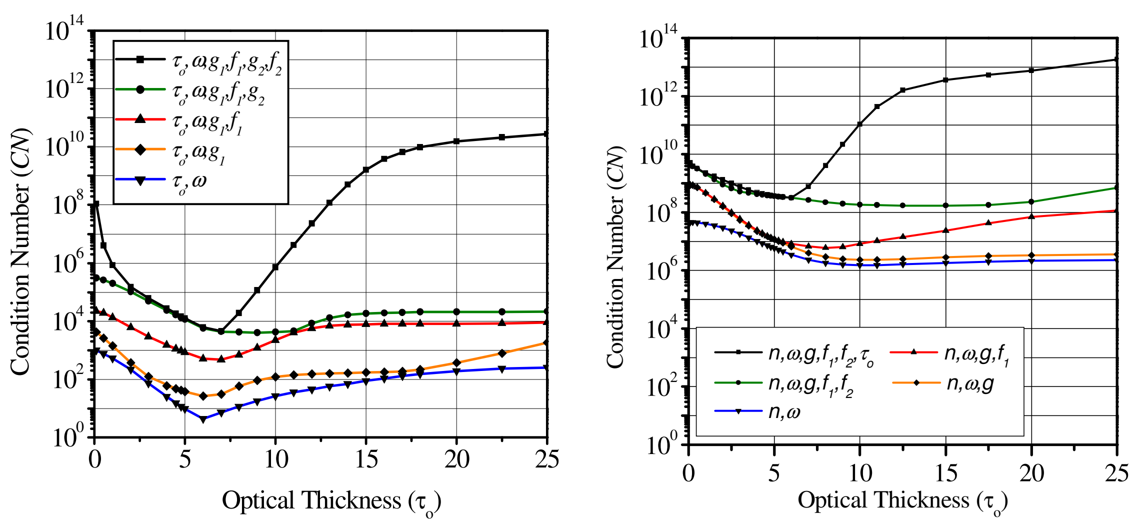


Figure 2. Parameters identification procedure: n, ω, f_1, f_2, g, τ_o

In order to evaluate the accuracy of the identification model, two case tests is carried for normal incidence. Bidirectional transmittances and reflectances are obtained starting from the direct code with known properties and increases of random errors (normal distribution and standard deviation, σ = 0.015% X 2.576; 99% confidence) with the same magnitude of the experimental noise, to a divergence beam of 2.5°. These values are used in the identification code to identify the properties and compare with the initial properties. The optical thickness is calculated by inverse method and second order model (Nicolau *et al.,* 1994), in order to analysis the identification error to each procedure. The error between correct value and the estimate value is defined as:

$$Error[\%] = \frac{\chi_k(correct) - \chi_k(estimate)}{\chi_k(correct)}.100 \tag{9}$$



(a) (b)

Figure 3. Condition number (CN) to n=1.0, ω=0.95, g=0.84, f_1=0.9 and f_2=0.95

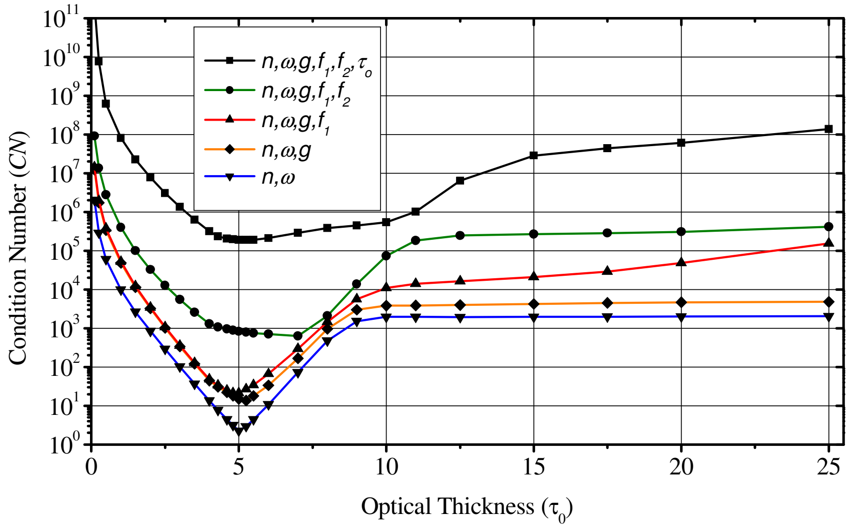


Figure 4. Condition number (*CN*) to different quantities of identified parameters (n=2.0, ω=0.95, g=0.84, f_1=0.9 and f_2=0.95)

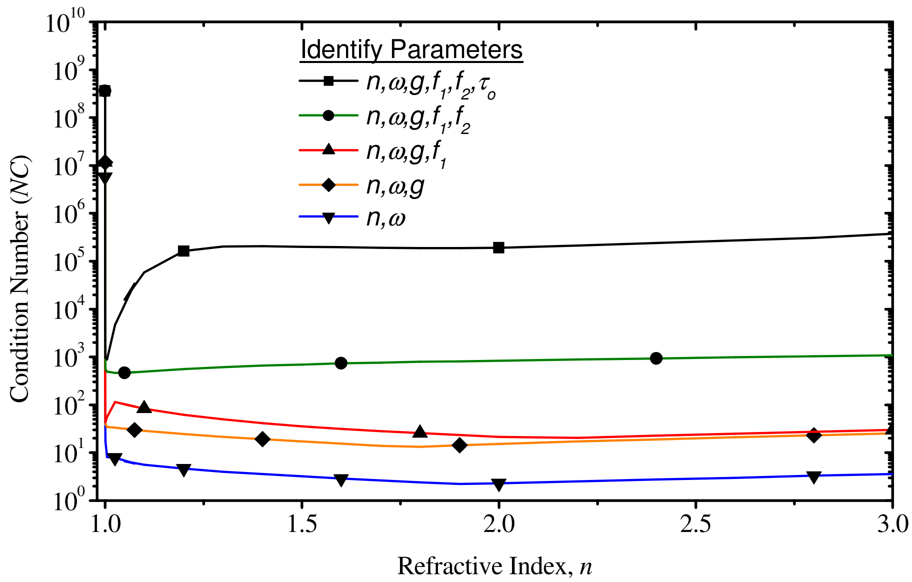


Figure 5. Condition Number (*CN*) function of refractive index to (τ_o=5.0, ω=0.95, g=0.84, f_1=0.9 and f_2=0.95)

4. Analysis

The results of identification analysis are presented for a case test. The **direct model** is solved starting from reference values (correct) of radiative properties. Bidirectional transmittances obtained from these values are increases of random errors and used in **inverse model**. To solve the inverse model it is necessary initial values of radiative properties. In principle these initial values can not change the final values of the parameter identifies. Using the identification procedure, 11 "numerical measurements" are added of random errors. The radiative properties are identified and they are compared with the initial values, determining the dispersion (random error) around of the average and the deviation (bias error).

Table 1 and Figures 6 and 7 present results to optical thickness, τ_o=5, and refractive index n=1.5, good concordance is observed between initial values and estimated values. Figures presents the radiative properties calculate to each iteration, starting from the initial values. A convergence is established to 50 iterations, but the method does not arrive to reduce the error to the convergence criteria adopted, that is 10^{-4}.

The parameter vector, $\hat{\chi}_{k=1,\ldots,6} = n, \omega, g_1, f_1, f_2, \tau_o$, to 200 iterations, optical thickness, τ_o=5, 140 control volumes and 24 directions is identified around 30 minutes using a Pentium 3-866 MHz. If the optical thickness is increase by a factor two the iteration time is also twice.

Radiative properties identify to optical thickness of τ_o=10 and refractive index of 1.5 are shown in Tab. 2 and Fig. 8. In this case, the errors are bigger; probably function of a more important optical thickness that results in smaller transmittance values ($0<\mu<1$), the noise having a more importance.

If the optical thickness is bigger than 10 the error increases function of low level signal in the transmittance measurements. In this case the radiative properties have a bias in order of 20%. The optimum range to the optical thickness is between 5 and 10.

Table 1. Difference between the identification procedure to τ_o=5 and n=1,5.

	Correct	**Initial Value**	**Identify Values**	**Random error (95%)**	**Bias [%]**
n	1,5	1,3	1,520	0,16	1,3
ω	0,95	0,85	0,950	0,009	–0,03
g	0,84	0,8	0,842	0,019	0,20
f_1	0,9	0, 8	0,902	0,016	0,19
f_2	0,95	0, 8	0,943	0,053	–0,67
τ_o	5,0	2° order	5,023	0,028	0,46

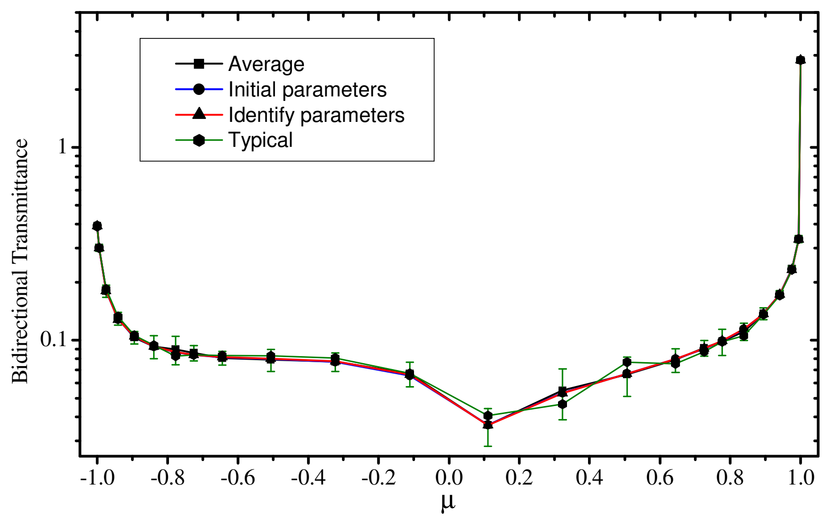


Figure 6. Bidirectional transmittances to n=1.5 and τ_o=5

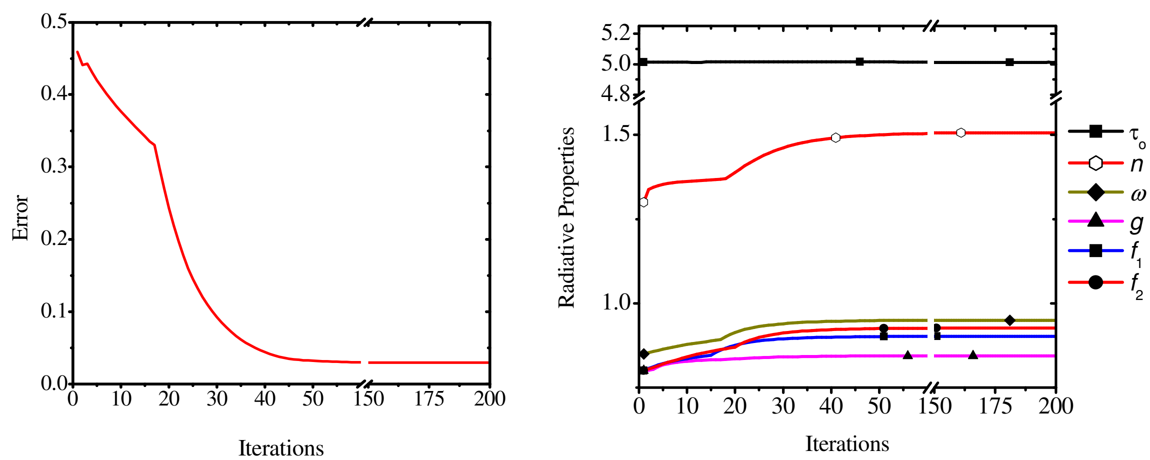


Figure 7. Radiative properties iterations to n=1.5 and τ_o=5

Table 2. Difference between the parameter identification methods to τ_o=10 and *n*=1, 5.

	Correct	Initial Value	Identify values	Random error (95%)	Bias [%]
n	1,5	1,3	1,465	0,339	–2,34
ω	0,95	0,85	0,951	0,009	0,10
g	0,84	0,8	0,833	0,042	–0,86
f_1	0,9	0, 8	0,892	0,042	–0,84
f_2	0,95	0, 8	0,954	0,063	0,43
τ_o	10,0	2° order	9,39	0,768	–0,64

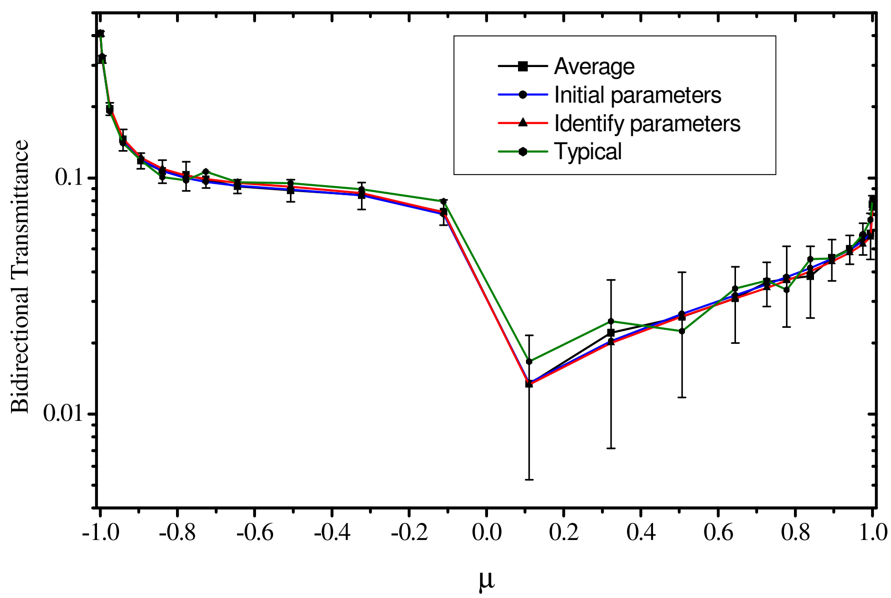


Figure 8. Bidirectional transmittances to τ_o=10 and *n*=1.5

5. Conclusion

The radiative properties identification is analyzed to a translucent and scattering medium, that is, the refractive index must be taken in account. The albedo, optical thickness and 3 parameters of a phase function are also considered. That is, analysis of the Condition Number (*CN*) and the "numerical measurements" allows verifying the uncertainties and the optimum optical thickness and refractive index ranges. It can be shown that to a refractive index, n=1, the *CN* is too high, but a little increase in refractive index reduce considerably the *CN*. Optimum optical thickness is between 5 and 10.

In sequence, this method will be improved to non-azimuthal symmetry of the radiation field, for obliquely incident collimated beam onto the sample. This change will increase the difficulty in the RTE solution. But some physical information must be improved to increase the success in identification analysis.

6. Acknowledgements

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