ACOUSTIC TRANSMISSION WITH MODE CONVERSION PHENOMENON

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Abstract. *If an acoustic wave impinges on an interface between two media with an oblique incidence, and if one of these media is a solid, in general, a complicated phenomenon occurs, that is referred to as mode conversion from longitudinal wave to shear wave, and vice-versa, for the reflected and transmitted waves. The relations between the reflected and transmitted waves and the incident wave, defined in terms of amplitude and intensity, depend on the characteristic acoustic impedances of the media, the angles of incidence and the types of the incident wave (longitudinal or shear wave). The determination of these relations, the reflection and transmission coefficients, can be made by using the transmission-line models for waves in fluids and isotropic solids. In solids, this resolution takes into account the continuity of stresses and the particle velocity at the interface, and can be difficult to solve. This work presents a simple form of the system of equations to calculate the reflection and transmission coefficients, using a solid-solid interface. Furthermore, the same system of equations can be particularized for a liquid-solid interface using a complex acoustic velocity of the shear wave propagating in the liquid. The model is written in a Matlab graphical user interface and some simulations are performed to compare to other modeling results. An excellent agreement between our results and theirs were found. Moreover, two important experimental and theoretical applications for nondestructive evaluation methods are presented using mode conversion: the characterization of solid materials by ultrasonic spectroscopy and the simulation of acoustic field evolution caused by interfaces between isotropic media.*

Keywords*: Mode conversion, transmission phenomena, acoustic wave propagation, ultrasonic transducer.*

1. Introduction

The acoustic transmission with mode conversion phenomenon is important in many ultrasonic transducer applications such as nondestructive testing and characterization of acoustical properties of solid materials. Generally, these applications have solid or liquid wedges between the transducer face and the test material. The transmitted or reflected ultrasonic beams through interfaces depend on the acoustic propagation velocity between the discontinued materials, the transducer geometry, and the angle of incidence at the interfaces. For this purpose, knowledge on the interaction of incident waves with interfaces becomes necessary.

The wave propagation in solid media is more complex than in liquid media, because both longitudinal and shear waves can simultaneously propagate in solids. The existence of both types of waves makes difficult the interpretation of the received signals and it is important to eliminate the undesired propagation modes (Kino, 1987; Krautkramer and Krautkramer, 1977). The propagation of shear waves in a non-viscous liquid is not possible because the energy is totally dissipated. In viscous liquids it is possible to have shear waves, but they are strongly attenuated and can travel a very small distance in the order of micrometers. Then, it is correct to say that only longitudinal waves can be propagated in liquids.

The angle of incidence is defined as the angle between the propagation direction of the wave and the normal vector of the reflecting surface. When normal incidence occurs at the interface, the transmitted and reflected waves are of the same type of the incident wave. By varying the angle of incidence, another type of wave begins to be generated, whose phenomenon is called mode conversion. For example, in the reflection process where a longitudinal wave propagating in glass is impinging on an interface in contact with air, if the angle of incidence is greater than 42º, only shear waves are reflected. This phenomenon, that is called total conversion, is important to convert a longitudinal wave to a shear wave without losing energy. Mode conversion permits to use longitudinal wave transducers, which have an easier fabrication process, for generating shear waves (Kino, 1987).

In nondestructive testing, it is necessary to generate waves of the highest frequency possible for obtaining better axial resolution (shorter wavelengths) and lateral resolution (less divergence of the beam). However, higher frequencies are more quickly attenuated. Therefore, selecting the optimal frequency often involves maintaining a balance between resolution and penetration. Furthermore, discontinuities perpendicular or tilted to the surface of a test material are usually difficult to be detected by straight beam. These kinds of flaws are more suitable to be detected with angle beam transducers using mode conversion phenomenon generating shear waves. As the shear waves have phase velocity in metals almost half of the longitudinal waves, the resolution is improved through the use of shear waves; the wavelength of a shear wave is approximately half of the wavelength of a longitudinal wave at a given frequency (Krautkramer and Krautkramer, 1977).

2. Transmission Phenomenon

Under the assumption that the media are isotropic, elastic and homogeneous, it is verified that an acoustic wave impinging on an interface between two media is partial or totally reflected. During the reflection or transmission process the amplitude and phase of the wave become different, in addition, it can happen mode conversion from longitudinal wave to shear wave and *vice-versa*. The relations between the reflected and transmitted waves and the incident wave, defined in terms of amplitude and intensity, depend on the characteristic acoustic impedances of the media, the angles of incidence and the types of the incident wave (longitudinal or shear wave).

Boundary value problems involving elastic waves in solids are generally intricate and difficult to solve. However, Oliner (1969) presents a model that establishes an analogy between transmission lines and acoustic waves for several interfaces. This approach makes the wide number of methods to solve transmission line problems available to solve acoustic waves problems in a systematic, simple, and direct manner. In the transmission line representations, the electrical analogies were made using the electrical current and electrical voltage representing the particle velocity and the mechanical stress vector, respectively. Using these analogies, the expressions for the characteristic impedances and the velocity and stress vector mode functions were derived. The method is presented extensively in Oliner's paper; it is possible to use this method for other wave types as Rayleigh, Leaky Rayleigh, Lamb and Love waves.

Three types of body waves are possible in solids: the *P* wave (longitudinal wave) and the *SV* (shear vertical) and *SH* (shear horizontal) waves. Although the particle displacements of both shear waves are perpendicular to the propagation direction, their polarizations are different. In the *SV* wave, particle motion is parallel to the *xz* plane, as shown in Fig. 1, while particle motion for the *SH* wave is perpendicular to this plane.

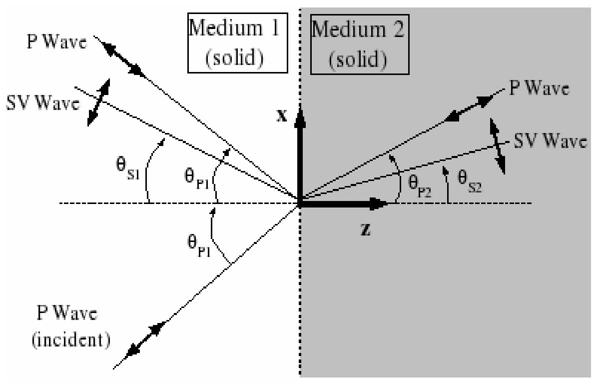

Figure 1. Longitudinal wave impinges a solid-solid interface.

The more general case of reflected and transmitted waves at an interface between two media with different impedances occurs when the incident wave strikes at an oblique angle. Considering that a longitudinal wave impinges on a solid-solid interface, the incident wave generates both reflected and transmitted *P* and *SV* waves. Fig. 1 shows these possible body waves and their particle motions. The reflected and refracted angles are calculated using Snell's law, which supplies information about the propagation directions. Using the transmission line model presented by Oliner (1969), the equations that rule the phenomenon can be easily obtained by taking into account the continuity of stress components and particle velocities at the interface. Eq. (1) relates the reflection (R_P) and transmission (T_P) coefficients of the longitudinal wave and the reflection (R_S) and transmission (T_S) coefficients of the shear wave with the properties of the media and the angles of incidence, reflection and refraction. The analytical model presented in this work is only valid for a longitudinal incident wave.

$$\begin{bmatrix} \frac{sen\,\theta_{P1}}{\rho_1 c_{P1}} & \frac{\cos\theta_{S1}}{\rho_1 c_{S1}} & \frac{-sen\,\theta_{P2}}{\rho_2 c_{P2}} & \frac{sen\,\theta_{S2}}{\rho_2 c_{S2}} \\ \frac{\cos\theta_{P1}}{\rho_1 c_{P1}} & \frac{-sen\,\theta_{S1}}{\rho_1 c_{S1}} & \frac{\cos\theta_{P2}}{\rho_2 c_{P2}} & \frac{sen\,\theta_{S2}}{\rho_2 c_{S2}} \\ -\cos(2\theta_{S1}) & sen(2\theta_{S1}) & \cos(2\theta_{S2}) & sen(2\theta_{S2}) \\ \frac{sen(2\theta_{P1})}{c_{P1}^2/c_{S1}^2} & \cos(2\theta_{S1}) & \frac{sen(2\theta_{P2})}{c_{P2}^2/c_{S2}^2} & -\cos(2\theta_{S2}) \end{bmatrix} \begin{bmatrix} R_P \\ R_S \\ T_P \\ T_S \end{bmatrix} = \begin{bmatrix} -\frac{sen\,\theta_{P1}}{\rho_1 c_{P1}} \\ \frac{\cos\theta_{P1}}{\rho_1 c_{P1}} \\ \cos(2\theta_{S1}) \\ \frac{sen(2\theta_{P1})}{c_{P1}^2/c_{S1}^2} \end{bmatrix} \tag{1}$$

where ρ_1 and ρ_2 are, respectively, the density of media 1 and 2; θ_{P1} , θ_{P2} , θ_{S1} and θ_{S2} are the angles between the normal vector of the surface and the propagation direction of the *P* and *SV* waves shown in Fig. 1; and c_{P1}, c_{P2}, c_{S1} and c_{S2} are the phase velocities of the *P* and *SV* waves in its respective medium. Solving the Eq. (1) for an angle of incidence (θ_{P1}), the amplitude reflection and transmission coefficients defined in terms of the amplitude can be found. The coefficients represent the amplitude value of the reflected and refracted waves normalized respect to the incident wave. Furthermore, these coefficients can also represent an energy balance of the transmission process; expressions (2) can be used to calculate the power transmission and reflection coefficients:

$$R_P^{Pot} = -R_P R_P^* ,\quad R_S^{Pot} = -R_S R_S^* \frac{real(1/Z_{S1}^*)}{real(1/Z_{P1}^*)} ,\quad T_P^{Pot} = T_P T_P^* \frac{real(1/Z_{P2}^*)}{real(1/Z_{P1}^*)} ,\quad R_S^{Pot} = -T_S T^{*_S} \frac{real(1/Z_{S2}^*)}{real(1/Z_{P1}^*)} . \tag{2}$$

where the * subscript represents the conjugated value of the complex variables, and *real( )* the real component of the impedance described in the following expressions:

$$Z_{P1} = \frac{c_{P1}\rho_1}{\cos(\theta_{P1})} ,\quad Z_{P2} = \frac{c_{P2}\rho_2}{\cos(\theta_{P2})} ,\quad Z_{S1} = \frac{c_{S1}\rho_1}{\cos(\theta_{S1})} ,\quad Z_{S2} = \frac{c_{S2}\rho_2}{\cos(\theta_{S2})} . \tag{3}$$

The angles of reflection and refraction are obtained by Snell's law and are represented by:

$$\frac{sen(\theta_{P1})}{c_{P1}} = \frac{sen(\theta_{P2})}{c_{P2}} = \frac{sen(\theta_{S1})}{c_{S1}} = \frac{sen(\theta_{S2})}{c_{S2}} \tag{4}$$

Equations (1) represent a general case and can be particularized for the liquid-solid interface, considering a complex phase velocity for the shear wave in the liquid, as shown in Eq. (5):

$$c_{complex} = \sqrt{\frac{j\omega\eta}{\rho}} \tag{5}$$

where ω is the angular frequency of the wave, η the medium viscosity, and ρ the medium density.

Because shear waves cannot propagate in liquids, when the longitudinal plane wave impinges the interface between a non-viscous liquid and a solid, three types of waves are generated: the reflected longitudinal wave with the same angle of incidence, and the transmitted longitudinal and shear waves, it can be seen in Fig. 2. The equations that solve the problem of a non-viscous liquid-solid interface are easily obtained from general case (solid-solid interface) using a very low viscosity in Eq. (5), for example, 10^{-10} Pa.s.

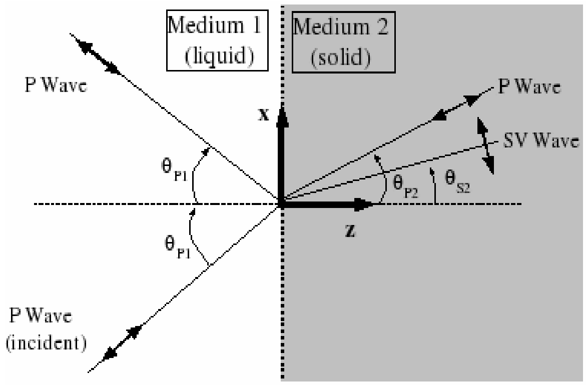

Figure 2. Longitudinal wave impinges on the interface between a non-viscous liquid and a solid.

2.1 Water-Aluminum interface results

The power transmission and reflection coefficients for a water-aluminum interface are calculated using the data presented by Kino (1987): c_{Pw}=1480*m/s* and $\rho_w = 1000\, kg/m^3$ in water, and c_{Pa}=6420*m/s*, c_{Sa}=3040*m/s* and $\rho_a = 2700\, kg/m^3$ in aluminum. The shear velocity in water is modeled using a viscosity of 10^{-10} Pa.s in Eq. (5), obtaining c_{sw}=0.0013+0.0013*j* m/s, this value corresponds to a strongly attenuated shear wave. The power transmission and reflection coefficients for the water-aluminum interface as function of the angle of incidence are shown in Fig. 3. These results are in agreement with the data presented by Kino (1987).

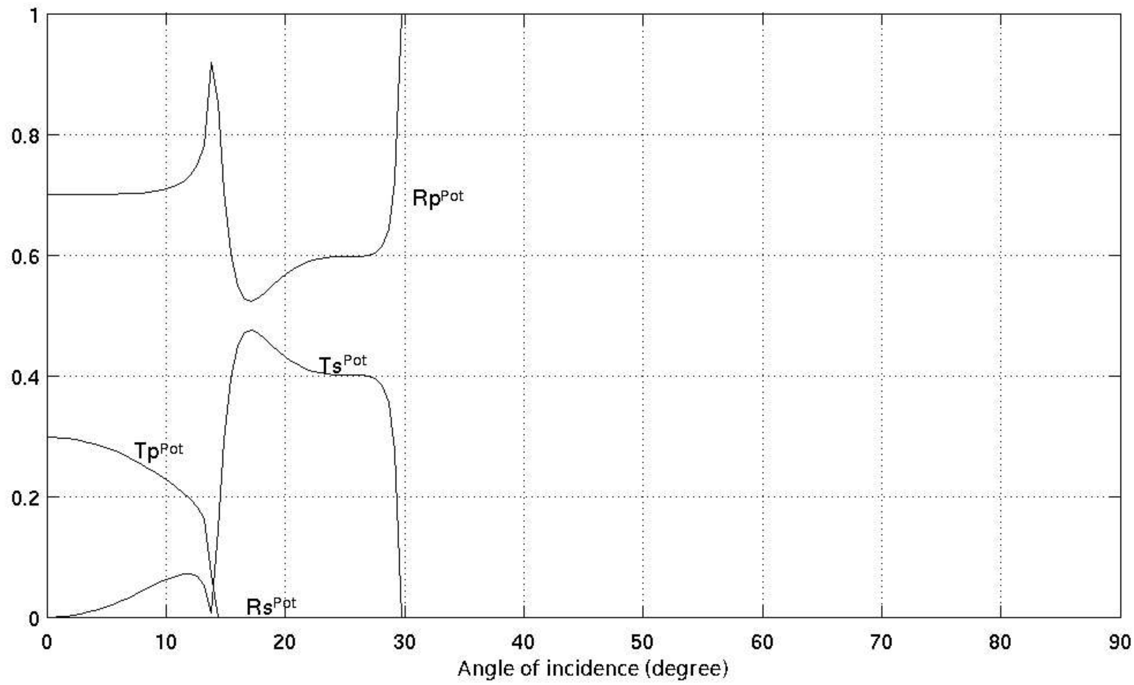

Figure 3. Power transmission and reception coefficients of the water-aluminum interface

3. Applications

3.1 Characterization of Solid Materials by Ultrasonic Spectroscopy

Measurements of acoustic properties of solid materials, such as phase velocity and attenuation coefficient, are important in many ultrasonic applications. The ultrasonic spectroscopy technique has been extensively used in determining these acoustic properties. By adjusting the angle of incidence, the mode conversion allows measuring both shear and longitudinal wave properties (Wang *et al.*, 2001). The technique uses broadband longitudinal transducers to measure the properties covering a continuous frequency range. Phase velocity and attenuation coefficient are the most important properties measured.

The technique use two transducers mounted coaxially, one is used as emitter and the other is a receiver, a sample of thickness d, whose wave velocity and attenuation needs to be measured, is inserted between the two transducers. The physical arrangement used in this technique is shown in Fig. 4. The phase velocity and attenuation are measured by the comparison of the spectra of the signals received in the case with and without the sample (Wu, 1996).

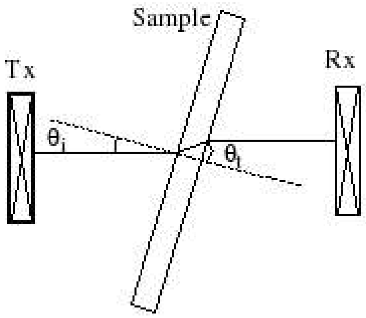

Figure 4 – Sample placed between the transducers

For longitudinal waves, the normal incidence ($\theta_i = 0$) is used. The measurement is performed in two steps. Firstly, in the case without the sample, the signal S_W is acquired and the FFT algorithm is used to calculate the magnitude (A_W) and the phase (ϕ_W) spectra. In the second step with the inserted sample between the transducers, the signal S_S is acquired and its amplitude (A_S) and phase spectra (ϕ_S) are obtained. The phase velocity (c_l) and the attenuation coefficient (α_l) of the longitudinal wave are calculated by (Wang *et al.*, 2001):

$$c_l = \frac{c_W}{1 + \dfrac{(\phi_S - \phi_W + 2\pi f \tau) c_W}{2\pi f d}} , \tag{6}$$

$$\alpha_l = \alpha_W + \ln\left(\frac{T_{Tp} A_W}{A_S}\right) / d . \tag{7}$$

where c_W is the acoustic velocity in water, f is the frequency, τ is the delay of the two time windows used to acquire the signals and d is the thickness of the sample. T_{Tp} is the total transmission coefficient for the longitudinal wave, which is equal to the product of the two transmission coefficients of the wave from water to the sample and from the sample to water (Wu, 1996).

For shear waves, it is used oblique incidence with an angle greater than the first critical angle for the water-sample interface. Thus, the phase velocity (c_s) and the attenuation (α_s) of the shear wave is calculated by (Wang *et al.*, 2001):

$$c_s = \frac{c_W}{\sqrt{\sin^2\theta_i + \left[\dfrac{(\phi_S - \phi_W + 2\pi f \tau) c_W}{2\pi f d} + \cos\theta_i\right]^2}} , \tag{8}$$

$$\alpha_s=\alpha_W\cos(\theta_r-\theta_i)+\ln\left(\frac{T_{Ts}A_W}{A_S}\right)\frac{\cos(\theta_r)}{d}\ , \qquad (9)$$

where θ_i is the angle of incidence, θ_r is the refractive angle of the shear wave calculated from Snell's law, and T_{Ts} is the total transmission coefficient of the shear wave (Wu, 1996).

By using two broadband transducers with diameter 6.3 mm and central frequency 5 MHz, the phase velocities of the longitudinal and the shear waves were measured in an aluminum sample (d=12.7 mm). Measurement of the shear wave velocity was carried out using an incident angle of 16°, because the first critical angle for the aluminum-water interface is approximately 14°, as shown in Fig. 3. The velocities calculated with Eqs. (6) and (8) at 5 MHz and temperature of approximately 20°C are c_l=6210 m/s and c_s=2999 m/s. By using Eqs. (7) and (9), the attenuation coefficients of $\alpha_p=22\,dB/m$ and $\alpha_s=127\,dB/m$ are obtained.

3.2. Simulation of Acoustic Field Evolution Caused by Interfaces between Isotropic Media

The study of the acoustic fields generated by ultrasonic broadband transducers through interfaces is important for nondestructive testing of structures because, typically, liquid or solid wedges of diverse geometry are used between the transducer face and the structure (Buiochi et al., 2004). The ultrasonic field is strongly dependent on the incident angle at the interface and the field simulation can yield valuable information about the phenomenon to interpret correctly the echographic data obtained.

The method proposed by Buiochi *et al*. (2004) uses the spatial impulse response (Stepanishen, 1971) and the discrete representation method (Piwakowski and Delannoy, 1989) to calculate the reflected and transmitted ultrasonic fields with interfaces of complex geometry, considering mode conversion at the interface. Both the transducer aperture and the interface are considered as a finite number of elementary sources, each emitting a hemispherical wave. The method is valid for all field regions and may be performed for any excitation waveform radiated from any arbitrary acoustic aperture.

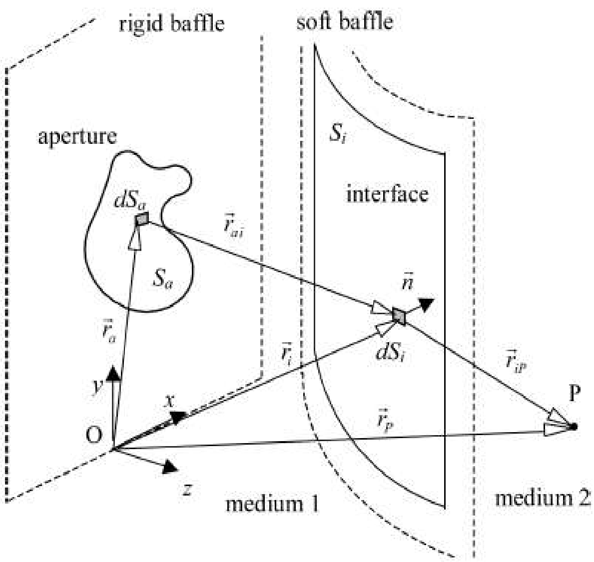

Figure 5. Geometry used to calculate the field produced by an arbitrary source through an arbitrary interface

Figure 5 shows an arbitrary aperture S_a embedded in an infinite rigid baffle. The acoustic pressure field at a point of an isotropic medium can be calculated in the time domain by the Rayleigh's integral:

$$p(\vec{r}_i,t)=\frac{\rho_1}{2\pi}\frac{\partial}{\partial t}\int_{S_a}\frac{v_n(\vec{r}_a,t-r_{ai}/c_1)}{r_{ai}}dS_a \qquad (10)$$

where c_1 and ρ_1 are the acoustic propagation velocity and the density of the medium 1, r_{ai} is the distance from the elementary area dS_a located at $\vec{r}_a$ to the point located at $\vec{r}_i$ and $v_n(\vec{r}_a, t)$ is the normal velocity in each point of the aperture.

The interface is large enough to intercept the major proportion of the incident energy of the acoustic field. Equation (10) is used to calculate the acoustic pressure at the interface and the reflected and transmitted pressure wave $p^{R/T}(\vec{r}_i, t)$ can be approximated by:

$$p^{R/T}(\vec{r}_i, t) = C^{R/T}(\theta_i)\, p(\vec{r}_i, t) \tag{11}$$

where $C^{R/T}(\theta_i)$ is referred to as the wave reflection and transmission coefficients at the interface which depend on the angle of incidence of a plane wave on each elementary surface at the interface.

Finally, considering that the acoustic pressure immediately after the interaction with the interface is known from Eq. (11), it is possible to calculate the reflected and transmitted acoustic fields applying the Rayleigh-Sommerfeld integral to the interface, assuming that it is embedded in an infinite soft baffle:

$$p(\vec{r}_i, t) = \frac{1}{2\pi c_M} \frac{\partial}{\partial t} \int_{S_i} \frac{|\cos(\vec{r}_{iP}, \vec{n})|}{r_{iP}} \frac{\partial}{\partial t} p^{R/T}\left(\vec{r}_i, t - \frac{r_{iP}}{c_M}\right) dS_i \tag{12}$$

where c_M is the acoustic propagation velocity of medium M (M=1 for reflected and M=2 for transmitted waves), r_{iP} is the distance from the elementary area dS_i located a $\vec{r}_i$ t to the field point located at $\vec{r}_p$, $\cos(\vec{r}_{iP}, \vec{n})$ is the cosine of the angle between the normal vector $\vec{n}$ and the vector $\vec{r}_{iP}$, and Si indicates the surface of the interface.

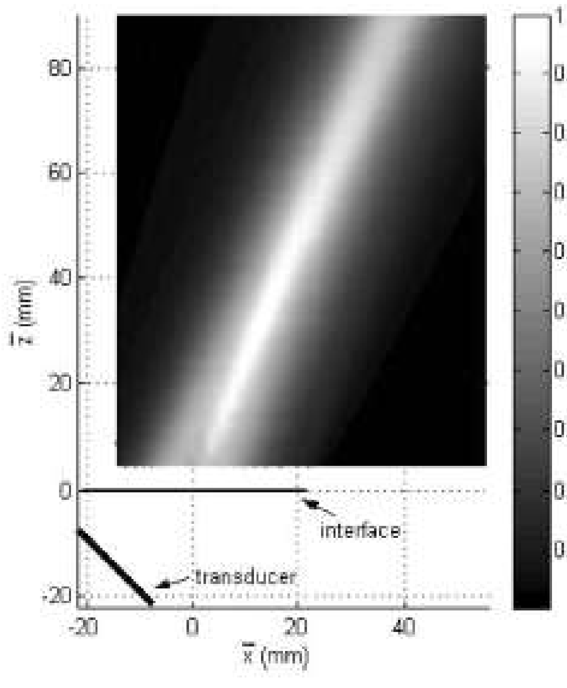

Figure 6. Simulated acoustic field through an interface

Figure 6 shows the *xz* scan of the maximum pressure distribution for the simulated fields refracted at the acrylic-water interface, with an angle of incidence of 45° and an axial distance Z = 20.7 mm. In Fig. 6, both interface and transducer used in the simulation are represented graphically.

4. Conclusions

Acoustic transmission with mode conversion is an important phenomenon in ultrasonic applications in nondestructive testing. The equations that model the complex phenomenon for a solid-solid interface with a longitudinal incident wave were presented. Furthermore, the particularized model for a liquid-solid interface was obtained using a complex acoustic velocity of the shear wave propagating in a liquid with very low viscosity.

Two important applications were described. Firstly, the method of characterization of solids using ultrasonic spectroscopy was discussed. An aluminum sample was characterized by measuring the shear velocity and attenuation coefficient. Secondly, the application to the study of acoustic fields generated by broadband transducers through interfaces was discussed and a simulation of the transmitted field through an acrylic-water interface was showed.

5. Acknowledgments

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