PROJECT OF A MODEL FOLLOWING FLIGHT CONTROL SYSTEM OF A VARIABLE STABILITY AIRCRAFT

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Abstract. *This work presents a model following flight control system of a variable stability aircraft that can be used to perform inflight simulations. First, it is presented the project of a feedforward control law and the utility of it. Later is shown that this control law is not capable to compensate effect of imprecisions of the model of the host aircraft and disturbances not predicted. Then it is shown the project of a feedback control law that can be added to the feedforward control law, in order to increase the system robusteness. This methodology of using simultaneous actuation of feedforward and feedback control law has shown excelent results in the simulations performed.*

Keywords: *inflight simulation, variable stability aircraft, model following, reference model, flight control system*

1. Introduction

During the development of new aircraft, it is necessary to make simulations in order to verify if the flight dynamics and handling qualities requirements defined in the preliminary design are satisfied. The tool utilized to verify it is the ground based flight simulator.

In some cases this tool has shown that there are limitations in motion representation during maneuvers in which there is a high gain of the pilot. Beyond that, the pilot does not have the physical and mental workload that he/she would have in a real flight. These facts can prejudice the qualitative evaluation of the test pilot. It was verified that these facts would not happen in case the pilot was commanding a simulation during a real flight. For doing it, it is utilized the inflight simulators.

Inflight simulators are variable stability aircraft with a model following flight control system that guarantees that this aircraft can track the dynamic of other airplane (reference model).

This work presents the project of a model following flight control system that can be used in inflight simulators.

It is presented the project of a feedforward control law based on the Method of Erzberger (1968) and the project of a feedback control law based on the method of Wu (2001). The feedforward control law performs the most part of the control action while the feedback control law is used to compensate effects of imprecision in the mathematical model of the host aircraft and disturbance not predicted acting on it. The simultaneous action of the feedforward and feedback control laws defined in this work has shown excellent results.

2. Project of the Feedforward Control Law

The project of the feedforward control law was based on the method of Erzberger (1968).

The linear dynamic of the plant (host aircraft) is defined by the Eq. (1):

$$\frac{dx_p(t)}{dt} = A_p * x_p(t) + B_p * u_p(t)$$
$$y_p(t) = C_p * x_p(t) \tag{1}$$

where: $x_p(t) \in R^n$, $u_p(t) \in R^m$, $y_p(t) \in R^p$, $A_p \in R^{nxn}$, $B_p \in R^{nxm}$ and $C_p \in R^{pxn}$

and the linear dynamic of the reference model is defined by the Eq. (2):

$$\frac{dx_m(t)}{dt} = A_m * x_m(t) + B_m * u_m(t)$$
$$y_m(t) = C_m * x_m(t) \tag{2}$$

where: $x_m(t) \in R^n$, $u_m(t) \in R^m$, $y_m(t) \in R^p$, $A_m \in R^{nxn}$, $B_m \in R^{nxm}$ and $C_m \in R^{pxn}$

The derivatives of order 0 and 1 of the output equations of the host aircraft and of the reference model must be equal in all instant of time t, to guarantee the tracking of the response of the reference model.

If the derivatives of order 0 of the output equations of the host aircraft and of the reference model are equal in all instant of time t, then:

$$C_p * x_p(t) = C_m * x_m(t) \tag{3}$$

If the derivative of order 1 of the output equations of the host aircraft and of the reference model are equal in all instant of time t, then

$$C_p * (A_p * x_p(t) + B_p * u_p(t)) = C_m * (A_m * x_m(t) + B_m * u_m(t)) \tag{4}$$

Using the Eq.(3) and the Eq.(4), the flight control law $u_p(t)$ of the host aircraft can be obtained

$$u_p(t) = (C_p * B_p)^{-1} * ((C_m * A_m - C_p * A_p * C_p^{-1} * C_m) * x_m(t) + C_m * B_m * u_m(t)) \tag{5}$$

This is the feedforward control law utilized to control the dynamic of the host aircraft. The dynamic of the reference model is implicit in the equation of the feedforward controller.

In case the matrices C_p and $C_p * B_p$ are not square, it should be calculated the pseudo-inverse of these matrices.

Many simulations were performed to verify the efficiency of this control law and some results are presented below.

It was used a host aircraft with characteristics similar to the aircraft Mirage III and a reference model with characteristics similar to the Airbus A-310.

Simulations of the non-linear dynamics of these aircraft were performed. Equations of the non-linear dynamics defined in Lewis and Steven (1992) were used.

The output vector used in the simulations is [V,α,β, ϕ ,θ,p,q,r,H] and the control vector is [$\delta elev$,thrust, δail , δrud]. But as the simulations performed in this work were associated only with latero-directional maneuvers, it was chosen only to show the output vector [β, ϕ , p, r] and the control vector [δail , δrud].

2.1. Dutch-Roll simulation

Figure 1 shows the simulation results of the response of the host aircraft and of the reference model after a perturbation in the initial conditions. The airplanes were initially trimmed in a wings level straight flight with true airspeed of 160 m/s and in an altitude of 2000 m. The airplanes suffered a perturbation of 1.7 deg in sideslip β and 1.7 deg/s in the roll rate (p) and yaw rate (r). The black plots show the values of the outputs and of the control surfaces of the reference model. The magenta plots show the same parameters of the host aircraft without any action of the model following flight control law, and the red plots show the same parameters of the host aircraft with the action of the flight control law defined by the method of Erzberger (1968).

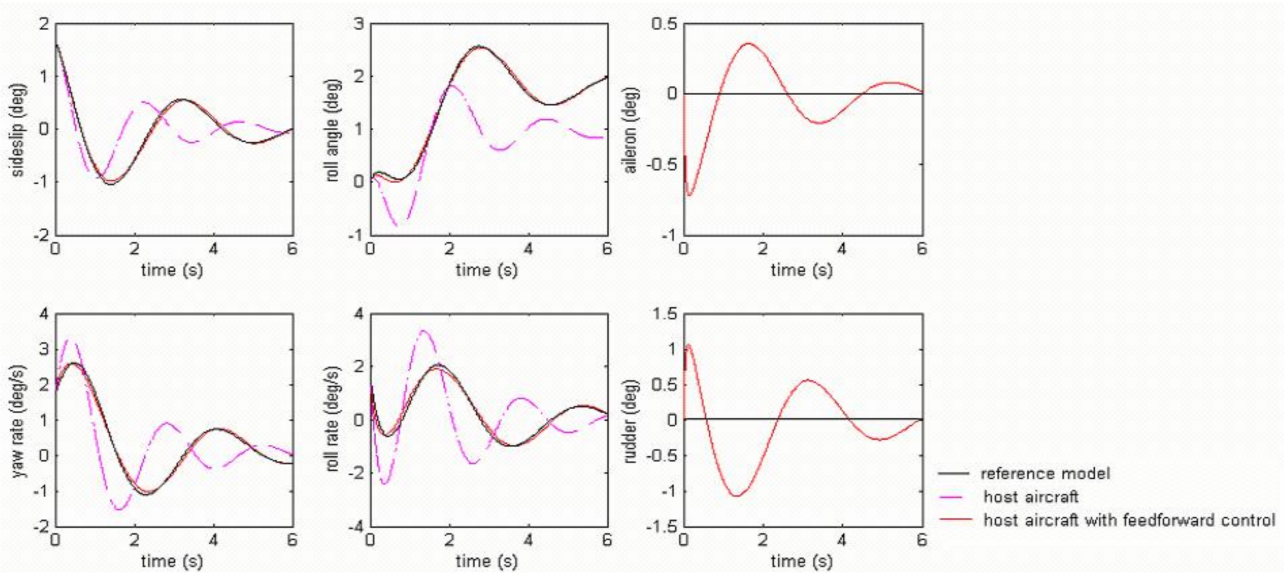

Figure 1- Dutch Roll simulation

The actuation of the flight control law defined with Eq.(5) permits the tracking of the autonomous response of the reference model. It can be seen that the red plots and black plots of the parameters [β, ϕ , p, r] are almost coincident.

2.2. Aircraft response to a aileron step deflection

The model following flight control law defined by Erzberger (1968) can be used to the tracking of the response of the reference model to inputs in the control vector $u_m(t)$. Figure 2 shows the response of the reference model to a step of 1 deg in the aileron deflection (black plots), and the response of the host aircraft with the actuation of the flight control law defined in Eq.(5) (red plots). The airplanes were initially in a wings level straight flight with true airspeed of 160 m/s and in an altitude of 2000 m.

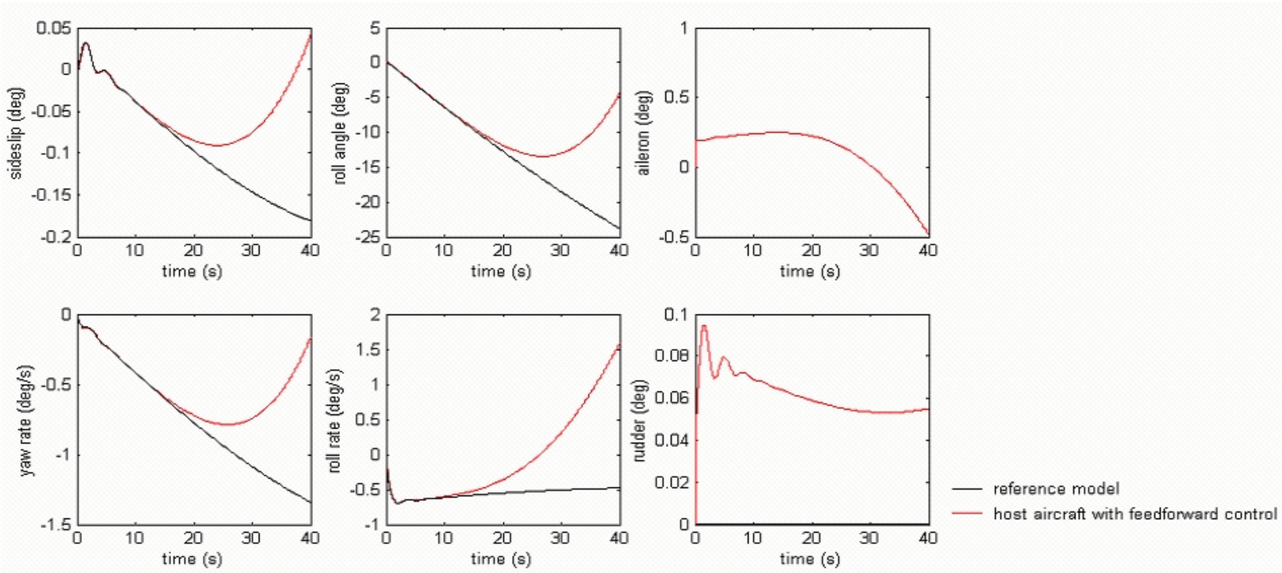

Figure 2- Aircraft response to an aileron step deflection

Figure 2 shows that the flight control law defined by Eq.(5) does not guarantee the tracking of the response of the reference model. It happens because the flight control law defined in Eq.(5) does not contain feedback action and is function of the linear dynamics of the host aircraft and of the reference model. The linearization of the two aircraft is valid only when the state parameters are relatively close to the initial equilibrium condition. In the simulation shown in Fig. 2, the output parameters [β, ϕ, p, r] deviate significantly from the initial condition and, for this reason the feedforward flight control law loose its efficiency. This lack of efficiency due to the significant deviation from the initial condition can be avoided in case it is added to Eq.(5) a feedback control law.

3 Project of the Feedback control law

The method of Wu, (2001) establishes a feedback control law that can be used to make a plant with uncertainties and disturbances, described by the Eq. (6) track a dynamic of a reference model described by the Eq. (7)

$$\begin{aligned} \frac{dx_p(t)}{dt} &= [A_p + \Delta A_p(\upsilon,t)] * x_p(t) + [B_p + \Delta B_p(\nu,t)] * u_p(t) + w(q,t) \\ y_p(t) &= C_p * x_p(t) \end{aligned} \qquad (6)$$

$$\begin{aligned} \frac{dx_m(t)}{dt} &= A_m * x_m(t) \\ y_m &= C_m * x_m(t) \end{aligned} \qquad (7)$$

where: $x_p(t) \in R^n$, $u_p(t) \in R^m$, $y_p(t) \in R^p$, $w(q,t) \in R^n$, $(\upsilon,\nu,q) \in \varpi'$ is the uncertainty vector,

$A_p \in R^{nxn}$, $B_p \in R^{nxm}$ and $C_p \in R^{pxn}$.

The matrices $\Delta A_p(.), \Delta B_p(.)$, represent the uncertainties of matrices A_p, B_p, and:

$$\Delta A_p(\upsilon,t) = B_p * N(\upsilon,t)\,,\ \Delta B_p(\nu,t) = B_p * E(\nu,t)\,,\ w(q,t) = B_p * \tilde{w}(q,t) \quad (8)$$

The feedback-feedforward controller proposed in this work is a combination of the controllers defined by the method of Erzberger (1968) and by the method of Wu (2001). The controller proposed is defined by the Eq.(9):

$$u_p(t) = (C_p * B_p)^{-1} * ((C_m * A_m - C_p * A_p * C_p^{-1} * C_m) * x_m(t) + C_m * B_m * u_m(t)) + \bar{p}(t) \quad (9)$$

The Eq.(10) to Eq.(22) explain how is defined the term $\bar{p}(t)$, that is used for the feedback action:

a) $\bar{p}(t) = \bar{p}_1(z(t),t) + \bar{p}_2(z(t),t)$ (10)

b) $\bar{p}_1(z(t),t) = -\frac{1}{2} * k_1(t) * B_p^T * P * z(t)$ (11)

c) $\bar{p}_2(z(t),t) = -\frac{k_2(t) * B_p^T * P * z(t)}{\|B_p^T * P * z(t)\| * \rho_i(t) + \varepsilon * \|z(t)\|^2 + s}$ (12)

d) $k_1(t) = \frac{\eta + \delta_1^2 * \rho_\upsilon^2(t)}{1 + \mu(t)}$ (13)

e) $k_2(t) = \frac{\rho_i^2(t)}{1 + \mu(t)}$ (14)

f) $z(t) = x_p(t) - G * x_m(t)$ (15)

g) G and H are solution of the Eq. (16)

$$\begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} * \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} G * A_m \\ C_m \end{bmatrix} \quad (16)$$

h) P is the solution of the algebraic Riccati Eq. (17)

$$A_p^T * P + P * A_p - \eta * P * B_p * B_p^T * P = -Q \quad (17)$$

i) $\rho_i(t) = max\{\|F(\upsilon,\nu,q,x_m)\|\}$, $F(\upsilon,\nu,q,x_m) = [N(.) * G + E(.) * H] * x_m(t) + \tilde{w}(.)$ (18)

j) $\rho_\upsilon = \max_\upsilon \|N(\upsilon,t)\|$, (19)

l) $\mu(t) = \min_\nu [\frac{1}{2} * \lambda min(E(\nu,t) + E^T(\nu,t))]$ (20)

m) δ_1, ε, and s are defined in order to satisfy the Eq. (21), shown below

$$\frac{1}{\delta_1^{\ 2}} < \lambda_{min}(Q) - 2*\varepsilon - 2*\frac{s}{\|z(t)\|^2} \tag{21}$$

n) The values used in this work are presented below:

$\varepsilon = 0.15$, $\delta_1 = 7.0711$, $\rho_\upsilon = 0.802$, $\rho_i = 0.435$, $\mu = 29.0$, $\eta = 4.0$, $s = 0.325$,

$Q = [1 \quad 4 \quad 3 \quad 1 \quad 7 \quad 1 \quad 9 \quad 4 \quad 2] * I_{9.x.9}$

Some simulations performed to show the efficiency of the feedforward-feedback control law is shown below.

3.1. Aileron response simulation

This item presents the same case simulated in the Item 2.2 (aircraft response to a step aileron deflection). The black and red plots of the Fig. 3 are the same plots presented in Fig. 2. The blue plots show the response of the host aircraft with the actuation of the feedforward-feedback control law defined in Eq.(9). The blue plots and the black plots of the state parameters are almost coincident. The feedback action increases significantly the efficiency of the model following flight control law.

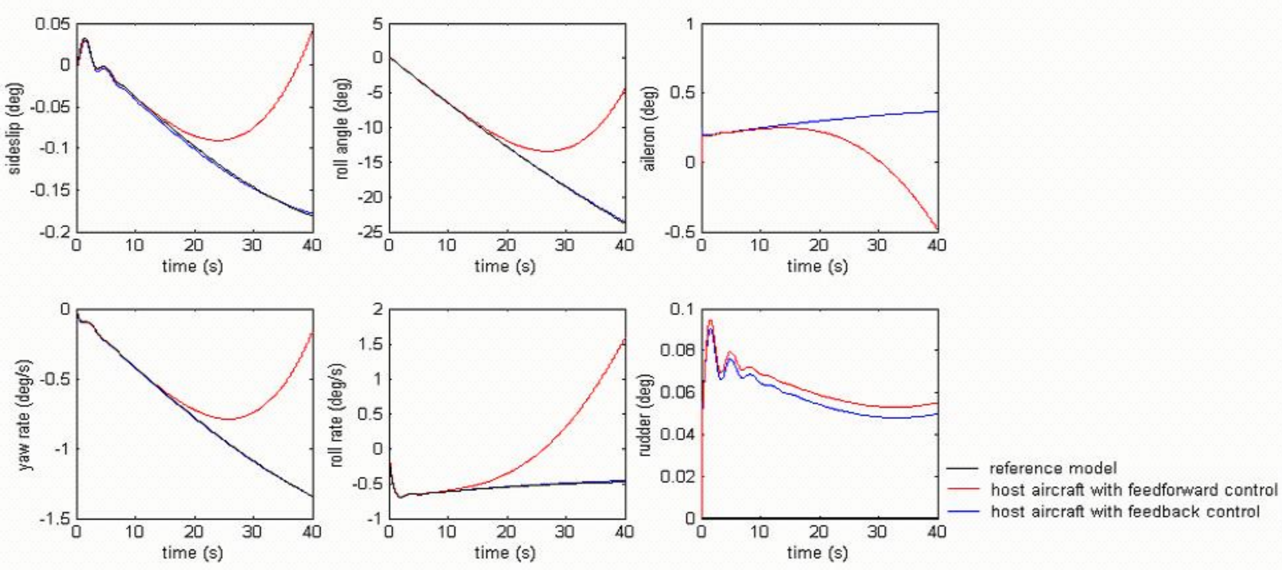

Figure 3- Aircraft response to an aileron step deflection

3.2. Dutch Roll simulation with disturbance not predicted

Figure 4 shows the response of the reference model and of the host aircraft to perturbations in the initial equilibrium conditions. The airplanes were initially in a wings level straight flight with true airspeed of 160 m/s and in an altitude of 2000 m and suffered perturbations of 1.7 deg in the sideslip angle and of 1.7 deg/s in the roll rate and in the yaw rate. The simulation presented in this item is different from the simulation presented in the Item 2.1 because it was considered disturbance $\tilde{w}(t)$ acting on the host aircraft during the simulation. The disturbance $\tilde{w}(t)$ considered in this work are described by Eq.(22)

$$\tilde{w}(t) = [-0.05 \quad 0.1] * 0.048 * (1 - cos(2.13 * \pi * t)) \tag{22}$$

The black plots show the autonomous response of the reference model, the red plots show the response of the host aircraft with the actuation of the feedforward control law defined in Eq.(5) and the blue plots show the response of the host aircraft with the actuation of the feedforward-feedback control law defined in Eq.(9).

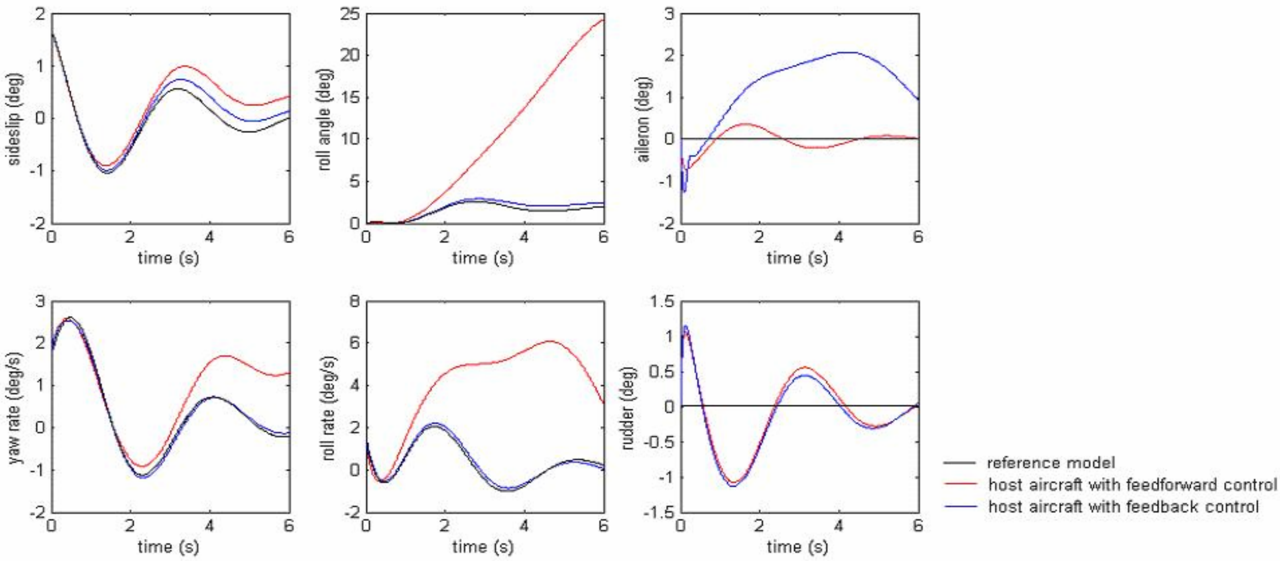

Figure 4- Dutch Roll simulation with disturbances

The results presented in Fig. 4 show the effects of the disturbances during the simulation. The response of the host aircraft with the feedforward control law (red plots) deviate significantly from the response of the reference model (black plots). The feedback action compensates the effects of these disturbances, as it can be seen that the host aircraft with the controller defined in Eq.(9) presents a better tracking of the response of the reference model.

4 Conclusions

This work presented a project of a model following flight control law that is capable of guaranteeing the tracking of the dynamic of a reference model, even when there are uncertainties in the mathematical model of the host aircraft and disturbances acting on it. The methodology used in the project consists in the simultaneous actuation of a feedforward and a feedback control laws. The first does the most part of the control action while the second is used to increase the system robustness against uncertainties and disturbances.

5 Acknowledgements

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6 References

Wu, H.,2001, "Robust Model Following Controllers Guaranteeing Zero Tracking Error for Uncertain Systems Including Delayed State Perturbations", IEEE International Conference on Control Applications, Mexico City, Mexico, pp. 1054-1059.

Lewis, F.L.and Stevens B.L., 1992, "Aircraft Control and Simulation", Wiley Interscience Publication, New York, USA, 617 p.

Erzberger, H.,1968, "On the Use of Algebraic Methods in the Analysis and Design of Model Following Control Systems", National Aeronautical and Space Administration – NASA-TN-D-4663, Ames Research Center, USA, pp. 1-28