A RISKS QUANTIFICATION PROCEDURE BASED ON BAYESIAN INFERENCE

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Abstract. *This paper has by objective to presents an approach of analysis of exceptional contingency in technical systems looking for imprecision and scarceness of historical data treatment. The basics goals of a contingency analysis are to identify, model and quantify in a proper manner the risks associated with a system operation and maintence. Unlike traditional methods of Probabilistic Risks Analysis, the approach used in this article tries to by-pass some specific problems found in these kinds of methodologies. The risk perception in these methods and its probabilistic modeling requires historical data and a precise formulations of risks and ways which take to a failure. In many situations, data are nor registered or are imprecisely registered. This paper proposal consists in presenting a risks quantification procedure based on bayesian inference, on which can be used a specialist knowledge for treating the lack of data. This subjective information is them actualized by a bayesian update. The event quantification result generates an inference tool which represents the probabilities distribution of identified exceptional contingence occurrence. This contingences analysis provides as results, probabilities that orient the establishment of emergency actions plans for contingences.*

Keywords: *Reliability, Bayesian update, Contingency analysis, Risk analysis*

1. Introduction

The world of statistics is traditionally divided into two areas: the “classic” and the “bayesian statistic”. In synthesis, what differs these two approaches is the fact that classic statistic treats the distribution parameters as specific values – fixed, while the bayesian statistic considers the parameters as random variables, that have their own distribution (RAC,2003). This make possible use, in bayesian statistic, prior knowledge.

The classic statistic tries to infer on collected data. However, it does not considers prior knowledge – except to suggest the choice of a particular population model to "fit" to the data, and this choice is later checked against the data for reasonableness (NIST/SEMATECH, 2003).

On the other hand, the bayesian statistic allows associates prior knowledge (prior distribution model) with observation data, making possible the knowledge update.

This bayesian statistic characteristic is especially important in case that data about an event is scarce, which is – fortunately – the case of the major part of accidents. In this case, it can be used the specialist knowledge, associated with the few observed data.

Every technical system is a danger carrier (Alonço, 2000). By Hazard, it’s meant “any act (omission or commission), condition or state of the system, or a combination thereof, with the potential to result in an incident or accident” (Mosleh et. al., 2004). The risk concept, in its turn, is associated with the chance of the danger become an accident.

An incident or an accident is a couple of actions that interfere negatively in the environment, human or technical system that in a extreme situation, can be a catastrophe. As shown in Figure 1, every accident has a casual chain. A condition in a technical system can, during its life time, can become a hazard condition - specially in using time and withdraw. But the hazard condition needs an activating or conditioning event (Trigger Event) to actually lead to a failure accident. This Trigger Event could be latent in the technical system, human system (organizational) or in the environmental. Barriers should be set into the casual chain to eliminate de hazard condition or reduce the risk - reducing the likelihood of occurrence, of the causes, reducing the likelihood of this causes chain to an accident, or even mitigate the unwanted consequences.

There are several approaches to deal with hazard and potential risk during the life cycle of a technical system. One of them is the accident management (or contingence management) is the one that will be worked in this paper. Contingence management is the process that, from a risk analysis, studies the possible scenarios and its consequences. Take actions in order to mitigate the consequences – which can be both in the project phase, relative to the danger condition, or in the barriers between the accident and the consequences. And update the scenarios with every new information. To manage this process, actions had to be taken during the whole life cycle, once every barrier has holes

(Figure 1) and these holes are dynamics. So they can arrange in a way that makes a path to the accident.

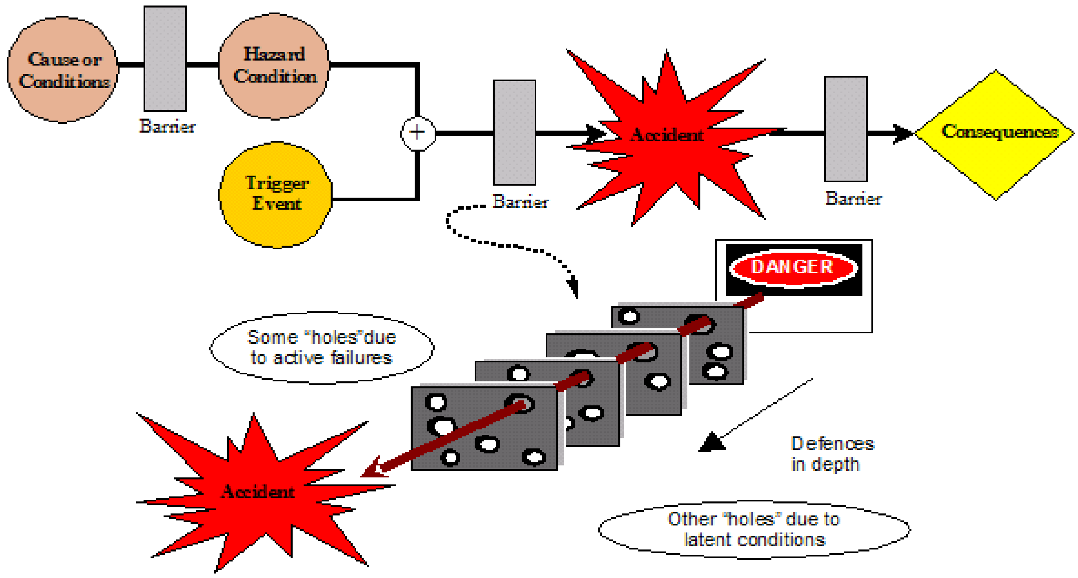

Figure 1 - Causal Chain and Role of Hazard passing through corresponding holes in the layesr of defences, barriesrs and safeguards (adapted from Mosleh et al., 2004 and Reason, 1997)

In order to provide support to the risk analysis, Kumamoto e Henley (1996) proposed a risk. So, every risk "i" would be defined by "Risk ≡ {(L_i, O_i, U_i, CS_i, PO_i) / i=1,.....n}", where:

- L_i (Likelihood) – is the likelihood of occurring the basic events that chain to the outcome;
- O_i (Outcome) – it's the event that is analyzed;
- U_i (Utility / Significance) – utility (which is opposite of Significance) describes the importance of the outcome;
- CS_i (Causal Scenario) – is the design of scenarios that analyses the casual chain of the outcome;
- PO_i (Population) – it's the population affected by the outcome.

Note that, more than the chance of occurring the outcome, the risk profile considers the utility, the population and the scenarios that would make the outcome happens.

The affected Population and Utility determination are very important for the Cost-Risk-Benefits (CRB) analysis, but it's beyond the scope of this paper.

The first step of design scenarios is understand the whole system in order to identify the critical casual chains. For this propose, techniques like Event Tree Analysis (ETA) and Fault Tree Analysis (FTA) can be used. These techniques help to understand causal-effects relationships, identify the probable paths and, than, design scenarios and failure causes deployment. In this analysis, from an initial event, all the possible outcomes are identified. Therefore, it's possible associate a probability to each event and, as consequence, a probability to each outcome, as shown in Figure 2.

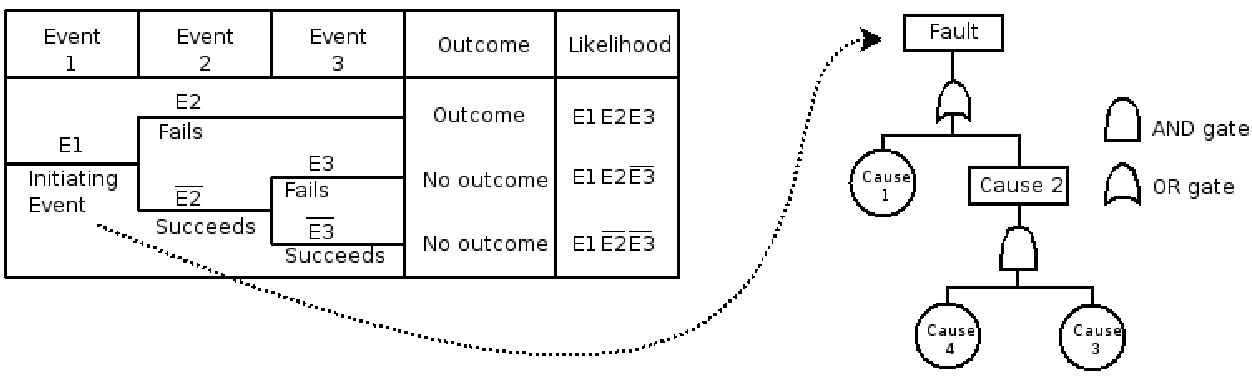

Figure 2 – Design of scenarios using ETA and FTA.

But, how to the probability of each one of the events failure or succeed?

When the event is a component failure and it's possible to make exhausting reliability tests in laboratory, a probability distribution can be fit on these dataset. However, the performance of this component in operation may not be exactly the one observed in laboratory,. Therefore is necessary an update of the probability distribution.

Another situation to be assessed is when is difficult to test the component in laboratory – or when the failure rarely occurs outside it's operational time. In these cases, generally, there is few data about the reliability of the component and make inference using the classic statistic approach would be very risky.

For this kind of situation and for events analysis which subjective judgment must be done, the bayesian approach has shown a better approach.

In this paper will be presented a process to establish the occurrence probability of an event using the bayesian approach. An application in reliability of a pressure reduction system of a natural gas delivery station will be present to illustrate the process. This example, presented in section 4, was design with hypothetical data and will be implemented in the future.

Section 2 presents some considerations about reliability, which are fundamental to understanding the model proposed. The bayesian approach will be presented in section 3. In section 4, it is presented the application example of the bayesian approach in reliability and, finally, in section 5 will be presented the results and final commentaries.

2. A perspective on Reliability

Due to the important role of satisfying consumer needs in the process of product design, quality theory advocates that consumer needs must be captured and understood for the specification of the design process. Considering this process is successful, the product quality can be assessed by the conformance of the products to that specification, which can be described, among other criteria, by the percentage of units that achieved the specification (NIST/SEMATECH, 2003).

The reliability specification of a product concerns the time after manufacturing and release in which the product will remain functioning properly, it is a attribute concerned with failures during the life of a product. Thus, reliability may be defined as the probability of an item to perform a required function under stated conditions for a stated period of time (Dias, 2004).

In practice it is known that if an item (component, subsystem, or system) does not fail within the predicted time, certainly it will fail in the future. The awareness of the fact that ever item will certainly fail in the future is very important for the understanding of the risks associated to technical systems.

Mathematically, the definition of reliability can be obtained from the specification of a statistical ramdom variable related to time "x" to failure – hours or number of cycles – and from its probability of failure density function $f(x)$. The cumulative distribution function $F(x)$ and the reliability function can be derived from $f(x)$ - Figure 4.

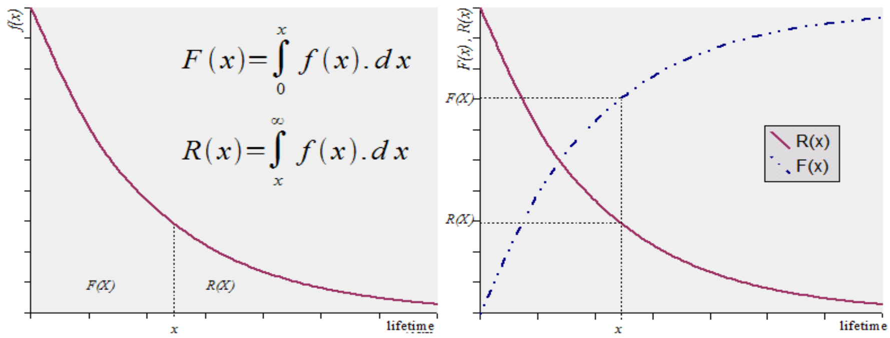

Figure 4 - Exponential probability of failures density function $f(x)$, cumulative distribution function $F(t)$ and reliability function $R(x)$ in terms of lifetime x.

Thus, for an initial life period, $x \to 0$, the cumulative function becomes $F(x \to 0) \to 0$. The reliability in this situation is assured by the quality of all the processes involved in the production of the items. For a very long period of life when x tends to infinity, $x \to \infty$, the failure probability tends to the unity, that is, $F(x \to \infty) \to 1$. The cumulative failure distribution function $F(x)$ increases from zero to unity as the random variable x changes its value from the lower to the higher value (Dias, 2004). The reliability function is complementary to $F(x)$, hence: $R(x)+F(x)=1$.

Concerning probability models of reliability representation, many types of density functions may be used to model failure probabilities, for instance, the Weibull or exponential models. Independent of the model, the characteristic parameter estimation is always necessary. For an exponential distribution, which is the most widely used in reliability assessment (Billinton e Allan, 1983), the parameter θ must be estimated.

The parameter θ is related to the mean life of the item (MTBF[1] or MTTF[2], if the item is not repairable) and can be obtained with life testing, failure database, or even expert opinion.

The central question of reliability assessment in this case is to know the beset value that represents θ. This parameter may be a fixed and deterministic value or can take lower and higher values than the value initially estimated by conventional means (databases and life testing).

In the classic approach θ is a fixed value but, in the bayesian approach, the parameter is treated as a random variable itself, also modeled with a specific probability distribution – as discussed in the next section. The value of θ, in these terms, can be found by an estimator, which is called bayesian point estimator.

1 MTBF – M*ean Time Between Failures*

2 MTTF – *Mean Time To Fail*

3. The bayesian approach

The bayesian approach is based on the known Bayes theorem (Bayes, 1763 apud Gill, 2002) later modified by Laplace, resulting the equation 1 (Kapur e Lamderson, 1977).

$$f(\theta|x)=\frac{f(x|\theta).f(\theta)}{\int_{-\infty}^{\infty} f(x|\theta).f(\theta).d\theta} \quad (1)$$

Another form of presenting the equation above omits the denominator form the right side of the equation, since it does not depend on θ (Gelman et al., 1995).

$$f(\theta|x)\propto f(x|\theta).f(\theta) \quad (2)$$

The equation can be understood as: the *a posteriori* distribution is proportional to the likelihood multiplied by the *a priori* function (Ehlers, 2003).

Thus, the bayesian approach uses previous information, and even subjective judgments, to construct a model of the *a priori* distribution of the parameter. This model is the initial evaluation of the likelihood of the parameter values. The observed data are then used (by the likelihood function) to update the initial evaluation, resulting the so called *a posteriori* model for the population of the parameter being modeled (NIST/SEMATECH, 2003). This process is called "bayesian updating".

It is worth stressing that the *a posteriori* function will become the *a priori* function in the next updating and the same way successively. Hence:

$$f(\theta|x_1)\propto f(x_1|\theta).f(\theta) \quad (3)$$

$$f(\theta|x_1,x_2)\propto f(x_2|\theta).f(\theta|x_1) \quad (4)$$

Thus:

$$f(\theta|x_1,x_2)\propto f(x_2|\theta).f(x_1|\theta).f(\theta) \quad (5)$$

The right side of the equality presents the product of the likelihoods of the first and second updates, the updating process is then independent of the order in which is performed. It is interesting to notice that the *a posteriori* function is obtained by the multiplication of two functions, the likelihood and the *a priori*, which may result a function whose integration does not have an analytical solution. An approach that facilitates the analysis, and avoids numerical integration, is to select conjugated *a prioris*. The result would consist of a *priori* and *posteriori* distributions of the same class.

In the case of the exponential distribution (likelihood) the conjugated function would be the Gama (*a priori*). Other examples of conjugated *a prioris* are: Beta and Binomial; Normal and Normal.

The updating of knowledge about the parameter θ is then given only by the variation of hiperparameters (Ehlers, 2003).

However, for the reliability evaluation of the component it is needed a value for θ. In order to obtain a representative value of θ from its posteriori function, statistics sucha as median a mean can be used. In the case of taking the mean as the statistic, the bayesian point estimator ($\hat{\theta}$) is calculated by (Kapur e Lamderson, 1977):

$$\hat{\theta}=E(\alpha|x)=\int_{-\infty}^{\infty}\theta.f(\theta|x).d\theta \quad (6)$$

4. Example of application

The bayesian approach is modeled in the analysis of an industrial pressure reduction and control system consisted of two control valves assisted by monitoring valves and blocking valves. The systems operates with two reduction subsystems, being one operant and the other in stand-by.

Each subsystem has a control valve which operates in according to a pressure set, when this valve fails, it fails open and passes the pressure control to the monitoring valve, physically disposed in series in the control subsystem. The monitoring valve is always adjusted for a slightly higher value of pressure set and when the control valve fails, the system outlet pressure is reduced to a different level.

If the monitoring valve fails, it also fails open a activates the blocking valve that shuts the operating subsystem and passes the control to the subsystem in stand-by. In the stand-by subsystem a valve operation follows the same logic. If finally all the control valves fail, the blocking valve of the stand-by subsystem shuts the entire system.

Additionally, when a blocking valve fails, these valves fail closed cutting the flow of the subsystem even if the control vales are functioning.

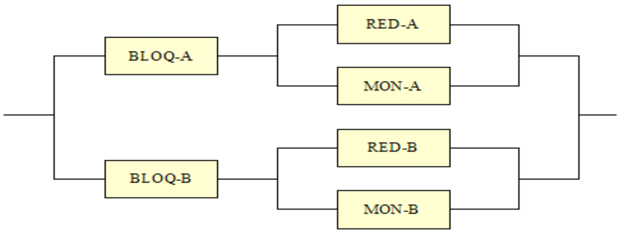

Figure 7 – Reliability Block Diagram of a pressure reduction system.

In reliability terms, the control valves may be understood as parallel components in the same subsystem, and each subsystem is parallel to the other. The blocking valves are component in series with the control valves since a failure in one of these valves shuts the flow of the subsystem. Figure 7 illustrates the reliability block diagram developed for the system.

4.1. Model for bayesian updating of valves in operation

The valves of the main pressure control subsystem operate until failure.

For the modeling in this case, the following considerations are adopted:

- The exponential distribution is adopted to represent the failure probability density function of the component for a life *"x"* and, normal distribution – distribution *a priori* $N(\mu_\theta, \sigma^2)$ – to represent the uncertainty related to the parameter θ (MTBF) of the exponential distribution;
- The maintenance times are neglected.

Thus:

$$f(x)=\frac{1}{\theta}.e^{-\frac{x}{\theta}} \quad (7)$$

$$f(\theta)=\frac{1}{\sqrt{2.\pi.\sigma^2}}.e^{\frac{-(\theta-\mu_\theta)^2}{2.\sigma^2}} \quad (8)$$

This signifies that θ does not have a fixed value, but can take any value with a probability associated. If "*n*" valves have been observed until its respective failures (being $x_1, x_2, \ldots x_n$ the life of each one of them, and $f(x_1)$, $f(x_2)$, ... $f(x_n)$ the respective probabilities of failure occurrence), the following conditional probability distribution is achieved:

$$f(x_1,\ldots,x_n|\theta)=f(x_1|\theta).f(x_2|\theta)\ldots f(x_n|\theta)=\frac{1}{\theta^n}e^{-\frac{\sum x}{\theta}} \quad (9)$$

Where $f(x_1, x_2, \ldots x_n \mid \theta)$ is the probability of finding the lives $x_1, x_2, \ldots x_n$ in the testing given the mean life is θ.

The *a posteriori* distribution of the parameter θ can be obtained by applying equation 1, and becomes the new mean life distribution taking into account the observed data.

$$f(\theta|x_1,\ldots,x_n)=\frac{1}{I}.\theta^{-n}.e^{-\frac{\sum x}{\theta}+\frac{-(\theta-\mu_\theta)^2}{2.\sigma^2}} \quad (10)$$

The bayesian point estimator of θ, acording to equation 6, can be obtained by[3]:

$$\hat{\theta}=\frac{1}{I}.\int_0^\infty(\theta^{1-n}.e^{-\frac{\sum x}{\theta}+\frac{-(\theta-\mu_\theta)^2}{2.\sigma^2}}).\partial\theta \quad (11)$$

Where:

$$I=\int_0^\infty(\theta^{-n}.e^{-\frac{\sum x}{\theta}+\frac{-(\theta-\mu_\theta)^2}{2.\sigma^2}})\partial\theta \quad (12)$$

[3] The equations of the model also adopt the integration limits of the failure equations which are 0 and ∞, and not those expressed in the Bayes equations (1 and 6) which are -∞ and +∞.

4.1.1. Application of the model

Supposing an expert has indicated that the MTBF of the pressure control valve in operation behaves according to a normal distribution (*a priori* distribution) with μ_0= 7,00 [10^3 hours] and σ= 1,50 and, that three failures have occurred during the operation period, as presented in the table below:

Table 1 – Assumed times between failures.

Failure	x_1	x_2	x_3
Time [10^3 hours]	6,00	8,90	7,80

The *a posteriori* distribution can be obtained by equation 10:

$$f(\theta|x_1,\ldots,x_n)=\frac{1}{3{,}95.10^{-4}}.\theta^{-n}.e^{-\frac{\sum x}{\theta}+\frac{-(\theta-\mu_0)^2}{2.\sigma^2}} \tag{13}$$

and the estimator by equation 11:

$$\hat{\theta}=7{,}148\,[10^3\,\text{hours}] \tag{14}$$

It has to be noted that with the classic statistics the MTBF would be the mean of the three times, 7900 hours.

4.2. Model adopted for the bayesian updating of the valves in stand-by

The stand-by valves are checked periodically and in the case of the detection of a failure, corrective measures are taken – the model was elaborated according to Lewis (1987).

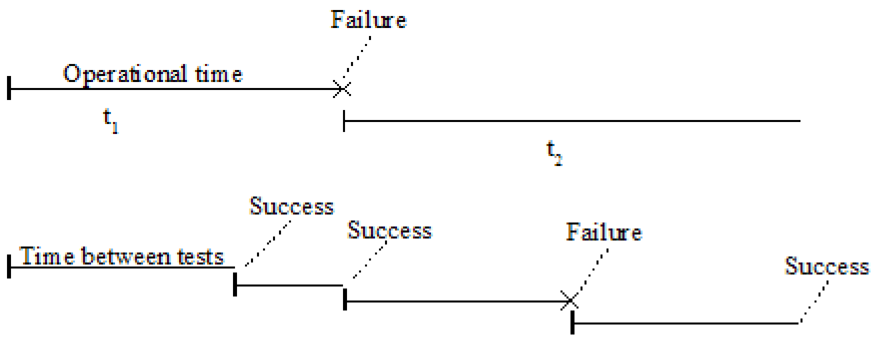

Figure 9 – Model of valves in operation stand-by.

For the modeling of this case the following considerations are adopted:

- The exponential distribution is adopted to represent the failure probability density function of the component for a life *"x"* and, normal distribution – distribution *a priori* $N(\mu_\theta,\sigma^2)$ – to represent the uncertainty related to the parameter θ (MTBF) of the exponential distribution;
- The failures are considered independent, which implies the adoption of a binomial distribution to the representation of the number of failures – *bin(r,n)*, for "*r*" failures in "*n*" items.

Thus:

$$f(x)=\frac{1}{\theta}.e^{-\frac{x}{\theta}} \tag{15}$$

$$f(\theta)=\frac{1}{\sqrt{2.\pi.\sigma^2}}.e^{\frac{-(\theta-\mu_0)^2}{2.\sigma^2}} \tag{16}$$

$$f(r|\theta)=C^{n}_{r}.F(x)^r.[1-F(x)]^{n-r}=C^{n}_{r}.F(x)^r.R(x)^{n-r}=C^{n}_{r}.\left(1-e^{-\frac{x_0}{\theta}}\right)^r.\left(e^{-\frac{x_0}{\theta}}\right)^{n-r}=C^{n}_{r}.\left(1-e^{-\frac{x_0}{\theta}}\right)^r.e^{-\frac{x_0.(n-r)}{\theta}} \tag{17}$$

Considering many testing periods, according to equation 5, the equation above can be expressed as:

$$f(r|\theta)=C\begin{matrix}n_0\\r_0\end{matrix}.\left(1-\mathrm{e}^{-\frac{x_0}{\theta}}\right)^{r_0}.\mathrm{e}^{-\frac{x_0.(n_0-r_0)}{\theta}}\cdots C\begin{matrix}n_j\\r_j\end{matrix}.\left(1-\mathrm{e}^{-\frac{x_j}{\theta}}\right)^{r_j}.\mathrm{e}^{-\frac{x_j.(n_j-r_j)}{\theta}} \qquad (18)$$

Where x_0 is the time between the n_0 tests that resulted in r_0 failures and, x_j the time of the n_j tests and r_j failures. The *a posteriori* distribution can be obtained by substituting equations 18 e 16 in the equation 1 . The result is:

$$f(\theta|r)=\frac{1}{I}.\mathrm{e}^{\frac{-(\theta-\mu_\theta)^2}{2.\sigma^2}}.\prod_{j=1}^{k}\left[\left(1-\mathrm{e}^{-\frac{x_j}{\theta}}\right)^{r_j}.\mathrm{e}^{-\frac{x_j.(n_j-r_j)}{\theta}}\right] \qquad (19)$$

Thus, the bayesian o estimator, according to Equation 6 can be obtained by:

$$\hat{\theta}=\frac{1}{I}.\int_0^\infty \alpha.\mathrm{e}^{\frac{-(\theta-\mu_\theta)^2}{2.\sigma^2}}.\prod_{j=1}^{k}\left[\left(1-\mathrm{e}^{-\frac{x_j}{\theta}}\right)^{r_j}.\mathrm{e}^{-\frac{x_j.(n_j-r_j)}{\theta}}\right].d\theta \qquad (42)$$

Onde:

$$I=\int_0^\infty \mathrm{e}^{\frac{-(\theta-\mu_\theta)^2}{2.\sigma^2}}.\prod_{j=1}^{k}\left[\left(1-\mathrm{e}^{-\frac{x_j}{\theta}}\right)^{r_j}.\mathrm{e}^{-\frac{x_j.(n_j-r_j)}{\theta}}\right].d\theta \qquad (21)$$

4.2.1. Application of the model

Supposing an expert has indicated that the MTBF of the pressure control valve in stand-by behaves according to a normal distribution (*a priori* distribution) with μ_θ= 20,00 [10^3 hours] and σ= 2,00 and, that the valves have had the behavior shown in the table below[4]:

Table 2 - Assumed testes outcome.

Outcome	Test	Test	Test	Failure Subsytem A	Test	Test	Test	Test	Failure Subsytem A	Test	Test	Test	Failure Subsytem A
[10^3 hours]	2,16	4,32	6,48	7,00	9,16	11,32	13,48	15,64	15,90	18,06	20,22	22,38	23,70
Success ou Failure	S	S	S	S	S	S	F	S	S	S	S	S	S

Thus, for:

- x_0 = 2,16 [10^3 hours], it is obtained n_0 = 10 and r_0 = 1;
- x_1 = 0,52 [10^3 hours], it is obtained n_1 = 1 and r_1 = 0;
- x_2 = 0,26 [10^3 hours] it is obtained n_2 = 1 and r_2 = 0; and
- x_3 = 1,32 [10^3 hours] it is obtained n_3 = 1 and r_3 = 0.

The *a posteriori* distribution can then be calculated by equation 1.

$$f(\theta|r)=\frac{1}{0{,}174}.\mathrm{e}^{\frac{-(\theta-\mu_\theta)^2}{2.\sigma^2}}.\prod_{j=1}^{k}\left[\left(1-\mathrm{e}^{-\frac{x_j}{\theta}}\right)^{r_j}.\mathrm{e}^{-\frac{x_j.(n_j-r_j)}{\theta}}\right] \qquad (22)$$

And the estimator by equation 20.

$$\hat{\theta}=20{,}031\,[10^3\,\text{hours}] \qquad (23)$$

It has to be noted that with the classic statistics the MTBF would be 23700 hours since during the entire period only one failure occurred.

[4] The times when the subsystem B was operating have been neglected, that is, the time of maintenance of the subsystem A.

5. Results and final remarks

From equations 19 e 22 it is observed that the *a posteriori* distribution are no long Normal distributions. This occurred because of the fact that conjugated distributions were not used, as discussed in the end of section 3.

Regarding the MTBF estimator, Table 3 presents a comparison of the values obtained by the classic approach and the bayesian approach. The bayesian approach results in different values of MTBF, this difference is a result of the influence of the subjective expert information.

Table 3 – Values of MTBF for classical and bayesian approach.

	Operational valves	Stand-by valves
Classical approach	**MTBF = 7900 hours**	**MTBF = 23700 hours**
Bayesian approach	**MTBF = 7018 hours**	**MTBF = 19841 hours**

The lower the number of experimental data, the greater is the influence of the *a priori* information on the results. This gives to the bayesian model a lower sensibility to new data. The example of the stand by valves can be observed. In the case the next test had failed, the value of the MTBF by the classic statistics would be 11850 hours, while the bayesian approach would result 19841 hours – in this last case the expert opinion (with a expectation of 20000 hours) is influencing the final result. Thus, in the search for better results, the bayesian approach is very important, and in some cases almost inevitable.

"It makes a great deal of practical sense to use all the information available, old and/or new, objective or subjective, when making decisions under uncertainty." (NIST/SEMATECH, 2003). Thus if the information is liable, a lower number of new tests will be needed to confirm the value of the parameter under study (NIST/SEMATECH, 2003).

However, if the *a priori* distribution is inappropriate, the bayesian approach can lead to erroneous results (RAC, 2003), since the results are influenced by the biased *a priori* information. Additionally the public may not accept the validity of the *a priori* information, mainly when this knowledge is originated from subjective expert opinion.

Therefore, the results of the bayesian approach cannot be evaluated alone, but within the framework of the entire analysis. These results can be used in the statistical treatment which aid in the establishment of the Likelihood and the Causal Scenario. The bayesian approach allows the quantification of the various events of the Causal Scenario, stressing the most likely paths to the occurrence of the Outcome. Consequently, it is possible to evaluate which barriers, illustrated in Figure 1, tend to be more effective.

6. Acknowledgements

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