

# ROTOR BALANCING IN NON-STATIONARY CONDITION BY THE USE OF ADAPTIVE FILTERING

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**Abstract.** *An adaptive filter is proposed for the identification and tracking mechanical system parameters. In this work, the adaptive filter method is applied to identify the rotor unbalance parameters in non-stationary condition, as find in run-up. The RLS adaptive filter is used to identify the modal parameters and unbalance, amplitude and angular position. Simulations and experimental analysis show that the balancing procedure can efficiently determine the unbalance and the modal parameters, like natural frequency and damping ratio.*

**Keywords** *Rotordynamics, Balancing, Non-stationary, Adaptive Filter, Recursive Least Square*

## 1. Introduction

Rotating machines are certainly among the most important equipment in the industrial area. Machines like generators, turbines, compressors, and pumps are, in many cases, the heart of the process they are involved in. For this reason (and many others), these machines require and deserve special attention of the manufacturers and maintenance people in order to guarantee their well functioning and availability.

The present work deals with one of the most common vibration problems in rotor dynamics: the unbalance excitation, which can be the reason for fail and defects, as the shaft fatigue.

Rotor balancing has been extensively studied for many years, usually, using influence coefficient or modal methods. These procedures requires, in many cases, a trial mass and/or the operation of the machine near to the rotor critical speed. These procedures are either time consuming, due to the use of trial mass, and inappropriate, due to the high level of vibration experienced by the rotor when it operates near the critical speed.

To avoid these problems, non-stationary techniques have been implemented in the last decades, Markert (1984 and 1988). These papers investigate the unbalance identification using the measurements performed during a single run-up, fast enough to avoid hazardous vibration levels, using tools as Fourier Transform and Modal Analysis. A similar approach was used in Menz and Gasch (1995) and Seidler and Markert (1999) for rigid and flexible rotors in time and frequency domain estimation.

These works used the Least Square (LS) identification method to estimate the unbalance parameters, which are time invariant. The method can be extended to a recursive form to identify parameters at each time instant. This approach enables the analysis and control of the balancing, for example, in real time, as proposed in Zhou and Shi (2000, 2001a and 2001b).

In this work, the application of adaptive methods for balancing rotors in non-stationary condition is proposed, through the use of adaptive filter theory. The Recursive Least Square (RLS) technique is used due to the facility of implementation and good precision, when compared with other filters like LMS (Least Mean Square) and Kalman Filters.

## 2. Adaptive Filter

Adaptive techniques have been employed very successfully in parameter identification since 60's, in areas such as sonar, radar, communication, control, image processing, adaptive antenna, noise cancellation, etc. These methods are based on signal processing algorithms and built in recursively.

System identification by adaptive algorithms is based on models and the closer the model approaches the real system the better the solution will be. The estimation of time invariant parameters means to track over a constant error surface, but if the system is time variant the surface changes. In this case, this analysis requires that the computational

method finds and tracks the minimum error point continually. To accomplish this, the model (this includes the numerical implementation) should be faster than the real system or should have equal velocity.

The RLS algorithm is based on the Least Square theory (Haykin, 1996). In the recursive form, the filter uses the whole information since the beginning of the measurements, what increases the rate of convergence compared to other algorithms as LMS, but also increases the computational complexity.

The problem is analyzed by the minimization of a cost function by Least Square. This cost function is a sum of the squared error vector, defined as the difference between the measured response ( $d$ , desired signal) and the filter estimation ( $\{w\}^H \{u\}$ ), as showed in the following equation:

$$\varepsilon(n) = \sum_{i=1}^n \lambda^{n-i} |e(i)|^2 \quad \text{and} \quad e(i) = d(i) - y(i) = d(i) - \{w(n)\}^H \{u(i)\}, \quad (1)$$

where  $\{w(n)\}$  is the filter coefficients and  $\lambda$  is the forgetting factor. This forgetting factor allows the filter to reduce the initial measurements importance for non-stationary environments. When its value is equal to one, the filter has an infinite memory and it becomes equal to the Least Square method.

The minimization of the error vector,  $e(n)$ , results in,

$$[\Phi(n)]\{\hat{w}(n)\} = \{z(n)\}, \quad (2)$$

where  $[\Phi(n)] = \sum_{i=1}^n \lambda^{n-i} \{u(i)\}\{u(i)\}^H$  is the autocorrelation matrix and  $\{z(n)\} = \sum_{i=1}^n \lambda^{n-i} \{u(i)\}d^*(i)$  is the cross-correlation vector between the filter input,  $\{u(i)\}$ , and the desire signal,  $d(i)$ . Eq. (2) is called deterministic normal equation.

The determination of the recursive parameters can be expressed by the following equation, using the matrix inversion lemma for the autocorrelation matrix:

$$\{\hat{w}(n)\} = \{\hat{w}(n-1)\} + \{k(n)\} \left( d^*(n) - \{u(n)\}^H \{\hat{w}(n-1)\} \right), \quad (3)$$

where the filter gain vector is  $\{k(n)\} = [\Phi(n)]^{-1} \{u(n)\}$  and the initial condition for the inverse of autocorrelation matrix is  $[P(0)] = \delta^{-1} [I]$ ,  $\delta$  is a positive small constant number. The filter requires an initial condition for each tracked parameter and the computation sequence can be written as:

- determination of the filter gain  $\{k(n)\} = \frac{\lambda^{-1} [P(n-1)] \{u(n)\}}{1 + \lambda^{-1} \{u(n)\}^H [P(n-1)] \{u(n)\}}$ ;
- estimation of the a priori error  $\xi(n) = d(n) - \{\hat{w}(n-1)\}^H \{u(n)\}$ ;
- updating of the parameters  $\{\hat{w}(n)\} = \{\hat{w}(n-1)\} + \{k(n)\} \xi^*(n)$ ;
- and calculation of the inverse of the autocorrelation matrix,  $[P(n)]$ ,  $[P(n)] = \lambda^{-1} [P(n-1)] - \lambda^{-1} \{k(n)\} \{u(n)\}^H [P(n-1)]$ .

### 3. Non-Stationary RLS Balancing

Consider a rotor modeled by a second order differential equation (equation of motion) and excited by an unbalance with amplitude  $u$  – which is the product of the unbalance mass and its eccentricity – and angular position  $\alpha$ . Defining  $\theta$  as the angular position of the rotor and using complex coordinates defined as  $\{z\} = \{x\} + j \{y\}$ , the equation of motion can be written as:

$$[M]\{\ddot{z}(t)\} + [C]\{\dot{z}(t)\} + [K]\{z(t)\} = - \left\{ u \frac{d^2}{dt^2} \left( e^{j(\theta(t)+\alpha)} \right) \right\}. \quad (4)$$

The right hand side of eq. (4) is the unbalance force, which is the second time derivative of the term  $e^{j(\theta(t)+\alpha)}$ . The physical parameters are the mass matrix,  $[M]$ , the proportional damping matrix,  $[C]$ , and the stiffness matrix,  $[K]$ .  $\{x\}$  and  $\{y\}$  are the displacements vectors of the rotor in two perpendicular directions. The vibration displacement,

velocity and acceleration (in complex coordinates) are, respectively,  $\{z\}$ ,  $\{\dot{z}\}$  and  $\{\ddot{z}\}$ . The gyroscopic effect is considered negligible.

Using the Euler relation, the complex exponential can be expressed as a function of sine and cosine terms, and eq. (4) becomes,

$$[M]\{\ddot{z}(t)\} + [C]\{\dot{z}(t)\} + [K]\{z(t)\} = \left\{ u \left( \ddot{\theta}(t) \sin(\theta(t) + \alpha) + \dot{\theta}(t)^2 \cos(\theta(t) + \alpha) \right) + \left( \dot{\theta}(t)^2 \sin(\theta(t) + \alpha) - \ddot{\theta}(t) \cos(\theta(t) + \alpha) \right) j \right\} \quad (5)$$

The complex eq. (5) can be split into its real and imaginary parts. Using the definition of the complex coordinates  $\{z\}$  and the real part of eq. (5), one can easily obtain the following equation:

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \left\{ u \left( \ddot{\theta}(t) \sin(\theta(t) + \alpha) + \dot{\theta}(t)^2 \cos(\theta(t) + \alpha) \right) \right\}, \quad (6)$$

or, developing the right-hand-side,

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \left\{ u \cos(\alpha) \right\} \left( \ddot{\theta}(t) \sin(\theta(t)) + \dot{\theta}(t)^2 \cos(\theta(t)) \right) + \left\{ u \sin(\alpha) \right\} \left( \ddot{\theta}(t) \cos(\theta(t)) - \dot{\theta}(t)^2 \sin(\theta(t)) \right) \quad (7)$$

Using the modal matrix,  $[\Phi]$ , and the coordinates transformation equation,  $\{x\} = [\Phi]\{q\}$ , the equation of motion (7) can be expressed in terms of the generalized coordinates  $\{q\}$ . It is worth noting that the matrix  $[\Phi]$  must be known in advance to apply this procedure. Considering the assumptions stated before (regarding to the damping model and the gyroscopic effect) and the orthogonality properties of the modal vectors, eq. (7) can also be written as a function of the modal parameters: mode shapes  $[\Phi]$ , natural frequencies,  $\omega$ , and damping ratios,  $\zeta$ :

$$\begin{aligned} \begin{bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \\ \vdots \end{bmatrix} + \begin{bmatrix} 2\zeta_1 \omega_1 & 0 & & \\ 0 & 2\zeta_2 \omega_2 & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \vdots \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 & & \\ 0 & \omega_2^2 & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \end{bmatrix} = \\ = ([\Phi]^T [M] [\Phi])^{-1} [\Phi]^T \begin{bmatrix} \ddots & 0 & 0 & \\ & u_i \cos(\alpha_i) & u_i \sin(\alpha_i) & \\ & 0 & 0 & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \ddot{\theta}(t) \sin(\theta(t)) + \dot{\theta}(t)^2 \cos(\theta(t)) \\ \ddot{\theta}(t) \cos(\theta(t)) - \dot{\theta}(t)^2 \sin(\theta(t)) \\ \vdots \end{bmatrix} \quad (8) \end{aligned}$$

In order to use the RLS method, the previous equation was slightly changed, and rewritten as:

$$\begin{aligned} \begin{bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \\ \vdots \end{bmatrix} = - \begin{bmatrix} 2\zeta_1 \omega_1 & 0 & & \\ 0 & 2\zeta_2 \omega_2 & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \vdots \end{bmatrix} - \begin{bmatrix} \omega_1^2 & 0 & & \\ 0 & \omega_2^2 & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \end{bmatrix} + \\ + ([\Phi]^T [M] [\Phi])^{-1} [\Phi]^T \begin{bmatrix} \ddots & 0 & 0 & \\ & u_{\xi,i} & u_{\eta,i} & \\ & 0 & 0 & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \ddot{\theta}(t) \sin(\theta(t)) + \dot{\theta}(t)^2 \cos(\theta(t)) \\ \ddot{\theta}(t) \cos(\theta(t)) - \dot{\theta}(t)^2 \sin(\theta(t)) \\ \vdots \end{bmatrix} \quad (9) \end{aligned}$$

The variables to be estimated are the natural frequencies, the damping ratios and the unbalance components:  $u_{\xi,i} = u_i \cos(\alpha_i)$  and  $u_{\eta,i} = u_i \sin(\alpha_i)$ . The other variables can be measured or computed, like the rotor displacement that can be differentiated to get the velocity. In this work, the angular displacement is obtained by an encoder and the modal mass and modal matrix are determined by a previous modal analysis. The experimentally estimated natural frequencies and damping ratios can be compared with the model estimation to verify the identification quality.

A model estimation prediction error is given by:

$$e_k = \ddot{q}(k) - \hat{\ddot{q}}(k), \quad (10)$$

where,  $\ddot{q}(k)$  is the model estimation for the rotor vibration at instant  $k$  and  $\ddot{\tilde{q}}(k)$  is the experimental measurement on the corresponding degree of freedom. The parameter identification is done by minimization of the prediction error squared:  $\min \left\{ \det \left( \sum_k e_k e_k^T \right) \right\}$ , resulting in the eq. (11).

The balancing method is based on eq. (11), where the variables are written as:  $\{\varphi\}_{1 \times n}$ , the measured input signals (eq. 12);  $\{\Theta\}_{n \times 1} = \left[ (2\zeta_r \omega_r) \quad \omega_r \quad u_{\xi,i} \quad u_{\eta,i} \right]^T$ , the parameters to be identified (damping ratio, natural frequency and unbalance components, respectively);  $\varepsilon$  is the model estimation error; and  $\ddot{\tilde{q}}$ , the desired signal, i.e., the rotor vibration measurement,

$$\{\varphi_k\} \{\Theta_k\} + \varepsilon_k = \ddot{\tilde{q}}_k, \quad (11)$$

$$\{\varphi_k\} = \left[ \begin{array}{cc} \dot{\tilde{q}}(t_k) & \ddot{\tilde{q}}(t_k) \\ \left( [\Phi]^T [M] [\Phi] \right)^{-1} [\Phi]^T \left\{ \begin{array}{c} \ddot{\theta}(t) \sin(\theta(t)) + \dot{\theta}(t)^2 \cos(\theta(t)) \\ \ddot{\theta}(t) \cos(\theta(t)) - \dot{\theta}(t)^2 \sin(\theta(t)) \\ \vdots \end{array} \right\} \end{array} \right]. \quad (12)$$

More information can be obtained in the reference Yang and Lin (2003). The cost function is defined, with a weight factor  $[W]$ , as,

$$J[\Theta_k, W] = [\ddot{\tilde{q}}_k - \{\varphi_k\} \{\Theta_k\}]^T [W] [\ddot{\tilde{q}}_k - \{\varphi_k\} \{\Theta_k\}], \quad (13)$$

where for  $[W] = [I]$  (identity matrix), the solution is given by the eq. (14), which is the normal equation,

$$\{\Theta_k\} = [\{\varphi_k\}^T \{\varphi_k\}]^{-1} \{\varphi_k\}^T \ddot{\tilde{q}}_k. \quad (14)$$

In the recursive form: the inverse of the auto-correlation matrix, the gain matrix and the estimated parameters are written as,

$$[P_{k-1}] = ([I] - [K_{k-1}] \{\varphi_{k-1}\}) ([\Lambda_{k-1}] [P_{k-2}] [\Lambda_{k-1}]^T), \quad (15)$$

$$[K_k] = ([\Lambda_k] [P_{k-1}] [\Lambda_k]^T) \{\varphi_k\}^T ([I] + \{\varphi_k\} ([\Lambda_k] [P_{k-1}] [\Lambda_k]^T) \{\varphi_k\}^T)^{-1}, \quad (16)$$

$$\{\Theta_k\} = \{\Theta_{k-1}\} + [K_k] (\ddot{\tilde{q}}_k - \{\varphi_k\} \{\Theta_{k-1}\}), \quad (17)$$

where  $[\Lambda_k]$  is a diagonal matrix of the adaptation factor, which varies between 0 and 1.

#### 4. Numerical Results

In order to study the balancing method, simulations were performed using a 34-degree of freedom finite element model, shown in Figure 1. It has three bearings, two dampers and two balancing planes with retainer bearings. This is a model of the rotor used in the experimental part of this work. The length of the shaft is 0.675 m and its diameter is 0.008 m. The two discs (placed at nodes 4 and 7) are equal and have radius of 0.055 m, the thickness is 0.026 m and the weight, 1.10 kg each. The unbalance forces are applied on the discs. The equation of motion of such system, operating in non-stationary condition, is:

$$[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{x\} = \{m\varepsilon \cos(\alpha)\} (\dot{\theta}^2 \cos(\theta) + \ddot{\theta} \sin(\theta)) + \{m\varepsilon \sin(\alpha)\} (\ddot{\theta} \cos(\theta) - \dot{\theta}^2 \sin(\theta)). \quad (18)$$

The system's first two natural frequencies are 25.64 and 60.56 Hz, and the corresponding damping ratios are 8.89 % and 1.86 %. The mode shapes of the system (at rotational speed zero) are illustrated in Figure 2. The dynamic response is obtained by a numerical integration, using Newmark method. An unbalance of 1.07 kg.m with a phase angle of 150° was applied at disk 1, node 4.

The initial conditions for the parameter estimation were: zero for the damping,  $2.2 \times 10^4$  N/m for the stiffness, and 0.001 kg.m for the unbalance. These values have showed to be important to obtain a fast and precise convergence of the

algorithm. The stiffness initial guess can be chosen by a previous critical speed evaluation (or by the natural frequency value) and the modal mass, applying the identity  $\hat{k}_r = \omega_{nr}^2 m_r$ .

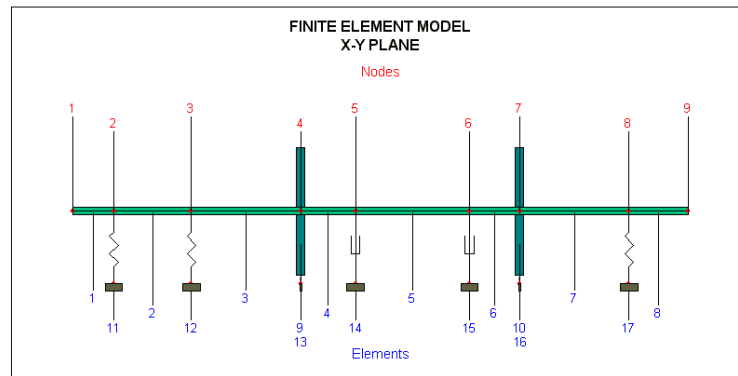


Figure 1 – Finite element model of the rotor.

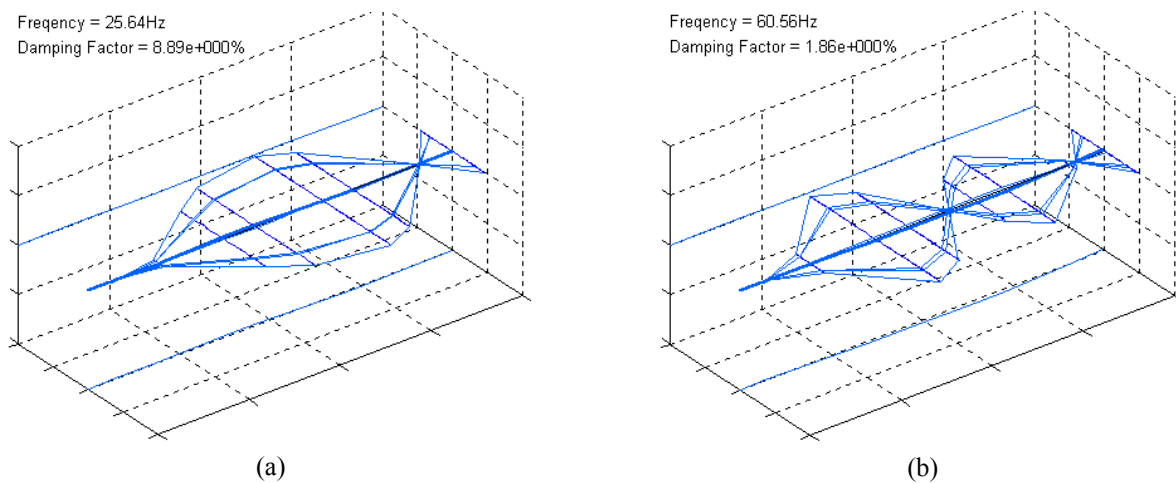


Figure 2 – (a) First and (b) second mode shapes of the rotor.

After evaluating the rotor vibration response with a constant acceleration at  $4 \text{ rad/s}^2$  and sampling rate of 1000 Hz, the results presented in Figure 3 were obtained where axis (a) shows the vibration response of disk 1, direction x, the rotation velocity and the unbalance estimative, magnitude and angle position, and (b) present the modal parameters, natural frequency and damping ratio. The values in the graphics indicate the estimation after 25 seconds (25,000 interactions) and the hatched line is the simulated parameters. The adaptation factor used is 0.9999.

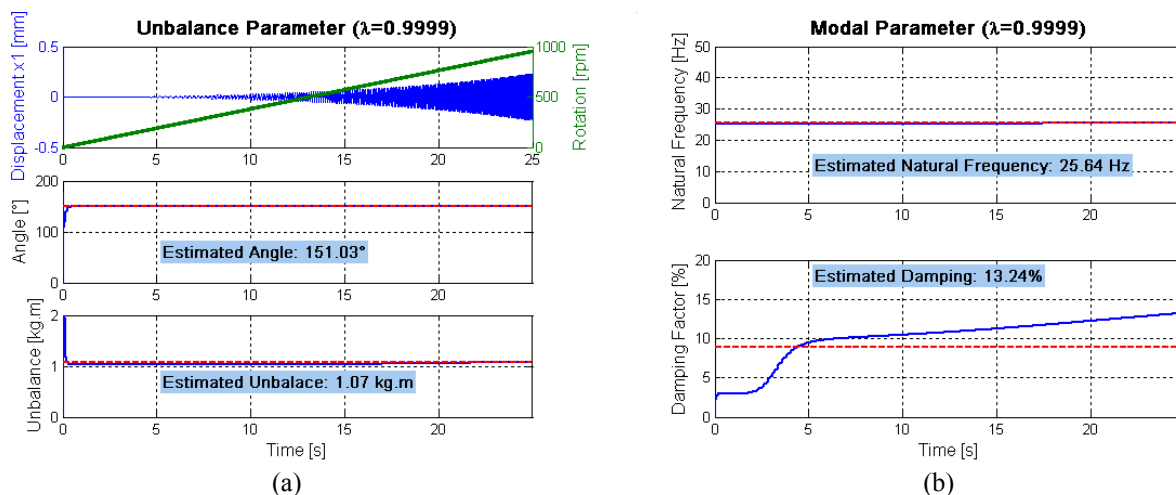


Figure 3 – (a) Unbalance estimation and (b) modal parameter estimation.

The two plots of figure 3 show a fast convergence for all the parameters with small error, less than one percent, except in the damping ratio estimation, which there is a divergence. During the simulations, it was verified that the velocity dependent parameters are harder to identify.

In order to investigate the influence of the adaptation factor on the quality of the identification process, the system was simulated with same conditions described before and the estimated parameters, after 25 seconds, are shown in Table 1. It is easy to see that when the adaptation factor decreases, the identification error increases. For low adaptation factor values, the algorithm converges very rapidly but to a wrong system condition. From this analysis one can conclude that the algorithm is very sensitive to variations in the factor  $[\Lambda_k]$ , which changes the convergence ratio and the tracking capability of the algorithm, increasing the filter stiffness as the factor decreases.

By this formulation, it would not be possible to balance two planes at the same time. The algorithm, as it is presented, cannot separate the excitations from the disks.

Table 1. Unbalance and modal parameter estimation for different adaptation factor values.

Adaptation factor ( $\lambda$ )	Angle ( $^\circ$ )	Unbalance (kg.m)	Natural Frequency (Hz)	Damping Ratio (%)
0.90000	179.52	0.5386	25.16	-0.0684
0.99000	136.46	1.2170	25.16	1.71
0.99900	149.52	1.0326	25.16	3.34
0.99990	151.03	1.0720	25.64	13.24
0.99999	151.43	1.0775	25.68	13.80
1.00000	151.47	1.0774	25.68	13.85

## 5. Experimental Results

The experimental part of this work was performed using the test rig at TUD presented on Figure 4. The flexible rotor is supported by three roller bearings, two pairs of dampers, two balancing disks with retainer bearings at each one. The test rig is instrumented with proximity probes, which measure the lateral vibration of the discs, and an incremental encoder placed at the free end of the shaft. The rotor dimensions are described in the Numerical Results section.

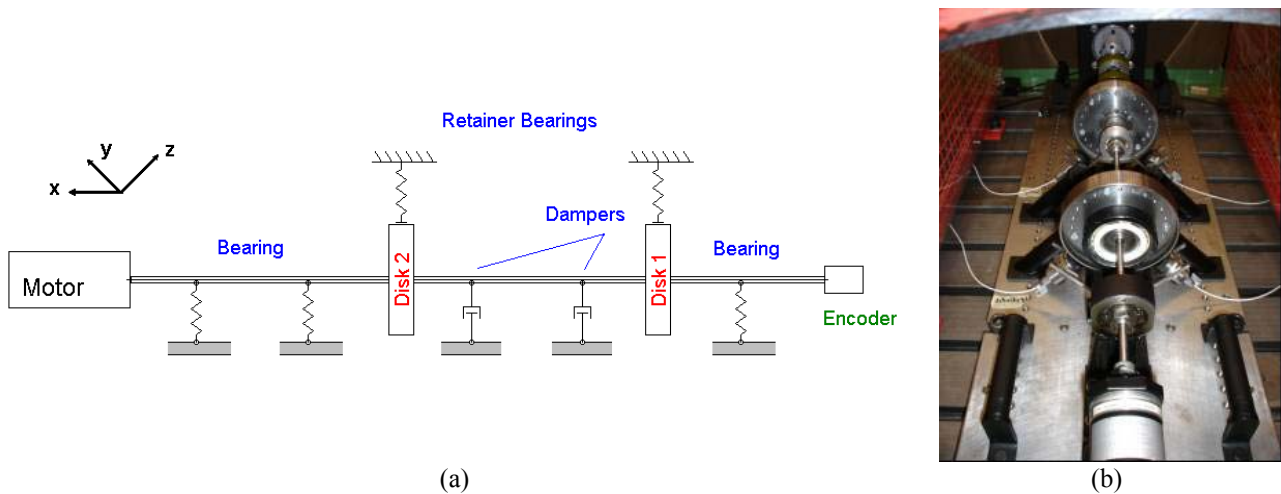


Figure 4 – Test rig scheme and experimental setup with two balancing planes

The first critical speed is about 25 Hz (1,500 rpm) and the second is higher than 60 Hz (3,600 rpm). The measured signals were disturbed by harmonic noises, as can be seen in the order maps, Figure 5 (a) and (b). The maps show a first order, one time the rotational speed, and other orders in the second, third, etc, with lower amplitudes. These higher order components, excited by misalignment or some bearing defects, can influence the balancing method accuracy.

An experimental modal analysis was performed on the rotor, not running, in order to obtain the modal mass and mode shapes at the discs location. The results were: natural frequencies 25.37 and 62.24 Hz, damping ratio 8.86 and 1.86%, and a modal matrix normalized by the mass matrix, Seidler (2002):

$$\Phi = \begin{bmatrix} 0.6710 & 0.5479 \\ 0.4637 & -0.7403 \end{bmatrix}. \quad (19)$$

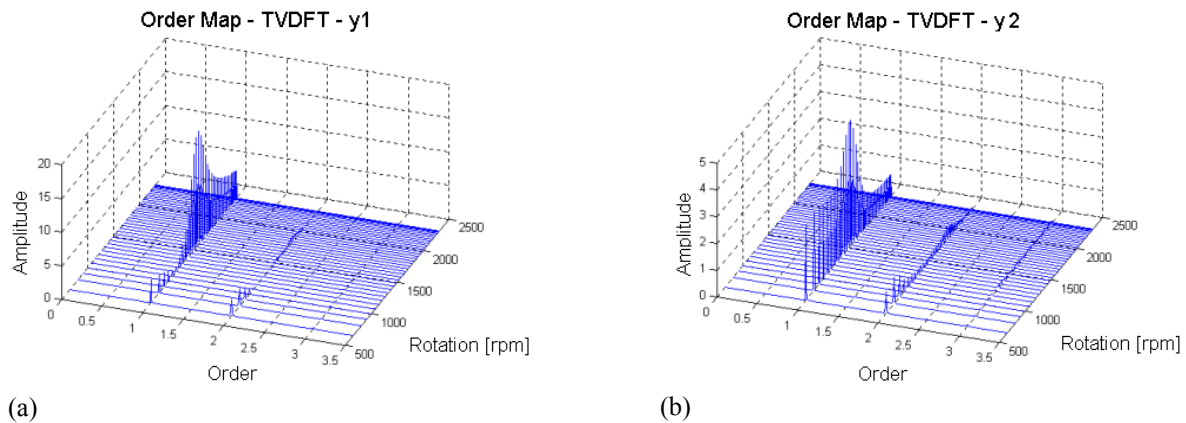


Figure 5. TVDFT Order Map: (a) direction y, disk 1 and (b) direction y, disk 2.

These mode shapes are used in the balancing method to transform the measured signals in generalized coordinate. For this analysis, the vibration signals from the probes were numerically differentiated to obtain the velocity and acceleration vibrations of the disks; and the rotor angular displacement, velocity and acceleration were determined by a polynomial approximation from the encoder signal. The sample rate was 2560 Hz.

An eccentric mass is positioned at disk 1, 15 degrees (from the encoder zero position) at 55 mm from the shaft center. The mass has 5.9 grams, resulting in an eccentricity of  $3.245 \times 10^{-4}$  kg.m. The plots in the Figure 6 show the estimation results for the (a) sensor displacement in y direction on the disk 1, unbalance angle position and magnitude; (b) the modal parameters (natural frequency and damping rate). The hatched lines indicate the expected values for each parameter. It can be seen that the estimations are far from these values and did not converged after 25 s.

It was expected that the noise in the measured data would contribute to increase the error in the balancing procedure. For this reason, a digital filter – Chebyshev type band pass filter, with frequency range between 10 to 45 Hz – was applied to the measured signals in order to reduce the undesired components. The results from this analysis are shown in Figure 7 and, as foreseen, much better estimations were obtained. The unbalance estimations are close to the expected values, especially near the first critical speed, due to the modal model used in the balancing. A similar convergence behavior can be observed for the modal parameter estimations, but the damping ratio estimation did not reach a stable value at the end of the simulated time.

Using the same condition of the previous test, except for the angular position of the eccentric mass placed at disk 2, which was changed to 210 degrees (–150 degrees), new measurements were done and the results are presented in Figure 8, without signal treatment, and Figure 9, signals filtered with Chebyshev band pass filter (10 – 45 Hz). The negative degrees are used when the estimation is over than  $180^\circ$ , because the angle estimation function (Matlab®) is restricted for  $\pm 180^\circ$ .

These estimations show the same behavior as the previous identification: a bad convergence for all the parameters when the signals are not filtered, except for the natural frequency that is near to the real value, mainly because of the initial condition. After the Chebyshev filter is applied, the estimated parameter gets closer to the expected values (hatched lines). A difference in the unbalance magnitude estimation is probably caused by a residual unbalance.

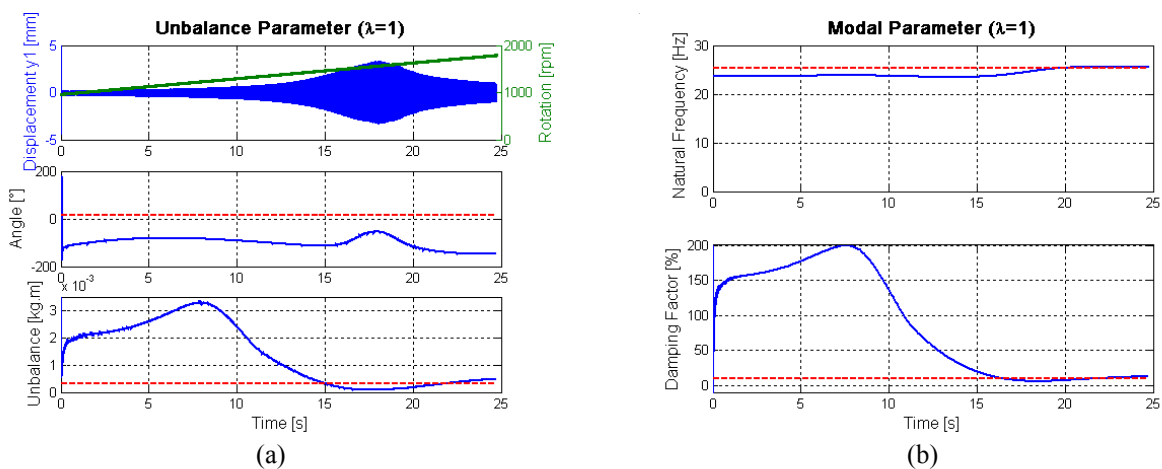


Figure 4 – Eccentric mass at disk 1: (a) unbalance and (b) modal parameter estimation. Unfiltered data.

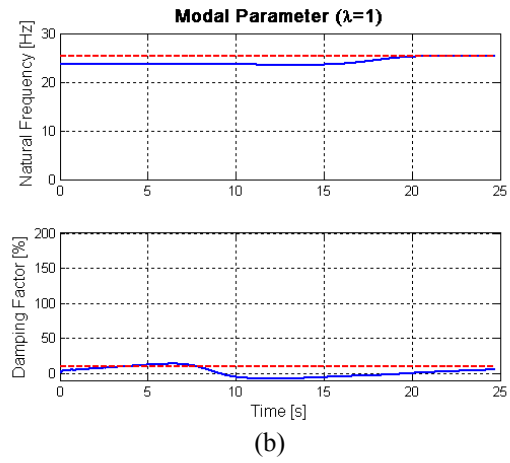
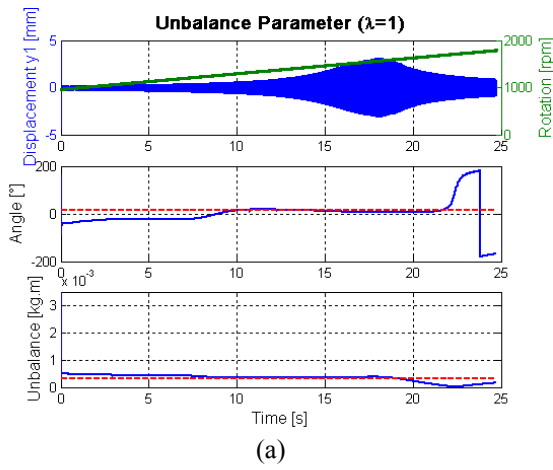


Figure 5 – Eccentric mass at disk 1: (a) unbalance and (b) modal parameter estimation. Filtered data.

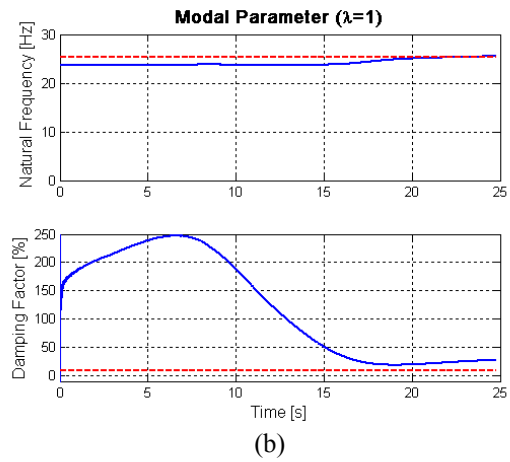
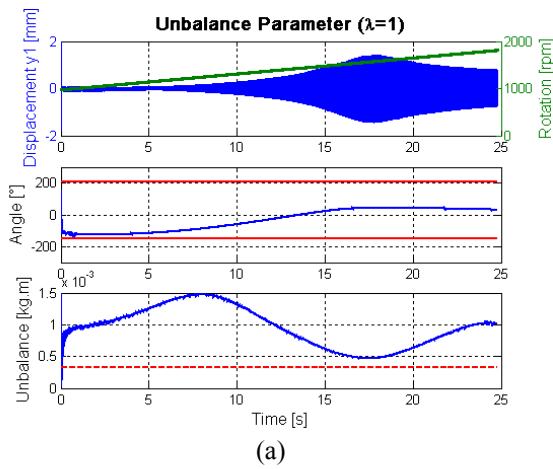


Figure 6 – Eccentric mass at disk 2: (a) unbalance and (b) modal parameter estimation. Unfiltered data.

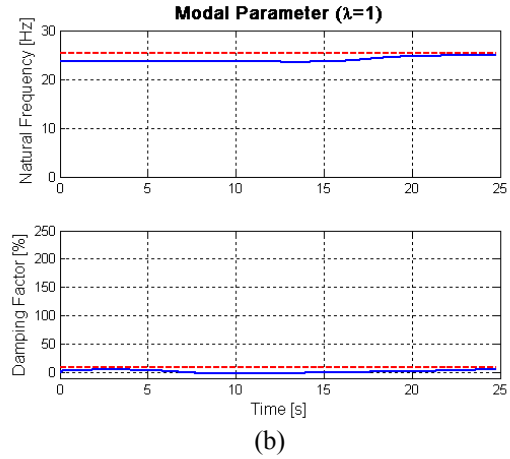
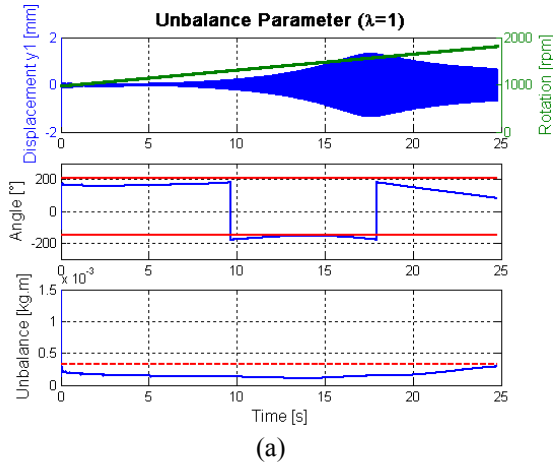


Figure 7 – Eccentric mass at disk 2: (a) unbalance and (b) modal parameter estimation. Filtered data.

## 6. Conclusion

A balancing method for a flexible rotor operating in non-stationary condition is proposed in this work. To apply the proposed methodology, the modal mass and mode shapes have to be known. These parameters can be obtained by an experimental modal analysis or by an analytical calculation. The method is based on the adaptive filter theory and it uses the RLS (Recursive Least Square) algorithm and a modal model of the rotating system. The numerical simulations show that the technique exhibits a good convergence behavior and provides pretty good estimative of the parameters. It was also verified that the identification of a stable value for the damping ratio is much harder to obtain. The filter control parameter – the adaptation factor – may change the convergence speed as well as the tracking stiffness. The



experimental results demonstrate the ability of the presented method to track the unbalance parameters, especially near the critical speed.

## 7. Acknowledgements

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## 10. Responsibility notice

The authors are the only responsible for the printed material included in this paper.