ESTIMATION OF THE HEAT TRANSFER COEFFICIENT OF CRYOGENIC SPRAY COOLING

Jian Su
Nuclear Engineering Program, COPPE, Universidade Federal do Rio de Janeiro, CP 68509, Rio de Janeiro, 21945-970, Brazil
sujian@con.ufrj.br

Allen T. Chwang
Department of Mechanical Engineering,The University of Hong Kong, Pokfulam Road, Hong Kong, China.
atchwang@hkucc.hku.hk

Abstract. *Cryogenic spray cooling has been applied as an effective method to reduce skin damage during laser therapy of some diseases. Applied during tens of milliseconds, the human skin can be instantaneously cooled for the subsequent application of laser therapy so that the skin will not be damaged by the laser heating. In this work, an inverse heat conduction problem was solved for the estimation of surface heat transfer coefficient of cryogenic spray cooling based on internal temperature measurements. The Alifanov's iterative regularization method was applied for the function estimation without any priori information of the functional form of the heat transfer coefficient. The effects of experimental errors of temperature measurements, the number and position of the temperature sensors were investigated.*

Keywords: *Cryogenic spray cooling, laser therapy, Port Wine Stain, inverse heat conduction problem, heat transfer coefficient.*

1. Introduction

Cutaneous laser therapy has been applied in the treatment of abnormal vasculature such as port wine stain (PWS), in hair and tattoo removal, and in facial wrinkle reduction. Port wine stains (PWS) are pink to purple vascular skin lesions characterized by an enlarged number or enlarged diameter of dermal blood vessels (Gabay et al., 1997). Laser treatment of PWS is based on selective heat deposition into the target dermal blood vessels to produce irreversible thermal damage in these target vessels. Ideally, laser heating energy should be confined to the target structures by choosing appropriately short pulse duration and wavelength. However, two competing irradiation absorbers have been identified: melanin, mostly located within the epidermis, and hemoglobin, located in the target blood vessels. The absorption of light by melanin over a broad spectrum causes thermal damage to the epidermis and underlying dermis, clinically resulting in blistering, dyspigmentation, and hypertrophic scarring.

Cryogen spray cooling (CSC) has been developed recently to cool the epidermis selectively during cutaneous laser therapies. A short spurt of cryogen is sprayed onto the skin surface to selectively cool the most superficial layers, immediately prior to laser exposure, without affecting the deeper target chromophores. In response to the subsequent laser pulse, the temperature within the superficial layer remains below the threshold for thermal damage while the targeted chromophores are thermally destroyed.

The heat transfer mechanism of cryogen spray cooling in conjunction with laser treatment of PWS has been investigated intensively. Torres et al. (1999) presented internal temperature measurements in an epoxy resin phantom in response to CSC and use the results in conjunction with a mathematical model of heat conduction to predict the temperature distribution within human skin for various cooling parameters. Pfefer et al. (2000) analyzed the thermal response of PWS skin to a combined treatment of pulsed laser irradiation and CSC through a series of simulations performed with a Monte Carlo optical model and a finite-difference heat transfer model that incorporates realistic tissue morphology. Pikkula et al. (2001) examined cryogen atomization and heat removal characteristics of various cryogen delivery devices, including fuel injectors, atomizers, and a device used in clinical settings. An inverse heat conduction problem (IHCP) was solved with the sequential function specification (SFS) method to estimate the time-varying boundary heat flux from internal temperature measurements. Choi and Welch (2001) analyzed tissue thermal relaxation following laser-induced heating and calculated the time required for a laser-induced temperature rise to decrease to near-baseline values. Aguilar et al. (2001) introduced a novel method to determine the heat flux and heat transfer coefficient of CSC at the surface of a sprayed object, based on measurements of steady-state temperature gradients along a thin copper rod during continuous cryogen spraying. Tunnel et al. (2002) formulated and solved an inverse heat conduction problem to predict the time-varying surface heat flux both during and following a cryogen spurt. Chang et al. (2002) compared the effectiveness of 585-nm vs 595-nm wavelength pulsed dye laser treatment of PWS in conjunction with CSC. Choi and Welch (2002) determined the radial temperature distribution created by CSC and evaluated the importance of radial temperature gradients upon the subsequent analysis of tissue cooling throughout the skin. Aguilar et al. (2002) introduced a definition of CSC efficiency based on the amount of heat removed per unit area of skin for a given cooling time and studied the feasibility of using multiple-intermittent cryogen spurts and laser pulses to improve PWS laser therapy. Aguilar et al. (2003) investigated the effects of spurt duration and spray distance on the dynamic behavior of CSC. Recently, Pikkula et al. (2003) presented an methodology for characterizing heat removal mechanism in human skin during cryogen spray cooling by solving an inverse heat conduction problem. Basinger et al. (2004) investigated the effect of skin indentation on heat transfer during CSC. Kao et al. (2004) evaluated the CSC exposure on *in vitro* model human skin. Shafirstein at al. (2004) developed a new mathematical approach to the diffusion approximation theory for selective photothermolysis modeling and its implication in laser treatment of PWS.

Inverse heat conduction problems have been studied extensively for estimation of unknown boundary or initial conditions, thermophysical properties, heat source strength, and geometrical configuration (Tikhonov and Arsenin, 1977; Beck et al., 1985; Hensel, 1991; Murio, 1993; Alifanov, 1994; Özişik and Orlande, 2000). A variety of numerical and analytical techniques have been developed for the solution of the inverse heat conduction problems, for example, the function specification method, the Tikhonov regularisation method, the mollification method, and the Alifanov's iterative regularisation method. The estimation of heat transfer coefficient has been studied by a number of researchers (Beck et al., 1996; Chantasiriwan, 1999; Flach and Özişik, 1993; Hinestroza and Murio, 1993; Kim and Lee, 1997; Colaço and Orlande, 1999; Martorano and Capocchi, 2000; Abou khachfe and Jarny; Huang et al.; Lin and Wang, 2002; Su and Hewitt, 2004).

In this work we solve an inverse heat conduction problem for the estimation of time varying heat transfer coefficients of cryogenic spray cooling by applying the Alifanov's iterative regularisation method, which has been successfully applied in a variety of inverse heat conduction problems. In this approach, an optimisation problem is solved in which a squared residue functional is minimised with the conjugate gradient method. A sensitivity problem is solved to determine the step size in the direction of descent and an adjoint problem is solved to determine the gradient of the functional. No prior information is used on the functional form of the heat transfer coefficient variation with time.

The paper is organised as follows. In Section 2 we present the mathematical model of the transient heat conduction problem of cryogen spray cooling. Then the mathematical formulation of the inverse heat conduction problem is presented in Section 3. Numerical results based on simulated experimental data are presented and discussed in Section 4.

2. Mathematical Model of the Direct Heat Conduction Problem

The human skin or skin phantom is modeled as an homogeneous medium with constant properties. As the linear dimensions of the spraying cooled surface ($\approx 1-1.5$ cm) are much larger than the depth of interests in the medium (≈ 1 mm), the heat transfer phenomenon of cryogen spray cooling in the medium is governed by the one-dimensional transient heat conduction equation, written as

$$\frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2}, \quad \text{in} \quad 0 < x < a, \tag{1}$$

with the following boundary conditions

$$-k\frac{\partial T}{\partial x} = h(t)(T - T_\infty), \quad \text{at} \quad x = 0, \tag{2}$$

$$T(0,t) = T_0, \quad \text{at} \quad x = a, \tag{3}$$

and the initial condition

$$T(x,0) = T_0 \quad \text{in} \quad 0 < x < a, \quad \text{for} \quad t = 0, \tag{4}$$

where T is the temperature, x is the spatial coordinate, t is the time, $\alpha = (k/\rho c_p)$ is the thermal diffusivity of the medium, k the thermal conductivity of the medium, T_∞ is the temperature of the cryogen film in contact with the medium surface, and h the heat transfer coefficient of the cryogen spray cooling. The medium is assumed initially at T_0, which is the temperature of the undisturbed boundary $x = a$.

When the material properties, the initial condition, and the boundary conditions are known, the temperature distribution, $T(x,t)$ can be determined. The mathematical model given by Eqs.(1 to 4) is then called the direct heat conduction problem. On the other hand, if any of these conditions, or a combination of them, is unknown, but instead experimentally measured temperatures are available somewhere in the space-time domain, an estimation of the unknown quantities may be attempted. This is known as the inverse heat conduction problem (IHCP).

3. Mathematical Formulation of the Inverse Heat Conduction Problem

We seek here to estimate the unknown time varying heat transfer coefficient of cryogen spray cooling $h(t)$ from the temperature measurements taken with several thermocouples in the interior of the medium using the Alifanov's iterative regularisation method, known also as the conjugate gradient method with an adjoint equation (Su and Silva Neto, 2001; Su and Hewitt, 2004).

We implement the Alifanov's iterative regularisation method for function estimation through the following steps: (i) the sensitivity problem; (ii) the adjoint problem and the gradient equation; (iii) the conjugate gradient method of minimisation; and (iv) the stopping criterion. We provide first a brief description of each step, and then present the solution algorithm linking these basic steps.

The sensitivity problem

By introducing a small perturbation on the heat transfer coefficient in the direct problem, that is, $h(t) \to h(t)+\Delta h(t)$, a small perturbation on the temperature field is expected, $T(x,t) \to T(x,t)+\Delta T(x,t)$. Subtracting from the resulting expression the direct problem, Eqs.(1), and neglecting second order terms, we have

$$\frac{1}{\alpha}\frac{\partial \Delta T(x,t)}{\partial t} = \frac{\partial^2 \Delta T(x,t)}{\partial x^2}, \quad \text{in} \quad 0 < x < a, \tag{5}$$

$$-k\frac{\partial \Delta T}{\partial x} = h(t)\Delta T + \Delta h(t)(T - T_\infty), \quad \text{in} \quad x = 0, \quad \text{for} \quad t > 0, \tag{6}$$

$$\Delta T = 0, \quad \text{in} \quad x = a, \quad \text{for} \quad t > 0, \tag{7}$$

$$\Delta T = 0, \quad \text{in} \quad 0 < x < a, \quad \text{for} \quad t = 0. \tag{8}$$

The adjoint problem and the gradient equation

The inverse problem is solved as an optimisation problem where we search for the solution $h(t)$ that minimises the functional

$$J \equiv J(h(t)) = \int_0^{t_f} \sum_{m=1}^{M} (T(x_m,t) - Y_m(t))^2 dt, \tag{9}$$

where $T(x_m,t)$ and $Y_m(t)$ are the computed and measured temperatures at position x_m, $m = 1,..,M$, M is the number of temperature sensors, and $[0,t_f]$ is the interval of time in which experimental data are acquired.

The adjoint problem is developed by defining the lagrangian

$$J = \int_0^{t_f} \sum_{m=1}^{M} (T(x_m,t) - Y_m(t))^2 dt + \int_0^{t_f}\int_0^{a} \lambda(x,t)\left[\frac{\partial^2 T(x,t)}{\partial x^2} - \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t}\right] dx dt, \tag{10}$$

where $\lambda(x,t)$ is the adjoint function.

Making the same perturbation as used in the sensitivity problem and neglecting second order terms, we obtain

$$\Delta J = \int_0^{t_f} \sum_{m=1}^{M} 2(T(x_m,t) - Y_m(t))\Delta T(x_m,t) dt + \int_0^{t_f}\int_0^{a} \lambda(x,t)\left[\frac{\partial^2 \Delta T(x,t)}{\partial x^2} - \frac{1}{\alpha}\frac{\partial \Delta T(x,t)}{\partial t}\right] dx dt. \tag{11}$$

The adjoint problem is obtained after some manipulation

$$-\frac{1}{\alpha}\frac{\partial \lambda(x,t)}{\partial t} = \frac{\partial^2 \lambda(x,t)}{\partial x^2} + 2\sum_{m=1}^{M} (T(x_m,t) - Y_m(t))\delta(x - x_m), \quad \text{in} \quad 0 < x < a \tag{12}$$

$$-k\frac{\partial \lambda}{\partial x} = h\lambda, \quad \text{at} \quad x = 0, \quad \text{for} \quad t > 0 \tag{13}$$

$$\lambda = 0, \quad \text{at} \quad x = a, \quad \text{for} \quad t > 0 \tag{14}$$

$$\lambda = 0 \quad \text{in} \quad 0 < x < a, \quad \text{for} \quad t = t_f. \tag{15}$$

The following integral term is left

$$\Delta J = \int_0^{t_f} -\frac{\lambda(0,t)}{k}(T(0,t) - T_\infty)\Delta h \, dt. \tag{16}$$

By the definition of gradient the following relation holds

$$\Delta J = \int_0^{t_f} J'\left[h(t)\right]\Delta h \, dt \tag{17}$$

A comparison of Eqs.(16) and (17) reveals that the gradient of the functional, $J'(t)$, is given by

$$J'\Big[h(t)\Big] \equiv J'(t) = -\frac{\lambda(0,t)}{k}(T(0,t) - T_\infty). \tag{18}$$

The conjugate gradient method of minimisation

The iterative procedure for the estimation of the unknown heat transfer coefficient $h(t)$ is given as

$$h^{k+1}(t) = h^k(t) - \beta^k P^k(t), \qquad k = 0, 1, 2, ..., \tag{19}$$

where the direction of descent $P^k(t)$ at step k is defined as

$$P^k(t) = J'^k(t) + \gamma^k P^{k-1}(t), \quad \text{with} \quad \gamma^0 = 0. \tag{20}$$

and the conjugate coefficient γ^k is given by

$$\gamma^k = \frac{\int_0^{t_f} \Big[J'^k(t)\Big]^2 dt}{\int_0^{t_f} \Big[J'^{k-1}(t)\Big]^2 dt}. \tag{21}$$

The step size β^k is determined by minimising the functional $J[h(t)]$ given by Eq.(9), that is

$$\frac{\partial J(h^{k+1})}{\partial \beta^k} = 0. \tag{22}$$

we obtain therefore

$$\beta^k = \frac{\int_0^{t_f} \sum_{m=1}^M (T(x_m,t,h^k) - Y_m(t))\Delta T(x_m,t,h^k)dt}{\int_0^{t_f} \Big[\sum_{m=1}^M \Delta T(x_m,t,h^k)\Big]^2 dt}. \tag{23}$$

The stopping criterion

The discrepancy principle is used to establish the criterion for stopping the iterations in the estimation of the heat transfer coefficient, as measurement errors are always present in real applications. Let the standard deviation σ of the measurement errors be the same for all measurements, that is

$$T(x_m,t) - Y_m(t) \cong \sigma. \tag{24}$$

Introducing this result into Eq.(9), we have

$$\epsilon^2 = \int_0^{t_f} \sum_{m=1}^M \sigma^2 dt = M\sigma^2 t_f. \tag{25}$$

The iterative procedure is interrupted when

$$J[h(t)] < \epsilon^2. \tag{26}$$

The solution algorithm

We now summarize the solution algorithm that implements the iterative procedure as follows:
Step 1. Choose an initial guess $h^0(t)$, for example $h^0(t)$ = constant;
Step 2. Solve the direct problem, Eqs.(1 to 4), to obtain $T(x,t)$;
Step 3. Solve the adjoint problem, Eqs.(12 to 15), to obtain $\lambda(x,t)$;
Step 4. Compute the gradient, $J'(t)$, with Eq.(18);
Step 5. Compute the conjugate coefficient, γ^k, with Eq.(21);
Step 6. Compute the direction of descent, $P^k(t)$, with Eq.(20);
Step 7. Solve the sensitivity problem, Eqs.(5 to 8), with the source term given by $\Delta h(t) = P^k(t)$, to obtain $\Delta T(x,t)$;
Step 8. Compute the step size, β^k, with Eq.(23);
Step 9. Compute a new estimate, $h^{k+1}(t)$, with Eq.(19);
Step 10. Interrupt the iterative procedure if the stopping criterion, Eq.(26), is satisfied; Otherwise, go back to Step 2.

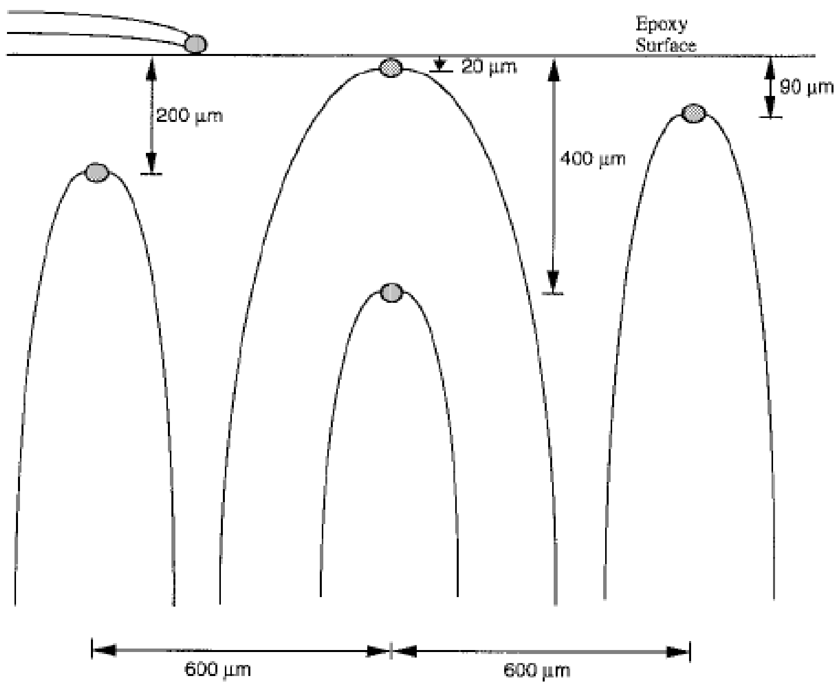

Figure 1. Microthermocouple placement within epoxy resin phantom (Torres et al., 2001).

4. Results and Discussion

We consider here the *in vitro* model described previously by Torres et al. (1999) and also studied by Tunnell et al. (2002) by the sequential function specification method. A skin phantom was constructed by creating a solid block of an epoxy resin, with a front face of 6 cm $\times$ 6 cm and a depth of 4 cm. Four thermocouples were positioned at the following depths: 20, 90, 200, and 400 μm, as measured from the epoxy resin front surface to the bead centers of the thermocouples. A fifth thermocouple was place on the phantom surface to measure the cryogen film temperature. The thermophysical properties of the epoxy are taken as the values given by Tunnell et al. (2002): the density ρ = 1019 kg/m^3, the specific heat c_p = 1631 J/kg°C, the thermal conductivity k=0.14 W/m °C. The thermal diffusivity is 0.843×10^{-7}m^2/s, which is within 25% of human skin 1.1×10^{-7}m^2/s.

It should be pointed out that the present mathematical model is not directly applicable for the estimation of heat transfer coefficient over the surface of human skin, since skin is a multilayer structure with variations in thermal properties among distinct layers (i.e. epidermis and dermis). The main differences between epidermal and dermal properties result from the low water content in the *stratum corneum* whose thickness is a fraction of the epidermis in most areas of skin. For short cryogen spurt durations used in this study (20–200 ms), undesirable phase change in tissue water (e.g. freezing) is not expected to occur, and the absence of water in the phantom should not affect extrapolation of results to human skin. Although the single layer homogeneous model of epoxy resin is expected to be a good approximation of human skin, a more complex multilayer model with water content should be more accurate (Torres et al., 1999).

We carried out a series of numerical simulations to evaluate the accuracy of the proposed inverse analysis for estimating the time varying heat transfer coefficient of cryogen spray cooling. We generated simulated transient temperature data, $Y_m(t_n)$, $n = 1, 2, ..., n_t$, by adding random errors to computed exact temperatures, $T_m(t_n)$,

$$Y_m(t_n) = T_m(t_n) + \sigma e, , \tag{27}$$

where σ is the standard deviation of measurement errors, assumed to be the same for all measurements, and e is a normally distributed random error. For normally distributed error, there is a 99% probability of the value of e lying in the range $-2.576 < e < 2.576$.

Following Tunnell et al. (2002), we simulated a 200 ms cryogen spurt on a medium initially at $T_0 = 33$°C. We first examine the effects of the experimental errors of the measured temperatures on the accuracy of estimated heat transfer coefficients with four thermocouples positioned at the the depths of 20, 90, 200, and 400 μm from the surface, as shown in Figure 1. We can see from Figure 2 that for σ = 0.0, 0.1, and 0.2 °C, the estimated heat transfer coefficients agree reasonably well with the exact heat transfer coefficient, while for $\sigma = 0.5$ °C, the estimated heat transfer coefficient deviates from the exact heat transfer coefficients.

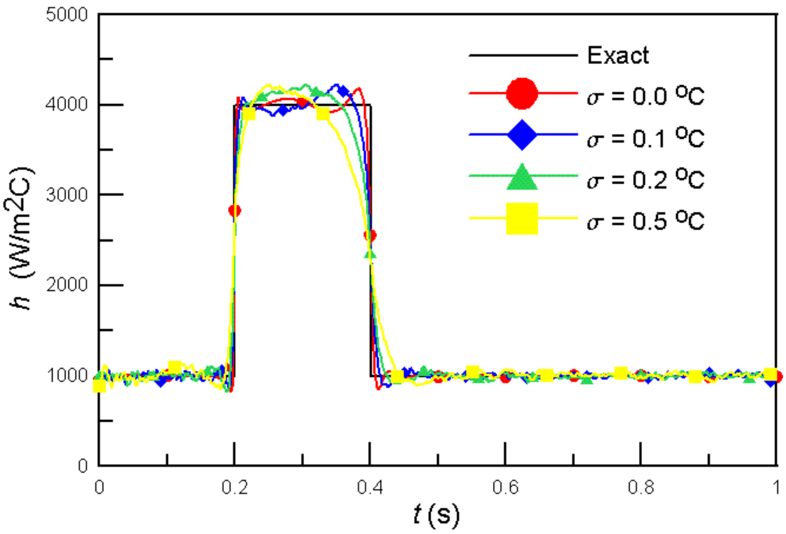

Figure 2. Estimated heat transfer coefficient of cryogenic spray cooling for different temperature measurement errors.

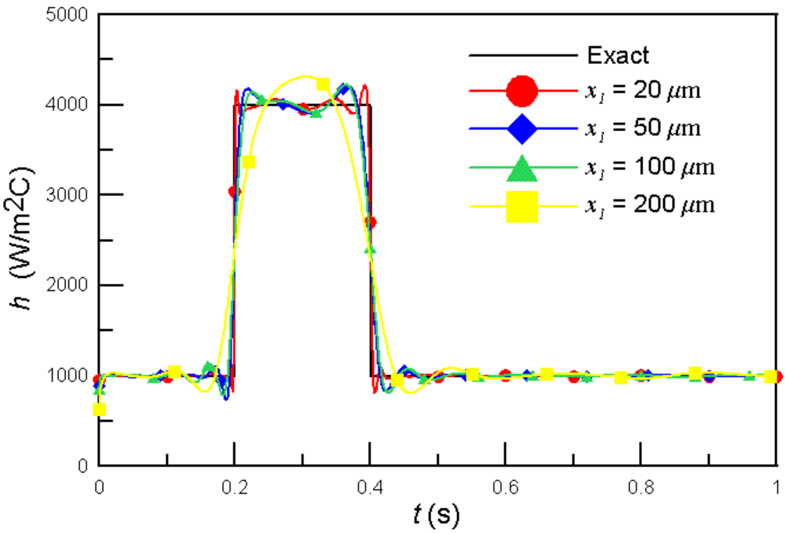

Figure 3. Estimated heat transfer coefficient of cryogenic spray cooling by a single thermocouple at different positions for $\sigma = 0.0\ ^{o}$C.

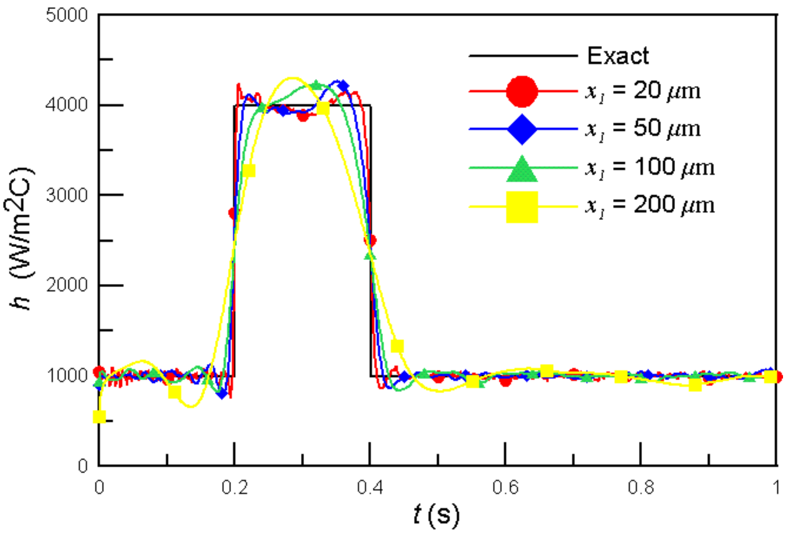

Figure 4. Estimated heat transfer coefficient of cryogenic spray cooling by a single thermocouple at different positions for $\sigma = 0.1\ ^{o}$C.

We examine also the effect of number of temperature sensors on the estimation of heat transfer coefficient of cryogenic spray cooling and found that if the first sensor is kept at the depth of 20 μm, there is practically no difference in the estimated heat transfer coefficients by using one, two, three, or four thermocouples. We thus concluded that the first sensor dominates the accuracy of estimation of heat transfer coefficients and that the other three sensors provide only redundant information.

In Figures 3 and 4 we examined the effect of the position of the single sensor on the estimation of the heat transfer coefficients. It can be seen that the heat transfer coefficients estimated by a single sensor located at a depth equal to or less than 100 μm agree reasonably well with the exact heat transfer coefficient, bothσ = 0.0 and 0.1. When the temperature sensor is positioned at a depth of 200 μm, the inverse solution differs significantly from the exact heat transfer coefficient.

5. Conclusions

We solved an inverse heat conduction problem for estimating time-varying heat transfer coefficient of cryogenic spray cooling of human skin prior to cutaneous laser therapy. We simulated numerically experimental conditions available in literature to investigate the possibility of obtaining time-vary heat transfer coefficient by the Alifanov's iterative regularization method. We concluded that it is feasible to estimate the time-varying heat transfer coefficient of cryogenic spray cooling solve the inverse heat conduction problem with the Alifanov's iterative regularization method. The estimated heat transfer coefficients with a single microthermocouple positioned within a depth of 100 μm from the surface agree reasonably well with the exact heat transfer coefficient, for temperature measurement errors with standard deviation for $\sigma = 0.1$ ^{o}C.

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