

## WAVELET PACKET FEATURE EXTRACTION FOR VIBRATION MONITORING IN HIGH SPEED MILLING

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**Abstract.** *Industrial machinery equipment requires a certain degree of monitoring techniques to assure successful operation over a long period of time. To achieve this objective, an automated condition monitoring system is needed. If the condition monitoring system is approached from a pattern classification perspective, it can be decomposed into three general tasks: data acquisition, feature extraction, and condition classification.*

*The Fourier-basis analysis provides a poor representation of signals localized in time; while wavelet bases are not well adapted to represent signals whose Fourier transforms have narrow high frequency support because of poor resolution at high frequency. In both cases, it is difficult to detect and identify the signal pattern from the expansion coefficients because information is diluted across the whole basis. The wavelet packet transform (WPT), on the other hand, uses a rich library of redundant bases with arbitrary time-frequency resolution. This work presents the application of the Wavelet Packet Transform as an alternative means of extracting time-frequency information from vibration signature in some high speed milling operations. Assuming some limited operation states, i.e. considering certain variations of the tool angular speed and some combinations between work-piece and tool, is verified that the resulting WPT coefficients provides a suitable feature selection criteria for a classification procedure. Additionally, is presented a methodology to extend the analysis to more complex operations in high-speed milling aiming a combination of parameters that minimizes chatter problems and maximize the operation speed.*

**Keywords:** *high speed milling, chatter, digital signal processing, pattern recognition and Wavelet Packet*

### 1. Introduction

The general principle behind using vibration signals for monitoring involves those components in mechanical systems that vibrate during operation. When some operation condition change, some of the system dynamics vary, resulting in significant deviations in the vibration patterns. By employing appropriate data analysis algorithms, it is feasible to detect changes in vibration signatures caused by changes in parameter values or operation states in order to make decisions about the status of the machinery (classification procedure).

Usually, the vibration signals considering different operation conditions are highly structured and can be grouped into two categories: sustained and intermittent (Paul, 1995). Condition monitoring of milling system based on vibration signatures has generally relied upon Fourier-based analysis as a means of translating vibration signals from time domain to frequency domain. However, Fourier transform provides a poor representation of signals well localized in time, making difficult to detect and identify the signal pattern from the expansion coefficients because the information can be diluted across the global basis. If the vibrations signatures are stationary and periodic, Fourier-based, which uses sinusoidal functions as basis functions, provides an ideal tool for extraction of these narrow-band signals. Instead, the vibration signals of end-milling process present a nonstationary and transient nature, and carry small informative components embedded in large repetitive signals. In this case, the Short Time Fourier Transform (STFT) can be employed to detect the localized transient. Unfortunately, the fixed window used in the STFT implies fixed time-frequency resolution in the time-frequency plane (Torrence and Compo, 1998). The difficulty remains in the accuracy of extracting frequency information that is limited by the length of the window relative to the duration of the sampled signal. To overcome the fixed time-frequency resolution problems, the Wavelet based analysis, which provides flexible time-frequency resolution, becomes an efficient alternative in dealing with this type of machinery transient signals. In particular, the wavelet packet transform (WPT), proposed in Coifman and Wickerhauser (1992), uses a rich library of

redundant bases with arbitrary time-frequency resolution. Therefore, it enables the extraction of features from signals that combine nonstationary and stationary characteristics.

Considering that wavelet packet coefficients contains too many elements to efficiently represent a signal, it is necessary to identify a suitable subset in order to manage the computational complexity in practical situations. For classification applications, a natural direction is to address the issue of finding a wavelet-packet-based feature set that offers maximum feature separability due to class-specific characteristics. This work explores the feasibility of the WPT as a tool in the search for features that may be used in the detection and classification of mechanical vibration signals.

In Section 2, the Fourier-based analysis for transient signal analysis is presented. Section 3 presents an overview of the proposed classification system based on wavelet packet features. In Section 4, the feasibility of the proposed wavelet-packet-based feature extraction technique is demonstrated through numerical analysis of sampled acoustic signal collected in the end-milling process. The concluding remarks are presented in section 4.

## 2. Time-Frequency Analysis of Vibration Signals

Figure 1 shows a block diagram of our reference of detection strategy described in Haykin and Thomson (1998) and Haykin and Bhattacharya (1997). It consists of three functional units:

- Time-frequency analyzer, which converts the time-varying waveform of the input signal into a picture with two coordinates, namely, time and frequency.
- Feature extractor, the purpose of which is to compress the two-dimensional data produced by the time-frequency analyzer by extracting a set of features that retain the essential frequency content of the original signal.
- Pattern classifier, which is trained to categorize the set of features applied to its input.

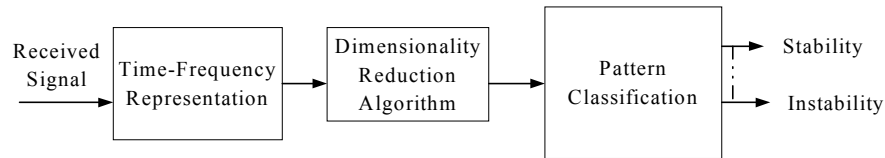


Figure 1. Block diagram of a detection strategy used in a nonstationary environment.

In sections 2.1, 2.2 and 2.3, three different approaches, based on frequency analysis, are presented in order to implement the described detection strategy.

### 2.1. Fourier-Based Analysis

Vibration signal classification generally requires windowing of the time-series vibration signals to form signal segments on which linear, bilinear, or nonlinear transformations are applied. The Fourier based methods, in particular, the short-time Fourier transform (STFT), are usually employed for the extraction of narrow-band frequency content in signals. The difficulty with STFT is that the accuracy for extracting frequency information is limited by the length of this window relative to the duration of the signal. Specifically, the STFT of is defined as

$$G(f, \tau) \equiv \int x(t) g^*(t - \tau e^{-j2\pi ft}) dt \quad (1)$$

where  $g(t)$  is a window function. The STFT decomposes a signal in the time domain into a two-dimensional function in a time-frequency plane  $(f, \tau)$ . At a given frequency  $f$ , (1) is equivalent to filtering a signal at all times with a band-pass filter having as an impulse response the window function modulated to that frequency. Alternatively, given a segment of signal windowed around time instant, one computes all frequencies of the STFT. Now, consider the ability of the STFT to discriminate between two pure sinusoids. Given a window function and its Fourier transform, define the bandwidth of the filter as

$$\Delta f^2 = \frac{\int f^2 |G(f)|^2 df}{\int |G(f)|^2 df} \quad (2)$$

Then, two sinusoids will be discriminated only if they are more than  $\Delta f$  apart. Similarly, the spread in time is given by  $\Delta t$  defined as

$$\Delta t^2 = \frac{\int t^2 |g(t)|^2 dt}{\int |g(t)|^2 dt} \quad (3)$$

So, two pulses in time can be discriminated only if they are more than apart  $\Delta t$ . Thus, the resolution in frequency of the STFT analysis is given by  $\Delta f$ , and the resolution in time is given by  $\Delta t$ . One important property, according to the uncertainty principle (Papoulis, 1963), is that for any suitably chosen window function, the time–bandwidth product of the window function has lower bound given by

$$\Delta t \Delta f = c \geq \frac{1}{4} \pi \quad (4)$$

Here,  $c$  is a constant dependent on the choice of  $g(t)$ . Note that once the window function  $g(t)$  is defined, the area (time–bandwidth product) of the window function in the time–frequency plane remains fixed. This means we cannot increase the time and frequency resolutions simultaneously. If we choose a window function with small  $\Delta t$  (good time resolution), then the corresponding frequency resolution will be poor ( $\Delta f$  will be large).

## 2.2. Wavelet Analysis

In order to overcome the resolution limitation of the STFT, a decomposition of square integrable signals  $L^2(\mathbb{R})$  has recently been developed under the name of wavelets (Mallat, 1989). These families of functions  $h_{a,b}$

$$h_{a,b}(t) = |a|^{1/2} h(t-b)/a \quad (5)$$

are generated from a single function  $h(t)$  by the operation of dilations and translations. The wavelet transform of a continuous signal can be defined as

$$CWT_x(b, a) = \langle x(t), |a|^{-1/2} h^*((t-b)/a) \rangle = |a|^{-1/2} \int x(t) h^*((t-b)/a) dt \quad (6)$$

where  $*$  represents the complex conjugation and where  $\langle \rangle$  represents the inner product. Equation 6 is interpreted as a multiresolution decomposition of the signal into a set of frequency channels having the same bandwidth in a logarithmic scale (i.e., constant Q or constant relative bandwidth frequency analysis by octave band filters). For the STFT, the phase space is uniformly sampled, whereas in wavelet transform the sampling in frequency is logarithmic, which enables one to analyze higher frequencies in shorter windows and lower frequencies in longer windows in time.

There are several families of wavelets, such as the Morlet, Haar, Daubechies, Biorthogonal, Coifflets, etc. (Misity et al., 1996). The Daubechies family is also referred to as  $D_n$ , where  $n$  is the order or the size of the mother wavelet, and  $D$  stands for the “family” of wavelet. This family has been extensively used, since its wavelet coefficients capture the maximum amount of the signal energy.

## 2.3. Wavelet Packet Transform (WPT)

Despite the flexible time–frequency resolution properties of WT, the frequency resolution could be poor in the high–frequency region. Therefore, it faces some difficulties for discrimination between signals having close high–frequency components. Wavelet packets, a generalization of wavelet bases, are alternative bases that are formed by taking linear combinations of the usual wavelet functions (Wickerhauser, 1994). These bases inherit properties such as orthonormality and time–frequency localization from their corresponding wavelet functions. A wavelet packet function is a function with three indices:  $W_{j,k}^n(t) = 2^{j/2} W^n(2^j t - k)$ . As with usual wavelets, integers and are index scale and translation operations, respectively

$$W_{j,k}^n(t) = 2^{j/2} W^n(2^j t - k) \quad (7)$$

The index  $n$  is called the modulation parameter or the oscillation parameter. The first two wavelet packet functions are the usual scaling function and motherwavelet function, respectively

$$\begin{aligned} W_{0,0}^0(t) &= \phi(t) \\ W_{0,0}^1(t) &= \psi(t) \end{aligned} \quad (8)$$

Wavelet packet functions for  $n=2,3,\dots$  are then defined by the following recursive relationships:

$$W_{0,0}^{2n}(t) = \sqrt{2} \sum_k h(k) W_{1,k}^n(2t - k) \tag{9}$$

$$W_{0,0}^{2n+1}(t) = \sqrt{2} \sum_k g(k) W_{1,k}^n(2t - k)$$

where  $h(k)$  and  $g(k)$  are the quadrature mirror filters (QMF), Akansu and Haddad (1992), associated with the predefined scaling function and mother wavelet function. To measure specific time-frequency information in a signal, we simply take the inner product of the signal and that particular basis function. The wavelet packet coefficients of a function  $f$  can be computed via

$$w_{j,n,k} = \langle f, W_{j,k}^n \rangle = \int f(t) W_{j,k}^n(t) dt \tag{10}$$

In Fig. 2, the usual wavelet decomposition is generalized to describe the calculation of wavelet packet coefficients  $w_{j,n,k}$  of a discrete-time signal. Computing the full wavelet packet decomposition (WPD) of a discrete-time signal involves applying both filters to the discrete-time signal  $[x_1, x_2, \dots, x_n]$  and then recursively to each intermediate signal. The procedure is illustrated in Fig. 3 a).

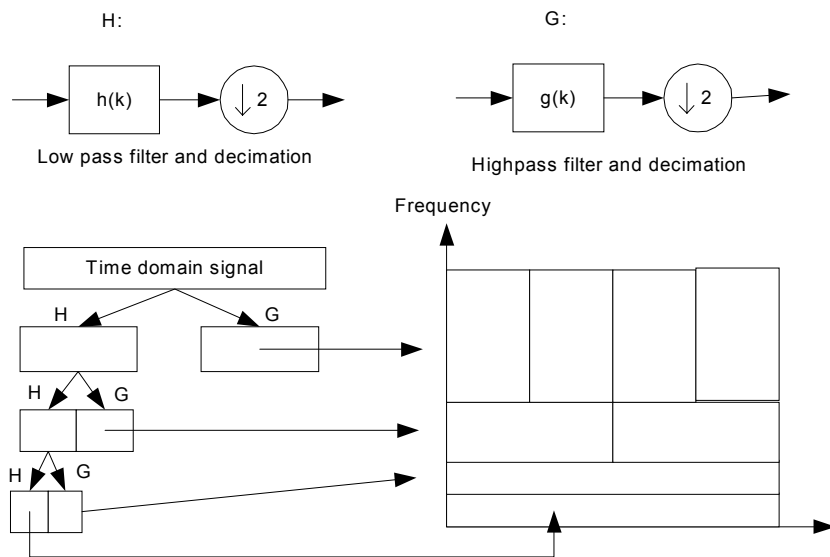


Figure 2. Wavelet decomposition of time-domain signal.

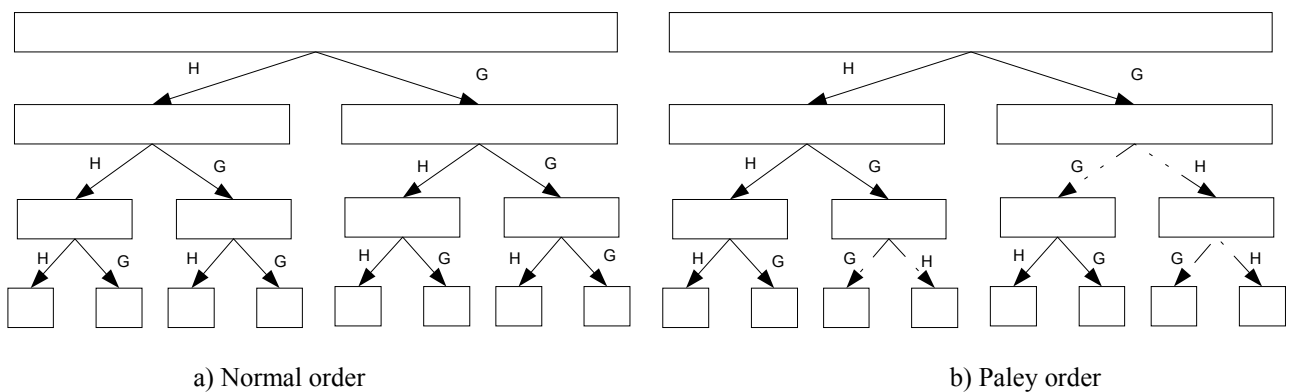


Figure 3. Implementation of discrete WPD

Note that the method of decomposition described above does not result in a WPT tree displayed in increasing frequency order. This is because aliasing occurs, which exchanges the frequency ordering of some nodes of the tree. A simple swapping of the appropriate nodes results in the increasing frequency ordering referred to as the Paley ordering (Wickerhauser M. V., 1994) of the tree, as shown in Fig. 3 b). The dashed lines highlight the differences with Fig. 3. In this way, the leftmost node at each level will correspond to the lowest frequency band. In following sections, we will

use this representation for easier interpretation. Whereas the FWT decomposes only the low-frequency components, WPT decomposes the signal utilizing both the low-frequency components and the high-frequency components. This flexibility of a rich collection of abundant information with arbitrary time-frequency resolution allows extraction of features that combine nonstationary and stationary characteristics.

### 3. Vibration Signal Feature Selection

A wavelet packet based method for end-milling state condition classification could be described as follows: a) Decomposition of the vibration data via the WPT to extract the time–frequency-dependent information; b) Features extraction from the WPD coefficients; c) Statistical processing based on discriminant analysis in order to identify a set of robust features that provides the most discrimination among the classes of vibration data; d) Classification of the data based on the reduced features set. Since the classification performance depends intensely on the feature extracting procedure, in this section is presented an analysis methodology in order to accomplish appropriately items a) and b). Additionally some comments related to the feature selections criteria and the classification system, items c) and d) respectively, will be exposed.

#### 3.1. Setup Configuration

Conclusive observations presented in Polly (2005) and Weingaertner et al. (2003), identify suitable stability conditions when the harmonics of the tooth passing frequency are distant from the system natural frequency in end milling process. In this sense, the stability evaluation used in this work is based on a sound pressure analysis and a workpiece texture test in order to verify the milling quality. Figure 4 presents the setup configuration used to sample the acoustic signal in all the tests.

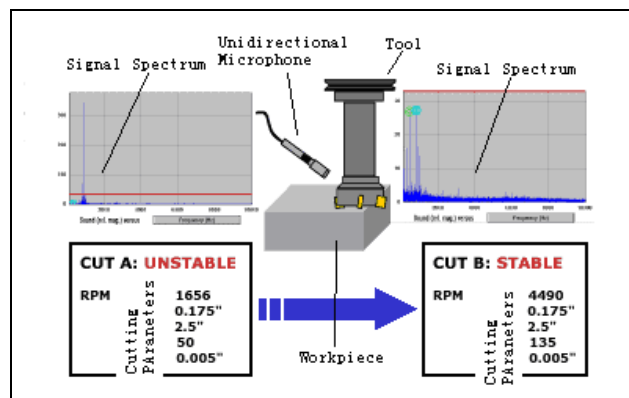


Figure 4. Experimental setup for the acoustic signal detection system

A microphone power supply (preamplifier type 2801 - Brüel and Kjaer, Copenhagen) and a unidirectional microphone (affixed to the turret machine) were used for acoustic vibration signal capture. The microphone bandwidth is 20 kHz; the amplifier gain was set to 20. All the sampled data were obtained of a 5 axes High Speed Hermle Machine (Model C6000) in the Center of Competence in Manufacture – CCM (Technological Institute of Aeronautics). A total of 84 tests were accomplished in order to analyze the WPT decomposition technique in the end-milling process, a group of 4 representative tests are presented in the following analysis. It must be pointed that the classes objectified in the following analysis consist in stability and instability milling conditions. Such stability and instability conditions were verified using a threshold value of 0.8  $\mu\text{m}$  in the surface roughness test ( $R_a$ ). A four flute with 12 mm of diameter end mill tool was used in the following cutting tests and the dynamic characteristics of each test are presented in table 1.

Table 1. Dynamic characteristics of the experimental implementation (figures 5-6)

Test	Condition	$f_d$ (Hz)	$f_v$ (Hz)	S (rpm)	$a_p$ (mm)	$f_z$ (mm/tooth)	L (mm)	$R_a$ ( $\mu\text{m}$ )
1	Stable	916	-	13750	0.5	0.1	56.5	0.46
2	Stable	783.33	-	11750	1	0.1	43	0.75
3	Unstable	750	1800	11250	0.5	0.1	56.5	1.2
4	Unstable	666.66	2350	10000	1	0.1	43	1.0

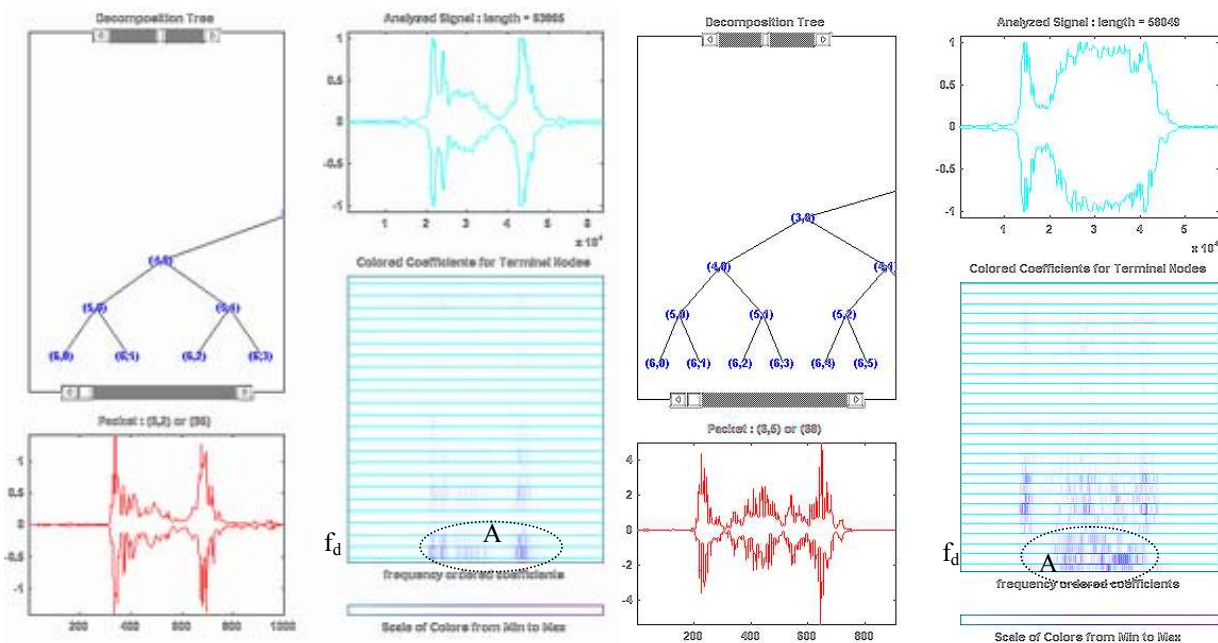
$f_d$ : tooth passing frequency,  $f_v$ : vibration frequency, S: spindle angular speed,  $a_p$ : axial depth,  $f_z$ : feed rate,  $R_a$ : Surface Roughness [ $\mu\text{m}$ ] (obtained by a portable Surftest SJ-201P - MITUYO). End milling operations were performed on aluminum workpiece.

### 3.2. WPT Decomposition and Feature Extraction of Acoustic Signals

As exposed in section 2.3, one advantage of using WPT to decompose a signal is that this analysis tool evidence different time-frequency resolution components in a certain signal. For example, by computing the full WPD on a signal segment with  $n=2^J$  points for  $\tau$  resolution levels (where  $J$  and  $\tau$  are positive integers), the result is a group of  $2^1 + 2^2 + \dots + 2^J = 2^{J+1} - 2$  sets of coefficients where each set corresponds to a wavelet packet node. Considering that a node could be used as a feature, we can obtain several feature candidates in a multiple level Wavelet Packet analysis and some careful must be taken in order to identify the features to be used in the classification system. It should be noted that the existence of undesired components in the node signal makes the classification unnecessarily difficult.

In order to discuss the advantages of the WPT for the vibration signal dimension reduction and the feature extraction procedure, a six level decomposition of the acoustic signals related to the stable and unstable tests (Table 1) are presented in figures 5a), 5b), 6a) and 6b). The ‘‘Daubechies’’ Wavelet function 2 is used in all the following analyses and each figure presents four relevant signal windows:

- The WPT decomposition tree (top/left).
- The envelope of the original acoustic signal to be decomposed (top/right).
- The envelope of the WPT decomposed signal relative to a particular node in the decomposition tree (bottom/left).
- The WPT coefficient relative to the power spectrum of the original signal (bottom/right).



a) Test 1 ( $f_d = 916$  Hz)

b) Test 2 ( $f_d = 783.33$  Hz)

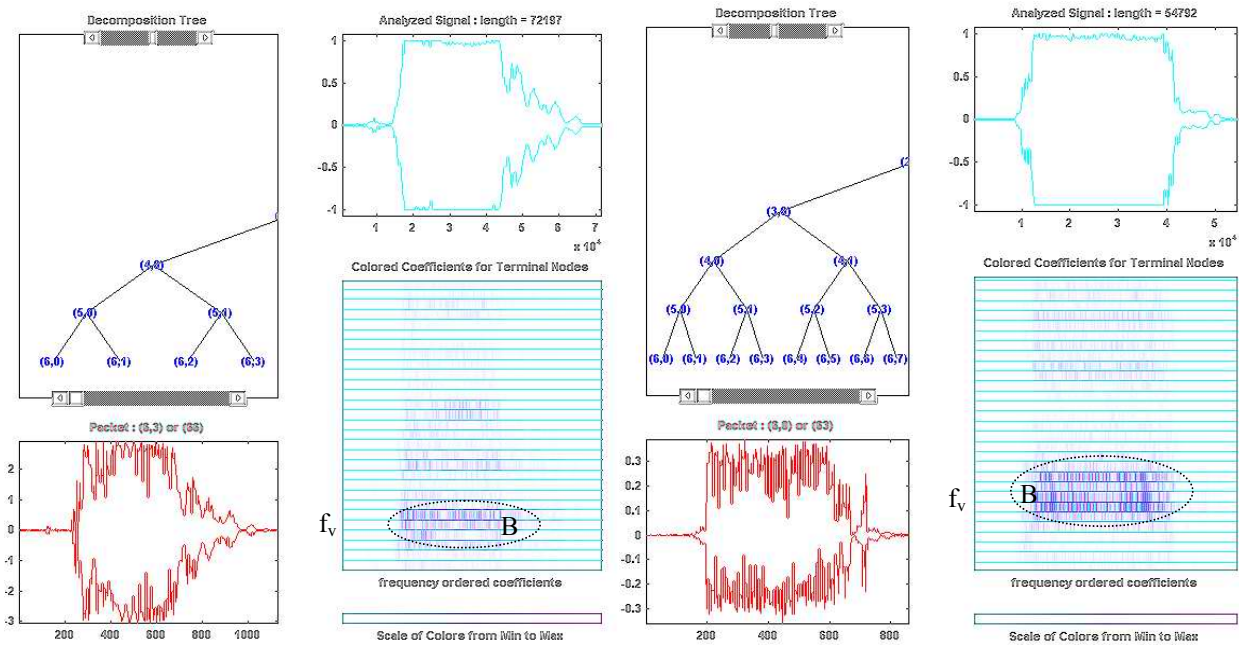
Figure 5. WPT decomposition and power spectrum of stable conditions signals

As region A in figures 5a) and 5b) shows, the stable conditions tests (1 and 2 in table 1) are characterized by dominant frequency component values with a multiple or sub-multiple relation with the tooth passing frequency ( $f_d$ ). As reported in Smith and Tlustý (1990) and verified in Weingaertner et al. (2003), the stability operation condition is related to the fact that the tooth passing frequency values are close to the most flexible mode natural frequency. On the other hand, figures 6a) and 6b) presents unstable results (tests 3 and 4) where the dominant component frequencies evidenced in region B, vibration frequencies ( $f_v$ ) in this case, are not related to the system natural frequency.

On the other hand, from the original signals in figures 5 and 6, is readily verified that there exists some overall similarity in the signals envelope shapes that can be used as features for classification. More precisely, in the stable cases, at least two abrupt variations are evidenced and in the unstable cases, a denser envelope shape is verified.

One of the main contributions of this work, by a detailed envelope shape analysis of each one of the 64 six-level nodes in the WPT decomposition, is the verification of a suitable reduction of the data dimension (from almost 60000 points in the first level to just less than 1000 points in the six WPT decomposition level) maintaining the principal characteristics of the original signal shape (features). It must be noted that the particular stable signals choice was made

in order to evidence two not similar envelope shapes, more precisely, most of the collected signals in stable milling process conditions present accentuated abrupt variations. As figure 5b) denotes, this abrupt variations were recovered by means of a suitable signal node choice in the WPT decomposition.



a) Test 1 ( $f_d = 750$  Hz)                      b) Test 2 ( $f_d = 666.66$  Hz)  
 Figure 6. WPT decomposition and power spectrum of unstable conditions signals

As figures 6a) and 6b) presents, the denser envelope characteristic of the unstable signals are preserved in the WPT decomposition. It must be stressed that for all the unstable cases, this feature was constantly repeated.

Considering a classification system for future steps in our research, more particularly a neural network classifier, it is desirable to use a lower dimensional vector as input to ease the design of the classifier and improve its generalization capability. In this sense, one popular technique in reducing the feature dimensionality is the Karhunen–Loève (K–L) transform. The K–L transform is optimal for "signal representation" in the sense that it provides the smallest mean-square error for a given number of data.

#### 4. Conclusion

This paper has investigated the feasibility of applying the WPT to the feature extraction acoustic signals related to vibration conditions in high-speed end-milling. Using the WPT, a rich collection of time-frequency characteristics in a signal can be obtained and examined for classification purposes. In this paper, we detailed a natural feature selection process that exploits signal class differences in the wavelet packet decomposition procedure. This results in a reduced-dimensional feature space compared to the dimension of the original time-series signal.

Finally, some suggestions for future works related to this article are listed: a) Compute the WPT considering only the transient state signal of vibration time series objectifying an instability detection system that could avoid the tool or material damage. b) Since the acoustic vibration signals of stable and unstable milling process present visible shape differences, with a simple neural network good results could be obtained and extended to identify new sub-classes originated from instability class.

#### 5. Acknowledgements

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#### 6. References

Akansu, A. N. and Haddad R. A., 1992, "Multiresolution Signal Decomposition: Transforms, Subbands, Wavelets". New York: Academic.

- Coifman, R.R. and Wickerhauser M. V., 1992, "Entropy-based algorithms for best basis selection", IEEE Trans. Inform. Theory, vol. 38, pp. 713-718.
- Gabor, D., 1946, "Theory of Communication", J. Inst. Elect. Eng., Vol. 93, No.3, pp. 429-457.
- Haykin, S. and Thomson D., 1998. "Signal Detection In A Nonstationary Environment Reformulated As An Adaptive Pattern Classification Problem", Proceedings of the IEEE, Vol. 86, No. 11, pp. 2325-2344.
- Haykin, S., Bhattacharya T., 1997, "Modular Learning Strategy For Signal Detection In A Nonstationary Environment", IEEE Transactions on Signal Processing, Vol. 45, pp. 1619-1637.
- Mallat, S., 1989, "A Theory For Multiresolution Signal Decomposition: The Wavelet Representation", IEEE Trans. Pattern Anal. Mach. Intell., Vol. 11, No. 7, pp. 674-693.
- Papoulis, A., 1962, "The Fourier Integral and Its Applications", New York: Mc- Graw-Hill.
- Paul D., 1995, "Condition monitoring and defect diagnosis in manufacturing process using DDS and wavelets," Ph.D. dissertation, Dep. Elect. Eng., Michigan Technol. Univ., Houghton, MI
- Polli, M.L., 2005, "Análise De Estabilidade Do Processo De Fresamento A Altas Velocidades De Corte", Tese de Doutorado, Universidade Federal de Santa Catarina (UFSC).
- Rioul, O. and Vetterli, M., 1991, "Wavelets and signal processing", IEEE Signal Processing Mag., vol. 8, pp. 1438
- Smith, S., Tlustý, J., 1990, "Update on High-Speed Milling Dynamics", Journal of Engineering for Industry, Transaction of the ASME, Vol. 112, pp. 142-149.
- Torrence, C. and G. P. Compo, 1998. "A practical guide to wavelet analysis". Bulletin of the American Meteorological Society 79 (1), 61--78.
- Weingaertner, W.L., Schroeter, R.B., Polli, M.L., Gomes, J.O., 2003, "Machining Parameters Selection To Avoid Chatter In High Speed Milling", 17th International Congress of Mechanical Engineering, COBEM 2003, São Paulo, SP.
- Wickerhauser M. V., 1994, Adapted Wavelet Analysis from Theory to Software, Natick, MA: Wellesley.