

## KINEMATIC ANALYSIS OF A FOUR LEGGED ROBOT SUSPENDED ON WIRE

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**Abstract.** *Some theoretical and experimental studies have been developed to obtain independent equipment for inspection and maintenance of electric and communication lines, allowing to increase the efficiency of the process and to reduce the risk of accidents with the employees who execute the task. This activity is perfectly adequate for mobile robots because they are capable to move in avoiding the existing obstacles on the cables. The more important mechanical characteristic of the mobile robots is related with its stability, that are in two types: the static stability that is its capacity to maintain a configuration from reaction forces; and the dynamic stability that is its capacity to maintain a configuration from reaction forces and inertia forces. In this paper we present the kinematic analysis and the project of a robot with four legs to move suspended on wire. The kinematic analysis have been developed by considering the capability of the robot in transposing obstacles on the wire. For that, two analysis have been made: the first one considers all legs having equal lengths and, the second one unequal lengths. The analysis enable to verify the maximum obstacle dimensions whose the robot can transpose. Simulations of the legs and the robot behavior have been carry out using graphical software. Results of the mathematical model and simulations are presented.*

**Keywords:** *Robotics, Mobile robots, Legged robots, Transmission lines*

### 1. Introduction

Many theoretical and experimental studies have been made in order to develop autonomous machines to travel along telecommunication lines and power transmission wires to perform inspection and/or repair work. These machines can improve the efficiency, reduce labor and are expected to reduce any danger to maintenance personnel. Locomotion is essential for these machines that can be achieved by several methods. In spite of theoretical researches and technological developments, problems related to stability, ability and autonomy still exist.

Wheels are the simplest way for locomotion but they cannot be used when obstacles exist on the wire, like rings and terminal boxes (Aoshima et al., 1989). Other interesting machine types had been studied like those proposed by Sawada, named as self-guide type (Sawada et al., 1991), the balancer type proposed by Higuchi (Higuchi et al., 1989), and the stride type proposed by Paula (Paula, 1989). These systems present problems related to stability, big dimensions and mechanical complexity.

By considering that a walking machine can locomote in highly unstructured environments (Angeles, 1997), it can be used to perform inspection and/or repair works on transmission/power lines, avoiding obstacles and, if it is suspended on wire, its own weight assure its equilibrium.

In this way Tsujimura and collaborators proposed and studied a legged robot that walks on aerial cables. The proposed robot has two pairs of legs. Each leg is made of slider-crank mechanism which one motor is coupled to the left legs and another to the right ones (Tsujimura and Morimitsu, 1997), (Tsujimura and Yabuta, 1989) and (Tsujimura et al., 1996). They obtained relationships between the length links of the mechanism legs in such way the robot can move most stably at regular speed. The experimental results confirmed the good behavior of the robot when obstacles are placed at equal intervals. If the obstacles are irregularly placed, an algorithm is necessary to adjust the gaits by changing the motor velocity in order to avoid the obstacle (Tsujimura and Morimitsu, 1997).

In this work we propose one alternative to provide the necessary mobility to Tsujimura's robot in order to transpose any kind of existing obstacles on wire, since the obstacle has a maximum predefined dimensions and using only one actuator to move all legs. In particular one degree-of-freedom is augmented on each leg enabling its length variation. From kinematic analysis the equations of the robot center of mass and the foot trajectories are presented, both for equal and unequal legs.

## 2. Kinematic Analysis

The structure of the proposed mobile robot was defined by considering characteristics such as stability, simplicity and controllability. It is composed by four slider-crank mechanisms, used as legs, with synchronized movement given by only one motor, as shown in Fig. 1. Thus the robot is composed by two pairs of identical legs, one at the front and another at the rear. Each pair of legs is composed by two slider-crank mechanism OABC and OA'BC' as shown in Fig. 2. The input angle  $\theta$  is defined as the angle between link OA and the horizontal line. The phase between link OA and link OA' is  $\pi$  radian.

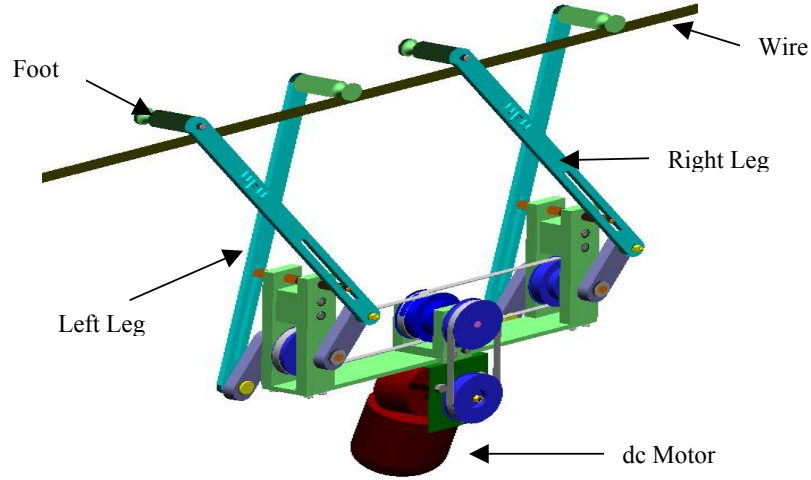


Figure 1. General configuration of the mobile robot suspended on wire.

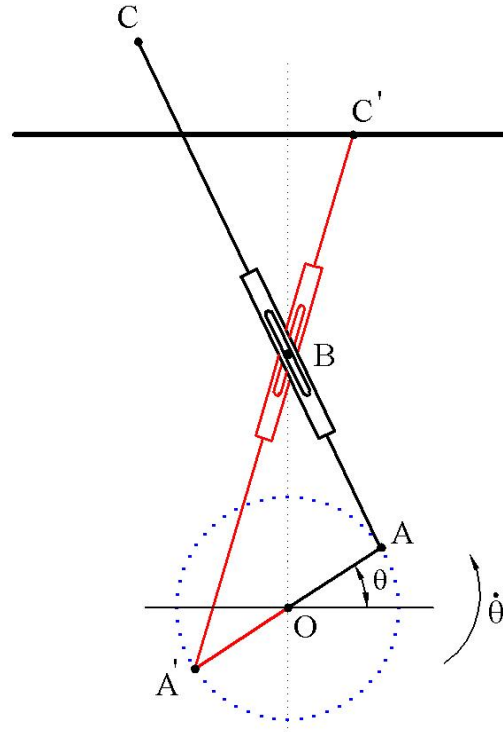


Figure 2. Pair of legs composed by two slider-crank mechanisms. The foot C' is on the wire.

Thus, the robot structure has two legs at right and two at left. The legs at same side have the same movement. The stability is assured by the contact of the same side feet with the wire, at each instant. A lateral view of the mobile robot is sketched in Fig. 3, where the left feet are in contact with the wire.

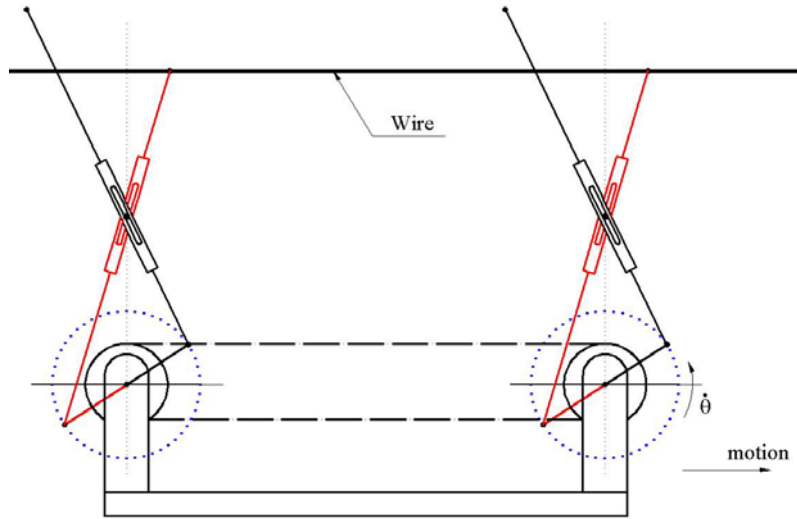


Figure 3. Lateral sketch of the mobile robot. The left feet are on the wire.

### 2.1. Kinematic Analysis for equal leg length.

When the legs have the same length one can observe that each foot keeps contact with the wire during a half of one rotational cycle. Then, the analysis of the robot motion can be obtained by considering two phases: the first one the foot leaves the wire and moves forward above the wire. In the second phase the foot is in contact with the wire and supports the robot weight. In the contact phase the robot moves forward. Thus each robot pair of legs repeats these two phases in turn assuring the continuous forward robot motion. The motion phase between the right and left legs is  $\pi$  radian then always have feet in contact with the wire.

By considering a reference frame attached on the wire and from the kinematic analysis of the slider-crank mechanism, the mobile robot trajectory can be described by the following equations:

$$x = x_{Oi} - \left\{ 1 - \frac{c}{\sqrt{a^2 + r^2 - 2ar \sin[\theta + 2(n - 1/2)\pi]}} \right\} r \cos[\theta + 2(n - 1/2)\pi] \quad (1)$$

$$y = - \frac{ac}{\sqrt{a^2 + r^2 - 2ar \sin[\theta + 2(n - 1/2)\pi]}} - \left\{ 1 - \frac{c}{\sqrt{a^2 + r^2 - 2ar \sin[\theta + 2(n - 1/2)\pi]}} \right\} r \sin[\theta + 2(n - 1/2)\pi] \quad (2)$$

Where  $x_{Oi}$  represents an initial position of the mobile robot; length of links OA, OB and AC are denoted by  $r$ ,  $a$  and  $c$ , respectively; and  $n$  is the number of gaits. A gait is defined by the distance, along the wire, between the position where the foot leaves the wire and where it contacts it again.

Theoretical and experimental studies carried out by Tsujimura and collaborators (Tsujimura and Morimitsu, 1997) and (Tsujimura, Yabuta and Morimitsu, 1996), shows that the optimum link lengths for a smoothness robot motion is obtained for:

$$\frac{a}{r} = 1,6 \quad (3)$$

$$\frac{c}{r} = 6,6 \quad (4)$$

The link ratios given by relations (3) and (4) enable the robot moves horizontally at regular velocity without movement fluctuations.

The mobile robot studied by Tsujimura has equal leg length and is useful in avoiding obstacles when they are equally spaced along the wire and, the maximum dimensions of the transposed object is function of the foot trajectory

above the wire. If the obstacle are irregularly spaced an algorithm is necessary to control the motor motion in order to change the gait to avoid the collision between the foot and the obstacle.

In order to optimize the object dimensions we propose to use legs with variable lengths. The kinematic analysis of the new mobile robot, moved by only one motor, is presented in the following sessions.

## 2.2. Kinematic Analysis for unequal leg length.

When the robot has unequal leg length it describe two unequal gaits: one gait corresponds to the contact of the long-leg's foot with the wire and the other gait when the short-leg's foot contacts the wire.

Figure 4 represents a pair of different legs in several situations and is used to analyze the robot motion. The long-leg AC is represented by a continuous line (in black) and the short-leg A'C' by a dashed line (in red). The initial position of the robot is considered when the feet contact the wire, as shown in Fig. 4a. In this situation the initial value for  $\theta$  is  $\theta_0$ .

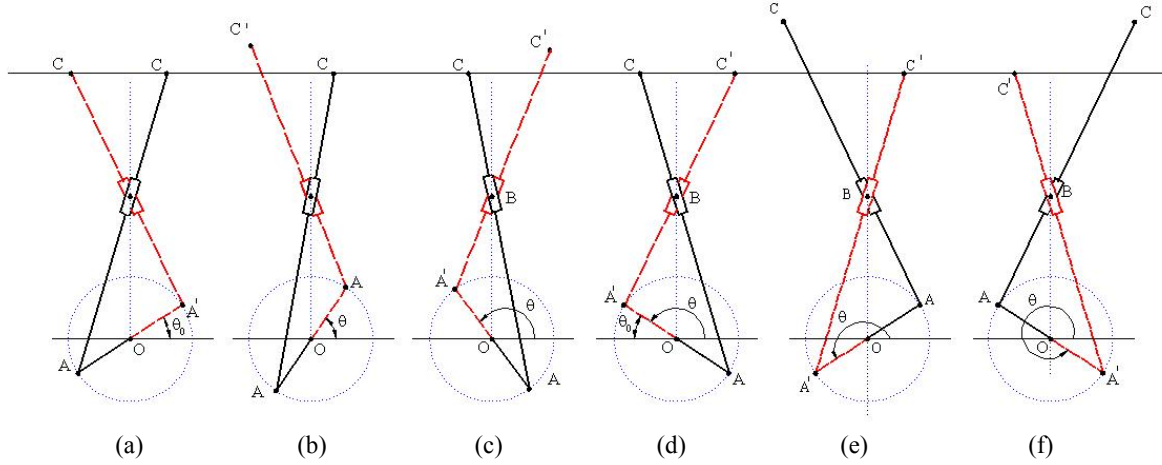


Figure 4. A pair of different legs in several situations for a motion cycle.

The first gait occurs when the long-leg's foot is in contact with the wire ( $\theta_0 < \theta < \pi - \theta_0$ ) and the short-leg's foot describes a trajectory above the wire. The second gait corresponds to the short-leg's foot contacts the wire and the long-leg's foot describes a trajectory above the wire ( $\pi - \theta_0 < \theta < 2\pi + \theta_0$ ). At  $\theta = \theta_0$  and  $\theta = \pi - \theta_0$  the two feet contact the wire simultaneously. The total cycle of the input link motion is  $2\pi$  radians.

From the kinematic analysis of the slider-crank mechanism the trajectory of the mobile robot can be given by coordinates  $x_{ul}$  and  $y_{ul}$  as:

$$x_{ul} = x_{O_i} + \sigma \left[ r \cos \theta_0 \left( \frac{d}{\sqrt{a^2 + r^2 + 2ar \sin \theta_0}} - l \right) - x_c \right] + (1 - \sigma) \left[ r \cos(\pi - \theta_0) \left( 1 - \frac{c}{\sqrt{a^2 + r^2 - 2ar \sin(\pi - \theta_0)}} \right) - x_{c'} \right] + 2(1 - \sigma) \left[ r \cos \theta_0 \left( \frac{d}{\sqrt{a^2 + r^2 + 2ar \sin \theta_0}} - l \right) \right] \quad (5)$$

$$y_{ul} = (1 - \sigma) \left\{ r \sin \theta + \frac{c[a - r \sin \theta]}{\sqrt{a^2 + r^2 - 2ar \sin \theta}} \right\} + \sigma \left\{ \frac{d[a + r \sin \theta]}{\sqrt{a^2 + r^2 + 2ar \sin \theta}} - r \sin \theta \right\} \quad (6)$$

Where  $\sigma$  is a binary parameter identifying if the short-leg's foot contacts the wire or not. When the short-leg's foot is above the wire,  $\sigma = 1$  and, if it contacts the wire,  $\sigma = 0$ . Segments OB, OA, AC and A'C' are denoted by  $a$ ,  $r$ ,  $d$  and  $c$ , respectively and,  $x_c$  and  $x_{c'}$  are given by:

$$x_c = r \cos \theta \left( \frac{d}{\sqrt{a^2 + r^2 + 2ar \sin \theta}} - l \right) \quad (7)$$

$$x_{c'} = -r \cos \theta \left( \frac{c}{\sqrt{a^2 + r^2 - 2arsen\theta}} - 1 \right) \quad (8)$$

In order to verify the maximum dimensions of an obstacle that the mobile robot can transpose it is necessary to analyze the feet trajectory when they describe a trajectory above the wire.

From the kinematic analysis of the mechanism the trajectory of the foot related to a reference frame fixed on the wire can be given by the coordinates  $x_{ulf}$  and  $y_{ulf}$  as:

$$x_{ulf} = (1 - \sigma) \left[ x_{ulc}' + r \cos \theta \left( \frac{c}{\sqrt{a^2 + r^2 - 2arsin\theta}} + \frac{d}{\sqrt{a^2 + r^2 + 2arsin\theta}} \right) - 2 \right] + \sigma x_{ulc} \quad (9)$$

$$y_{ulf} = (1 - \sigma) y_{ulc} + \sigma y_{ulc}' \quad (10)$$

Where

$$x_{ulc} = r \cos(\theta) \left( \frac{c}{\sqrt{a^2 + r^2 - 2arsin(\theta)}} + \frac{d}{\sqrt{a^2 + r^2 + 2arsin(\theta)}} - 2 \right) \quad (11)$$

$$y_{ulc} = \frac{d(a + r \sin(\theta))}{\sqrt{a^2 + r^2 + 2arsin(\theta)}} - \frac{c(a - r \sin(\theta))}{\sqrt{a^2 + r^2 - 2arsin(\theta)}} - 2r \sin(\theta) \quad (12)$$

$$x_{ulc}' = r \cos(\theta) \left( 2 - \frac{c}{\sqrt{a^2 + r^2 - 2arsin(\theta)}} - \frac{d}{\sqrt{a^2 + r^2 + 2arsin(\theta)}} \right) \quad (13)$$

$$y_{ulc}' = 2r \sin(\theta) - \frac{d(a + r \sin(\theta))}{\sqrt{a^2 + r^2 + 2arsin(\theta)}} + \frac{c(a - r \sin(\theta))}{\sqrt{a^2 + r^2 - 2arsin(\theta)}} \quad (14)$$

### 3. Simulations and Analysis

Simulations have been made using the obtained kinematic equations and graphical software. Results have been compared by considering the foot and the mobile robot trajectories. Both trajectories were compared for the mobile robot with equal and unequal legs.

The trajectories could be verified by using a graphical simulator. Figure 5 represents feet trajectories for an equal leg length robot, both obtained from kinematic equations as by the graphical simulator. The dimensions correspond to the constructed prototype, described on section 4. The curves are quite similar.

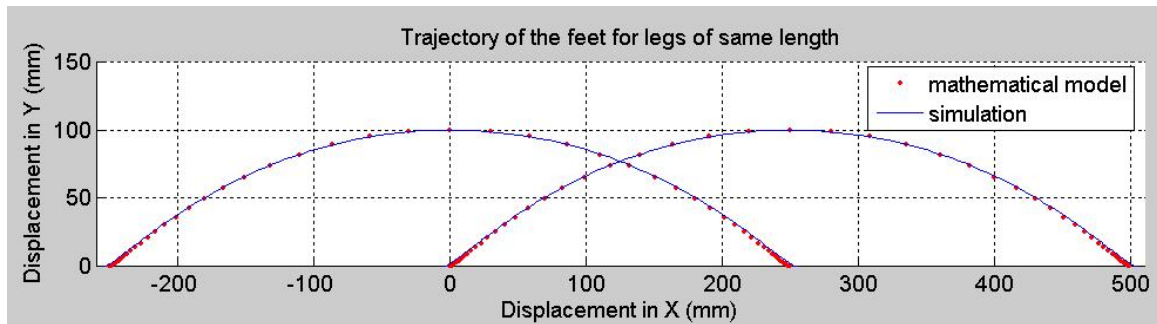


Figure 5. Example of graphical and numerical simulations for feet trajectories for an equal legs robot with ( $a = 80\text{mm}$ ,  $r = 50\text{mm}$ ,  $c = 330\text{mm}$ ).

In order to analyze the obstacle dimensions that the mobile robot can transverse firstly it was analyzed the feet trajectories for equal and unequal leg lengths. Figure 6 represents the results of the analysis. Feet trajectories for equal leg length are represented by a continuous line (brown line). One can note that the gaits are equals, both for the right and left feet. The maximum obstacle dimension placed on the wire is limited by the collision possibility between the foot and the obstacle. Considering the obstacle as a circle, its maximum dimension is represented by the red line. For unequal leg length, the trajectory described by the short-leg's foot is smaller then the equal leg length. This condition makes the obstacle dimension smaller then the obstacle dimension for equal legs. In Figure 6 feet trajectories for unequal leg length are represented by blue dotted lines. The indicated dimensions are those from the prototype. Other link length gives similar results.

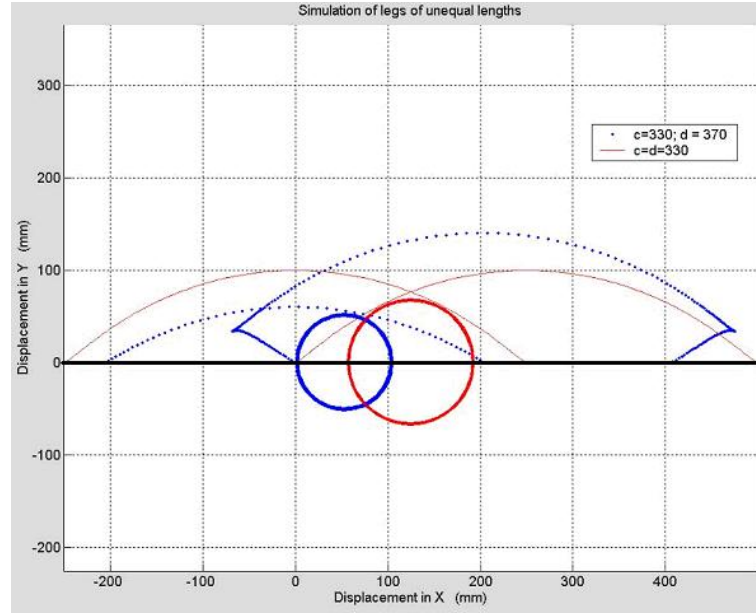


Figure 6. Feet trajectories and obstacles dimensions for equal and unequal leg length robot.

Other disadvantages of this solution are: a) the foot trajectory is not smooth presenting brusque variations in trajectory and velocity, introducing undesirable vibrations; b) the robot horizontal forward motion presents important fluctuations whose can compromise its stability. For trajectories represented in Fig. 6, the equal legs have 330mm and the unequal leg one leg length (link  $c$ ) is augmented from 330mm to 370mm and the other leg maintaining its 330mm. Thus, the maximum circle diameter reduces from 150mm to 100mm.

Then the first analysis doesn't enable an augmentation of the obstacle dimensions that the robot can transverse and the robot motion isn't stable. A second analysis can be done by considering that the leg length change only when its foot leaves the wire. This consideration assures the robot motion stability i.e.; the horizontal forward movement has minimum fluctuations as if have had equal legs. For example, Figure 7 shows the vertical robot motion for the prototype dimensions, the maximum oscillation peak is 0.35mm. This solution enable also augments the obstacle dimension that the robot can transverse.

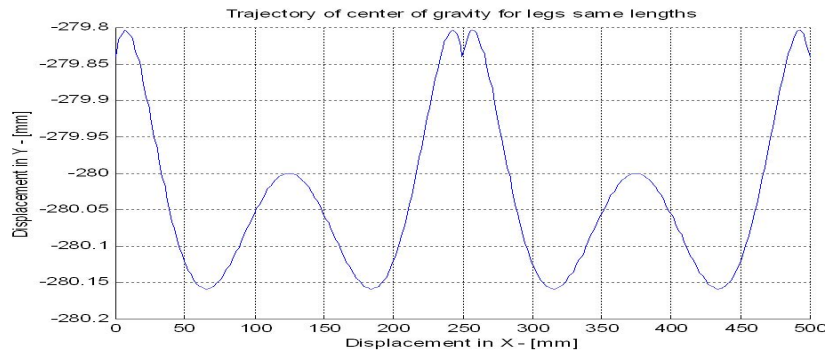


Figure 7. Trajectory of the robot center of mass for equal legs ( $c = 330\text{mm}$ ,  $r = 50\text{mm}$ ,  $a = 80\text{mm}$ ). Maximum vertical displacement: 0.35mm

When the foot leaves the wire one can control the leg length variation in such way the circle diameter is maximum as shown in Fig. 8. From Figure 8 one can see that the maximum circle diameter is the distance between the points where a foot comes back to the wire and the point where the other foot leaves the wire.

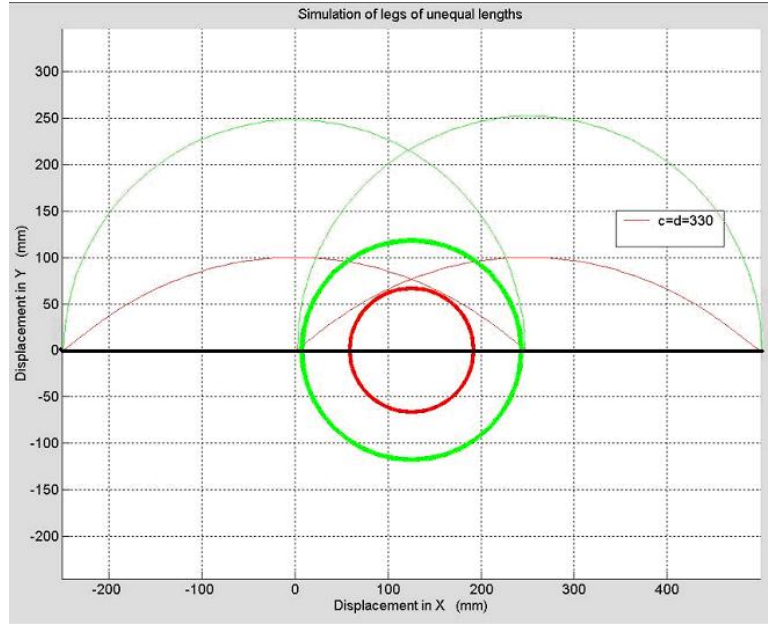


Figure 8. The maximum obstacle dimension that the mobile robot can transpose. Red line: maintaining equal leg length; Green line: varying the leg length when the foot leaves the wire.

From kinematic analysis of the mechanism the maximum circle diameter is given by:

$$D_{max} = 2r \left( \frac{c}{\sqrt{a^2 + r^2}} - 1 \right) \quad (15)$$

The leg length variation function must be sufficient to enable the foot over pass the maximum diameter. For the cited example ( $c = 330\text{mm}$ ,  $r = 50\text{mm}$ ,  $a = 80\text{mm}$ ) the circle diameter rises from  $150\text{mm}$  to  $250\text{mm}$  i.e.; 66,6% bigger. In Figure 8 the leg length variation enable the foot describe a semi-circular trajectory. Optimum leg length variation function is still in study.

#### 4. Experimental System

A mobile robot was constructed to validate the numerical results. The links ratio respect those obtained by Tsujimura i.e.;  $a:r = 1,6$  and  $c:r = 6,6$ , that allows an horizontal stable motion. For the constructed robot the ratio adopted are:  $a:r:c = 80:50:330$ .

The mobile robot structure is in aluminum and the synchronism of the motion legs are given by a dc motor, pulley and timing belt.

The constructed mobile robot is shown in Fig. 9 with same leg length. Experimental tests are still carrying out by considering the variation of the leg length.

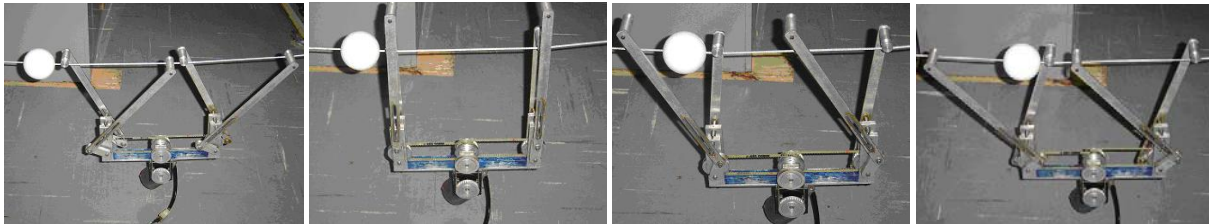


Figure 9. Photos of the mobile robot prototype suspended on wire avoiding an obstacle.



## 5. Conclusions

Kinematic analysis of a four legged mobile robot suspended on wire showed that an optimized dimensions of an obstacle, in which the robot can transpose, can be obtained by varying the leg length when it leaves the wire. This maximum dimensions can be obtained by assuring the robot motion stability too. Another advantage refers to use only one actuator to move the four robot legs. A prototype was constructed to verify the capabilities of the four legged robot.

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