Low-Cost Alternative Paths for Direct Earth-Moon Transfer

Cristiano Fiorilo de Melo
Centro Universitário de Barra Mansa - UBM, CP 85926, Barra Mansa, RJ, 27030-550, Brazil
Programa de Pós-graduação em Engenharia e Tecnologia Espaciais - INPE, CP 515, São José dos Campos, SP, Brazil
e-mail cristianofiorilo@terra.com.br

Othon Cabo Winter
Grupo de Dinâmica Orbital e Planetologia - UNESP, CP 205, Guaratinguetá, SP, 12500-000, Brazil
Programa de Pós-graduação em Engenharia e Tecnologia Espaciais - INPE, CP 515, São José dos Campos, SP, Brazil
e-mail ocwinter@feg.unesp.br

Abstract. *The restricted three-body problem predicts the existence of periodic orbits around the Lagrangian point of equilibrium L1. Considering the Earth-Lunar-probe system, some of these orbits pass very close to the surfaces of the Earth and the Moon. These characteristics make it possible for these orbits, in spite of their instability, to be used in transfer maneuvers between Earth and Lunar parking orbits. The main goal of this article is to explore this scenario, adopting a more complex and realistic dynamical system, the four-body Sun-Earth-Moon-probe problem, to which we have aggregated the eccentricity of the Earth's orbit, and the eccentricity and inclination of the Moon's orbit. We then defined and investigated a set of paths, derived from the orbits around L1, which are capable of achieving transfer between low-altitude Earth and lunar orbits, including high-inclination lunar orbits, at a low cost and with flight time between 13 and 15 days. The high instability of these orbits was also investigated to control the final approach of the probe relative to the Moon through the application of a corrector impulse in the trajectory's apogee.*

Keywords: *Celestial Mechanics, Periodic Orbits and Orbital Maneuvers.*

1. Introduction

After more than a decade without receiving visits from space vehicles, the Moon is once again one of the focal points of space exploration, principally due to the discovery of signs of water ice in its polar regions made by the North American probes Clementine (1994) (Spudis, 2004; and Jeffry, 2000) and Lunar Prospector (1998) (Feldmann *et al.* 1998). At the present time, the European probe SMART 1 (ESA, 2003-2004) is orbiting the Moon, examining its surface with its spectrometers and cameras to confirm this finding. Once the presence of water ice on the lunar poles has been confirmed, our satellite will have an even greater role in the future of space exploration, furnishing water, essential for life and for rocket fuel, thus making the Moon a natural trampoline for future interplanetary trips. In this context, a special type of path, capable of making a transfer between lunar and Earth parking orbits would be of great interest. In this article, we investigated a set of paths capable of making such a transfer at a relatively low cost.

The restricted three-body problem, circular and planar case, applied to the Earth-Moon-particle system, predicts the existence of periodic orbits around the Lagrangian equilibrium point L1. Broucke (1968) called these orbits the G Family. Some of the orbits in this family pass very close to the Earth's and Moon's surfaces, and despite their instability, they may be conveniently used for transfer maneuvers between the Earth and the Moon.

To explore this scenario, we have initially adopted the circular, planar, restricted three-body problem Earth-Moon-probe as a dynamical system to define the set of paths we are interested in. As a criteria to define these paths, we have chosen those that are tangent to terrestrial parking orbits with altitudes between 160 to 60,000 km and lunar parking orbits with altitudes less than 100km. Next, we adopted a more complex and realistic dynamical system, the four-body problem Sun-Earth-Moon-probe, to which we added the eccentricity of the Earth's orbit, and the eccentricity and inclination of the Moon's orbit. Although the set of paths capable of making the transfer continues to exist, in general, the introduction of this new dynamical system "removes" the probe from the plane of its initial orbit, which also corresponds to the Moon's orbital plane. This fact can also be conveniently taken advantage of, since the orbits are unstable, to control final approach and to insert the probe into high inclination lunar orbits.

This article is organized as follows: in the next section, we describe the two dynamical systems utilized in this study. In section 3, we discuss the principal properties of the Family G orbits for the restricted three-body problem and the four-body problem. In section 4, we present the set of paths capable of making the maneuver, through an empirical mathematical expression. In section 5, we present some examples of missions that could be made using these paths, and finally, in section 6, we present our conclusions.

2. Dynamical Systems Considered

2.1. Restricted Three Body Problem (R3BP)

This problem, well known in the literature – see, for example, (Murray and Dermott, 1999), considers three bodies, m_1, m_2 and m_3, with m_3 being a particle with negligible mass that does not influence the other two bodies, who have preponderant mass and are called primary. These, in turn, have circular, co-planar orbits around the center of mass that is common to both; they maintain a constant distance from each other, and also have the same angular velocity in relation to this center of mass. Due to these characteristics, it is useful to study the R3BP adopting a system of references whose origin is fixed in the center of the mass common to the primary bodies, with axes x and y rotating with the same angular velocity as the first bodies. This system is called a barycentric rotating or synodic system, and in it, bodies m_1 and m_2 remain fixed over the x-axis, while m_3 moves on the xy-plane. The system is normalized, considering its reduced mass, μ, as unitary mass, that is, $\mu = \mu_1 + \mu_2 = G(m_1 + m_2) = 1$, where G is the universal gravitational constant. The constant distance between the mass m_1 and m_2 is also considered equal to 1. Thus, the coordinates of m_1 and m_2 are $(-\mu_2, 0)$ and $(\mu_1, 0)$, respectively. The equations of motions for the third body in the synodic system are:

$$\ddot{x} = 2\dot{y} + x - \left[\mu_1 \frac{(x-\mu_2)}{r_{13}^3} + \mu_2 \frac{(x-\mu_1)}{r_{23}^3} \right]$$
$$\ddot{y} = 2\dot{x} + x - \left[\frac{\mu_1}{r_{13}^3} + \frac{\mu_2}{r_{23}^3} \right] y \tag{1}$$

with

$$r_{13}^2 = (x+\mu_2)^2 + y^2 \tag{2}$$

$$r_{23}^2 = (x-\mu_1)^2 + y^2 \tag{3}$$

where, considering the Earth-Moon-probe system, $\mu_1 = \mu_{Earth} = 0.9878494$ and $\mu_2 = \mu_{Moon} = 0.0121506$ are the mass parameters for the Earth and the Moon, respectively, r_{13} is the Earth-probe distance and r_{23} is the Moon-probe distance. The Equations (1) do not have an analytical solution, but they do have symmetry properties that guarantee the existence of periodic orbits in the synodic system (Broucke, 1968; and Murray and Dermott, 1999), such as the G Family orbits that we investigated in this study.

The R3BP also has another interesting property, demonstrated by Lagrange, which corresponds to the existence of five equilibrium points called Lagrangian equilibrium points, symbolized by the letter L. When a particle is placed on these points with a null velocity in relation to the origin of the synodic system, it remains in them indefinitely. The "Figure 1" illustrates this synodic system and the relative location of the five Lagrangian equilibrium points.

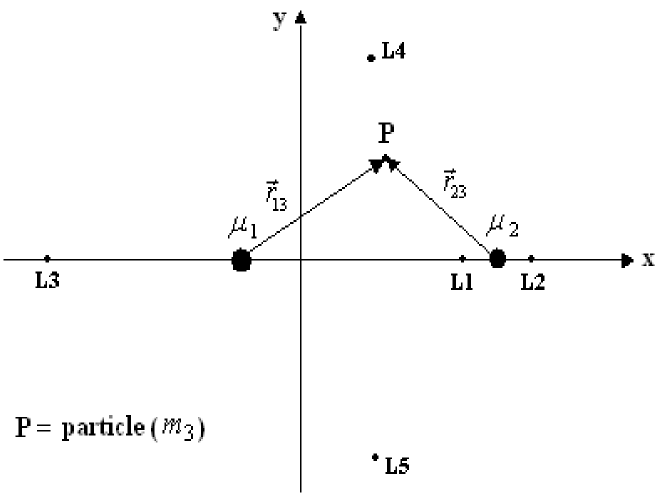


Figure 1. Synodic reference system and the qualitative and relative location of the Lagrangian equilibrium points.

2.2. Four-body Problem

In our numerical simulations, we also considered as dynamical system the four-body problem in three-dimensional space. Thus, for a system of fixed Cartesian coordinates, in which the position of a certain body is given by the vector $\boldsymbol{x}_{ki} = (x_{k1}, x_{k2}, x_{k3}) \in \Re^3$, the equations of motion are given by:

$$\ddot{x}_{ki} = \sum_{\substack{j=1 \\ j \neq k}}^{4} \frac{\mu_j}{r_{jk}^3} (x_{ji} - x_{ki}) \tag{4}$$

where k = 1,2,3,4; and

$$r_{jk} = |\boldsymbol{x}_j - \boldsymbol{x}_k| = \left[\sum_{i=1}^{3} (x_{ji} - x_{ki})^2 \right]^{1/2} \tag{5}$$

is the distance between the k-th and the j-th bodies, μ_j is the mass parameter for the j-th body and with i = 1,2,3 denoting the three coordinates of the fixed Cartesian system. The "Eq. (4)" represents 12 differential equations of the second order and expresses the fact that acceleration of a given body is the result of the sum of the forces exercised by the other three bodies. This means that the "Eq. (4)" takes into account the mutual interactions between the four bodies in the system. Now, if we associate indexes 1 to the Sun, 2 to the Earth, 3 to the Moon and 4 to the probe, and still consider that the system is normalized based on the Earth's and Moon's mass, so that $\mu_2 + \mu_3 = 1$, then we will have μ_1 = 328904.4747; μ_2 = 0.987849396 and μ_3 = 0.012150604. The probe's mass was assumed to be equal to 307kg, the same as the lunar probe of the European Space Agency, SMART 1, launched in August 2003. In this manner, $\mu_4 = 5.0763\text{x}10^{-23}$, and even with "Eq. (4)" taking into account the mutual interactions between the four bodies considered, the probe would not influence the motion of the other three. For this reason, the terms of "Eq. (4)", which contains μ_4, were suppressed. This is the four-body Sun-Earth-Moon-probe problem, and the word restricted could precede it, without a loss of generality, given the order of the size of μ_4. The aforementioned normalization is completed adopting the average distance between the Earth and the Moon, 384400km, as a unit of measurement.

The eccentricity of the Earth's orbit, e_2, the eccentricity, e_3, and the inclination, i_3, of the Moon's orbit were included in the system via initial conditions, thus bringing it ever closer to reality. The values for these elements are: e_2 = 0.0167; e_3 = 0.0549 and $i_3 = 5.1454^o$ (relative to the ecliptic).

3. Properties of the G Family Orbits

Considering the R3BP, the G Family generally has short range orbits around L1, long range orbits that pass a few kilometers from the Earth's surface and a few dozen kilometers from the Moon's surface, and even orbits that present a loop. The two first kinds of orbits have initial conditions of the following type:

$$(x_0, 0, 0, \dot{y}_0) \tag{6}$$

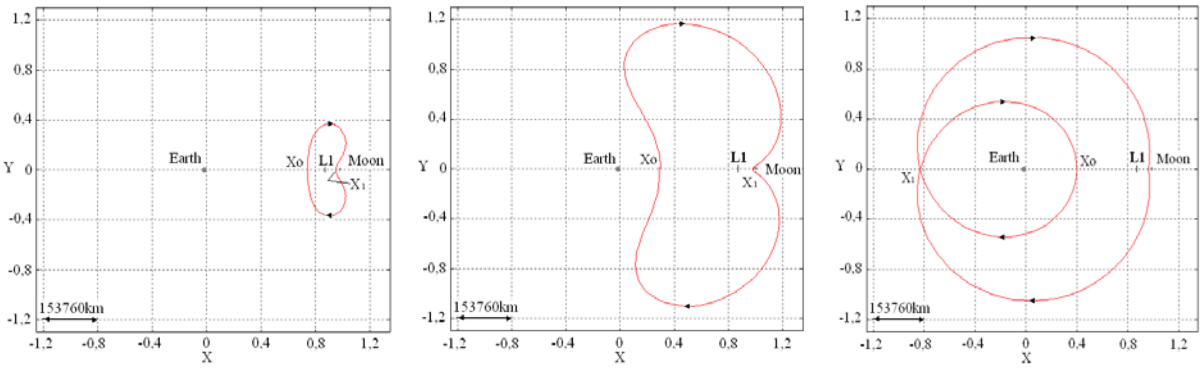
While the third kind has initial conditions of the following type:

$$(x_0, 0, 0, -\dot{y}_0) \tag{7}$$

The "Figure 2" exhibits an example of each type, seen in the synodic system. We can observe that for the first two types of orbits, point x_0 is between the Earth and L1; and point x_1, which corresponds to the first passage by the particle along the x-axis of the coordinate system under consideration, is located between L1 and the Moon. For the third type, x_0 is also between the Earth and L1, and x_1 is located to the left of the Earth, between it and L3. All three types of orbits are unstable and a large number of them diverge before completing the first period.

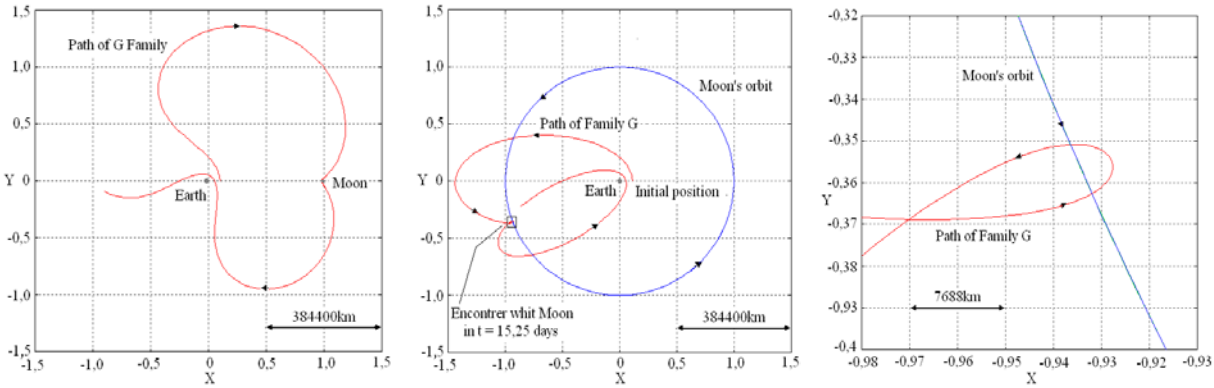
The types of paths illustrated in "Fig. 2.b" are those paths capable of making a direct transfer between the Earth and the Moon. "Figure 3" exhibits, for the restricted three-body problem, a path of this type. In "Fig. 3.a", it is seen in the synodic system, in "Fig. 3.b", in the geocentric system, together with the Moon's orbit, and in "Fig. 3.c", we have a zoom showing a loop given by the path of the Moon's orbit. The "Figure 4" shows, for the four-body problem, the orbit obtained with the same initial path conditions as "Fig. 3". Note that with the final approach to the Moon, the probe leaves the Moon's orbital plane. This fact allows the probe to be inserted into a highly inclined lunar orbit.

Note that in the initial conditions (6) and (7), the particle is always on the x-axis, between the Earth and the Moon, in t = 0. In this manner, considering a probe in a terrestrial parking orbit, the injection impulse to acquire a transfer path could only be applied when the Earth, Moon and probe were all lined up, in this order. For example, for a terrestrial parking orbit with an altitude equal to 200km, there would be a launching window open every 1.47h, a timeframe which is not restrictive to using these orbits for transfer maneuvers between the Earth and the Moon, even when the eccentricity of the Moon's orbit is considered.



(a) (b) (c)

Figure 2. G Family periodic orbits obtained considering the R3BP: (a) short range, and (b) long range with x_0 between the Earth and L1 and with x_1 between L1 and the Moon. (d) Orbit that have loops with x_0 between the Earth and L1, and with x_1 between the Earth and L3.



(a) (b) (c)

Figure 3. Unstable path of the G Family and Moon's orbit (R3BP): (a) Earth-Moon rotating Barycentric System (Synodic system), (b) Geocentric System, (c) zoom showing the paths of the probe and the Moon in the encounter region.

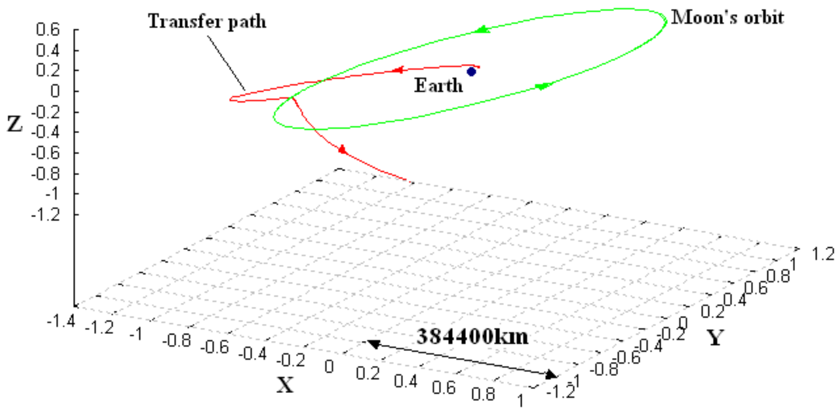


Figure 4. Path of transfer spatial view in the geocentric system obtained considering the four-body problem Sun-Earth-Moon-probe.

4. Definition of Set of Unstable Transfer Paths

In order to select the paths capable of achieving transfer between a terrestrial and lunar parking orbits, we consider that the probe depart always of a circular orbit around the Earth. We consider altitudes for this orbit varying between 160 and 60,000 km. Then, we selected only those paths that reach the perilune with altitudes between 0 and 100km. This procedure us permitted to find a graphical relation between the injection speed for acquisition of a transfer path, V_I, and the terrestrial circular parking orbit, H_T, for transfer path's perilune altitude, H_L, less than 100km from Moon's surface, including collision paths. But if we consider $160 \le H_T \le 700$km, an empirical mathematical expression that relates V_I and H_T for $H_L \le 100$km can be write. This mathematical expression is:

$$V_I = -\ 0.008380 H_T + 11{,}1010000 + \Delta \tag{8}$$

The interval $160 \le H_T \le 700$km was chosen taking a count the capacity of present launch vehicles. The set of transfer paths can be seen in "Figure 5.a", and the set defines by "Eq. (8)" in "Fig. 5.b", which is a zoom of "Fig. 5.a" for $160 \le H_T \le 700$km, note that this interval, we have a black band defining the region and not a line, this fact justifies the last term, Δ, in "Eq. (8)". The "Figure 6" shows a zoom of the diagram in "Fig. 5.b" for $240 \le H_T \le 245$km. In this Figure it is possible to verify the extreme sensitivity of perilune altitude with injection speed in achieving the transfer path. This structure exists in all the intervals studied, and can be expressed taking into account small variations in Δ. These variations are presented in table 1.

Table 1. Relation between Δ and perilune altitude.

Interval of variation of Δ in km/s	Altitude of perilune in km
$\lvert\Delta\rvert \le 1.7\text{x}10^{-4}$	100 e 1000
$\lvert\Delta\rvert \le 2.3\text{x}10^{-4}$	1000 e 2000
$\lvert\Delta\rvert \le 2.8\text{x}10^{-4}$	2000 e 3000
$\lvert\Delta\rvert \le 3.4\text{x}10^{-4}$	3000 e 5000
$\lvert\Delta\rvert \le 6.1\text{x}10^{-4}$	5000 e 10000

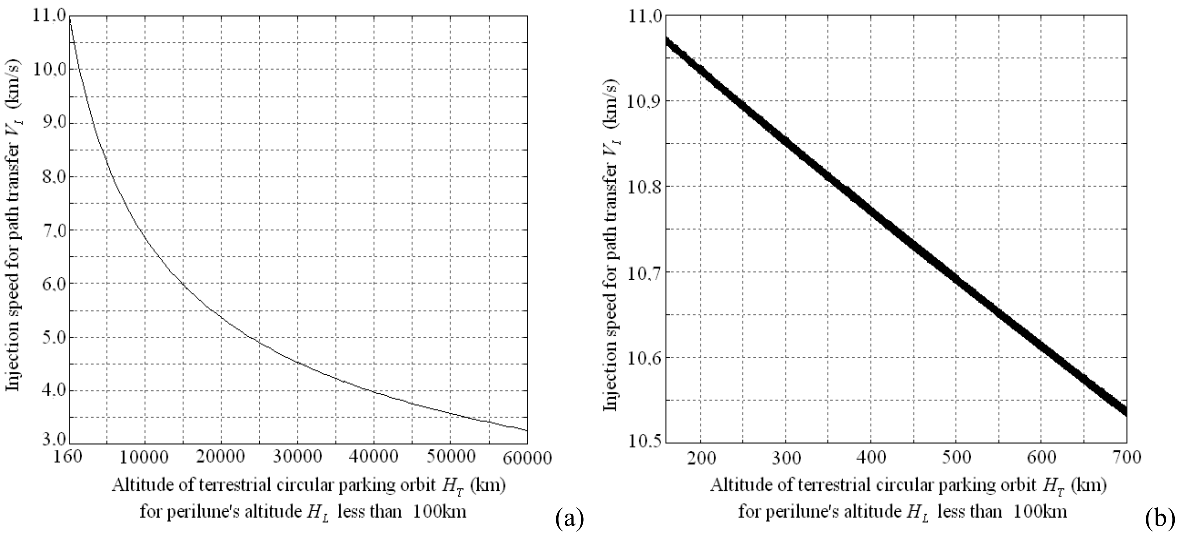


Figure 5. Injection speed, V_I, versus the altitude of the earth's parking orbit, H_T, to paths with pelrilune, H_L, bellow 100km. (a) $160 \le H_T \le 60{,}000$ km and (b) $160 \le H_T \le 700$km.

5. Mission Flexibility

5.1. Direct Transfer

"Equation (8)" defines the set of paths capable of making a direct transfer between a terrestrial parking orbit, with an altitude of $160 \le H_T \le 700$km, and a lunar parking orbit, with an altitude of $0 < H_L \le 100$km, with only two impulses. The ΔV_1 required for the first impulse, the injection impulse that will place the probe into the transfer path, is the difference between V_I and the velocity of the terrestrial circular parking orbit, that is:

$$\Delta V_1 = V_I - \sqrt{\frac{\mu_2}{R_1}} \tag{9}$$

where $R_I = R_T + H_T$, with R_T being the Earth's average radius (6370km). The ΔV_2 required for the second impulse, the insertion impulse into the lunar orbit, will correspond to the difference between the velocity of the probe in the perilune of the transfer path, V_P, and the velocity of the lunar orbit desired at the point of its application. Supposing that the desired orbit around the Moon is circular, we have:

$$\Delta V_2 = V_P - \sqrt{\frac{\mu_3}{R_P}} \tag{10}$$

The value for V_P is found through numerical integration, $R_P = R_L + H_L$, with R_L being the average radius of the Moon (1738km). In the perilune of the transfer path, the probe's velocity is always perpendicular to the Moon-probe vector position, which facilitates the insertion into a circular lunar orbit. As an example, we can imagine a transfer between a terrestrial circular parking orbit with an altitude of H_T = 200km. According to the "Eq. (8)", the value of V_I (200km) = 10,930km/s. These initial conditions are integrated, considering the four-body Sun-Earth-Moon-probe problem, with the addition of the eccentricity of the Earth's orbit, the eccentricity and inclination of the Moon's orbit. The path defined by these values of H_T and V_I is capable of taking the probe to 37.8km from the lunar surface in conditions to insert it in a circular orbit with an inclination of 148.5° and Ω = 20.34°. The magnitude of the ΔVs are: ΔV_I = 3.135km/s, ΔV_2 = 0.915km/s and ΔV_{Total} = 4.050km/s. The flight time is 13.95 days. With respect to the cost of the maneuver, in direct comparison with conventional methods, which consider the dynamics of only two bodies, like Hohmann (ΔV_{Total} = 3.970km/s) and Patched-conic (ΔV_{Total} = 4.250km/s), we can observe that the unstable orbits have a ΔV_{Total} that is quite close to that Hohmann's, nonetheless, simple numerical simulations considering the restricted three and four-body problems, for example, show that Hohmann's ΔV_{Total} is not sufficient to conclude the maneuver, requiring additional ΔVs for this purpose. With relation to Patched-conic, the transfer via unstable orbit offers savings of some 5%, but this number could be greater, since this method also makes use of the two-body dynamic, however, in a more refined way. With respect to flight time, the conventional methods are unbeatable, with 4.9 days for Hohmann and 5.0 days for Patched-conic, but a flight time between 13 and 15 days could be acceptable for logistical missions and even for some manned missions, given the savings observed.

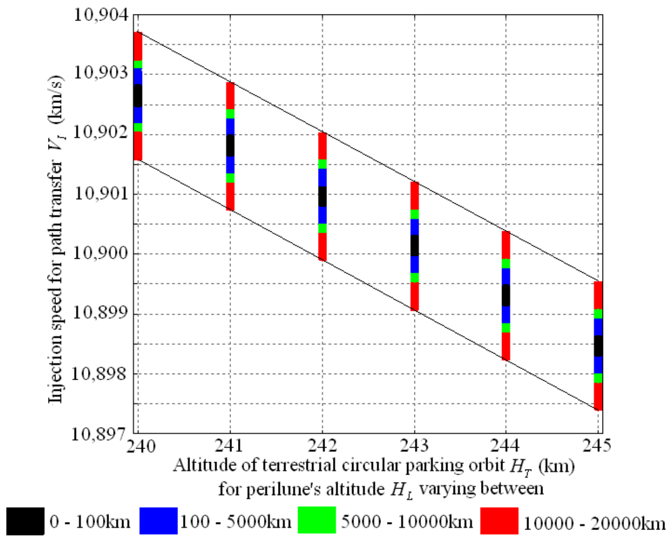


Figure 6. Injection speed, V_I, versus the altitude of the earth's parking orbit, H_T, to paths with various altitude of pelrilune, H_L, indicated of colors code.

5.2. Guided Transfer

The instability inherent with the orbits herein studied, associated with the increase in inclination during the final approach with Moon, allows us to guide the probe by applying a small corrector impulse during the apogee of the transfer path. The direction and the magnitude of the corrector impulse are calculated numerically according to the parameters of the initial terrestrial orbit and the final lunar orbit desired. To illustrate, let us imagine that leaving the Earth's orbit from an altitude of H_T = 200km discussed in the previous subsection, we want to insert a probe into a circular lunar orbit with an altitude of $H_L \leq$ 100km and with inclination greater than or equal to 80°. The diagrams in "Figure. 7" correspond to the numerical results obtained based on integration of the terrestrial parking orbit for H_T = 200km, V_I = 10.930km/s and considering as dynamical system, the four-body Sun-Earth-Moon-probe problem, with the addition of the eccentricity of the Earth's orbit, the eccentricity and inclination of the Moon's orbit. The diagram in "Fig. 7.a" shows that the required ΔV_{Total}, already taking into account the corrector impulse, to insert the probe into a lunar orbit with $i \geq 80^{\circ}$ varies between 4.100km/s and 4.117km/s. Discounting $\Delta V_1 = 3.135$km/s from this number, we have:

$$0.965 \leq \Delta V_{corrector} + \Delta V_2 \leq 0.982\text{km/s} \tag{11}$$

The diagram in "Fig. 7.b" shows that the $\Delta V_{corrector}$ capable of making it possible for the probe to be inserted into a lunar orbit with $i \geq 80^{\circ}$ varies between 0.024km/s and 0.031km/s. If we opt for $\Delta V_{corrector} + \Delta V_2 = 0.965$km/s, and, observing the diagram in Fig. 7.c, that for ($\Delta V_{Total} = 4{,}100$km/s) we find that $\Delta V_2 = 0.935$km/s, we can then conclude that insertion into a circular lunar orbit with $i \geq 80^{\circ}$ is thus obtained:

$$\begin{aligned} \Delta V_1 &= 3.135\text{km/s} \\ \Delta V_{corrector} &= 0.030\text{km/s} \\ \Delta V_2 &= 0.935\text{km/s} \end{aligned} \tag{12}$$

The straight ascension of the ascending node and the altitude of the final orbit around the Moon can be obtained from the diagrams in Figures 7.d and e. Figure 8 illustrates the final orbit around the Moon for $i = 80^{\circ}$ and $\Omega = -75^{\circ}$ (105^{o}), H_L = 60km and the transfer path on the final approach.

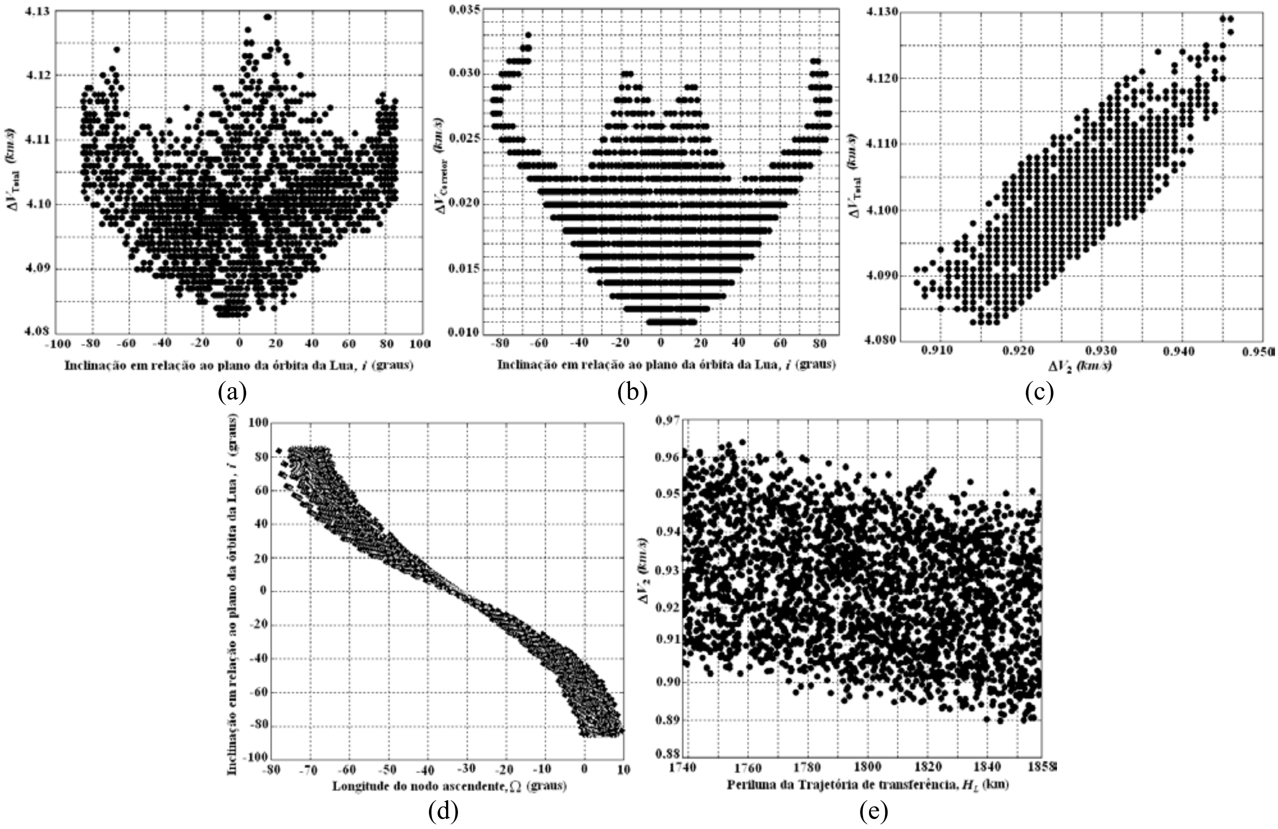


Figure 7. (a) Diagram ΔV_{Total} versus inclination of probe's path in relative to the Moon's orbital plane, (b) Diagram $\Delta V_{\text{corrector}}$ versus inclination, (c) Diagram $\Delta V_{\text{Corretor}}$ versus ΔV_2 , (d) Diagram ΔV_2 versus H_L.

6. Conclusions

The results of this study allow us to affirm the existence of a well defined set of paths that are capable of making a direct transfer between a terrestrial parking orbit and a lunar parking orbit, both low altitudes orbits and at a low cost. This set of paths can be expressed using an empirical mathematical expression, "Eq. (8)", for $160 \leq H_T \leq 700$km, and by the curve in "Fig. 5.a". With respect to the cost of the maneuver via an unstable orbit, it is some 0.010km/s (10m/s) greater than those obtained by Hohmann, but as mentioned in subsection 5.1, the ΔV's of Hohmann's transfer are obtained via a two-body dynamic, and are insufficient to complete the entire maneuver, requiring additional ΔV's. Comparing the transfer cost via unstable orbits with the cost using Patched-conic orbits, also obtained from a two-body dynamic, the gain is around 5%.

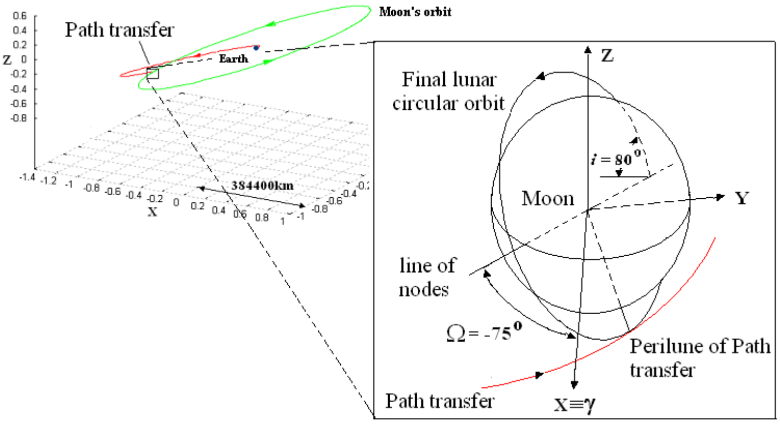


Figure 8. Path of transfer spatial view in the geocentric system shows, in zoom, the final circular orbit around the Moon

With relation to flight time, a transfer via an unstable orbit takes from 13 to 15 days, from 8 to 10 days more than the time required for conventional transfers, however, the savings provided may be convenient in some logistical missions, or even for some manned missions. On the other hand, comparing flight time via an unstable orbit with the time required by the methods that use the gravitational capture phenomenon (Koon *et al.* 2000 and 2001; Belbruno and Carrico, 1996; Miller and Belbruno, 1991; Winter *et al.* 2003; and Yamakawa, 1992), which can run into months, transfers via unstable orbits are much faster, although they are slightly more expensive. Nevertheless, the great majority of studies in the literature on the phenomenon of gravitational capture for Earth-Moon transfer consider the dynamical system to be the restricted, circular, planar, three-body problem, while we considered, in this study, a more complex and realistic dynamical system, based on the three-dimensional four-body Sun-Earth-Moon-probe problem, to which was added the eccentricity of the Earth's orbit, the eccentricity and inclination of the Moon's orbit.

Finally, we would like to point out that the exit of the probe from the Moon's orbital plane, in its final approach, associated with the instability of the set of paths herein defined, can be conveniently exploited to insert probes in highly inclined lunar orbits, and as we have seen, without a large increase in the final cost of the maneuver.

7. Acknowledgements

The authors are grateful to the Foundation to Support Research in the São Paulo State (FAPESP) and to CNPq (National Council for Scientific and Technological Development) – Brazil.

8. References

Spudis, P. D., 2004, Scientifc American, "Mistérios ainda não decifrados da Lua", No. 20 Brasil.
Jeffry, T. G., august 2000, Planetary Science Research Discovery, "A New Moon for the Twenty-first century".
Feldmann, W. C. et al, 1998, Science, "Major Compositional Units of the Moon: Lunar Prospector Thermal and Fast Neutrons", 281, 1496-1504.
http://www.esa.int/SPECIALS/SMART-1
Broucke, R. A., 1968, Report 32-1168, JPL, "Periodic Orbits in the Restricted Three-Body Problem With Earth-Moon Masses".
Murray, C. D and Dermott, S. F., 1999, Cambridge University Press, "Solar System Dynamics".
Bate, R. R., Mueller, D. D., and White, J. E., 1971, "Fundamentals of Astrodynamics", Dover, New York.
Koon, W. S., Lo, M. W., Marsden, J. E. E Ross, S. D., 2000, "Constructing a Low Energy Transfer Between Jovian Moons".
Koon, W. S., Lo, M. W., Marsden, J. E. E Ross, S. D., 2001, "Low Energy Transfer the Moon", Kluwer Academic Publishers, Netherlands.
Belbruno, E. A. and Carrico, J. P., 2000, "Calculation of weak Stability Boundary Ballistic Lunar Transfer Trajectories", AIAA-2142.
Miller, J. K., Belbruno E. A., 1991, "A Method for the Construction of a Lunar Transfer Trajectory Using Ballistic Capture", AAS-91-100.
Winter O. C., Vieira Neto, E. and Prado, A. F. B. A., 2003, "Orbital Maneuvers Using Gravitational Capture Times", COSPAR 2003, Vol 31, No. 8, pp. 2005- 2020.
Yamakawa. H., 1992, "On Earth-Moon Transfer Trajectory with Gravitational Capture", PhD Dissertation, University of Tokyo.

9. Responsibility notice