A PIECEWISE NONLINEAR CONTROL APPLIED TO VEHICULAR DYNAMICS

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Abstract. *Great part of the works in control theory applied to vehicle dynamics use linear models, valid in small regions of the domain of system parameters, representing part of the behavior of a complex vehicular system. To cover the domain more thoroughly, integrating the different local controllers, it is necessary to demonstrate that commutation among controllers keeps stability and other properties of interest of the controlled system. Another way is to obtain more complete and computationally feasible models, designing new and more complex control strategies, aiming at efficacy and reachability. The present work approximates both ways, searching for new controller comprising bigger valid domains. Using nonlinear models with better comprehension, it is possible to suggest the division of the domain of the system into distinct regions, designing a controller for each one of them – either by piecewise linearization, or piecewise nonlinear approximations, complemented in each region to deal with specific problems that may come up. Local controllers are then united to form a unique nonlinear controller, able to acting in different vehicle behaviors. The tests allow checking for stability maintenance and tracking of a desired trajectory.*

Keywords*: piecewise linearization, nonlinear control, vehicle dynamics, optimal control*

1. Introduction

This work intends to promote an initial study on homeostatic controllers, which are able to keeping a system inside predetermined stability regions or, whenever outside those regions, take the system back to stability, as an effort to control the controller itself. The study comprised reference research and attempts to apply one technique found to vehicle dynamics.

The present study is necessary due to inconsistencies detected during the test phase of the master dissertation work (Spinola 2003), with respect to the performance of the vehicle. During Spinola (2003) two distinct nonlinear models were developed and served as basis for the design of nonlinear controllers based on state feedback. With these two models it was possible to notice limitations, which impeded the achievement of a correct operation.

These limitations, which deal with emergency limit conditions when driving, as slip, obstacle avoidance and hard turns at high speed, determine validity regions for the developed controllers. Apart from imposed physical limitations, there are some introduced by the modeling process, which can make the design of an accurate model unfeasible, if it is too complex. So, from the results obtained in Spinola (2003) it is possible to reach some observations.

One first observation relates to model accuracy. The first one, simpler, corresponds to a specific type of vehicle behavior, at constant speed. There are no effects of acceleration or brake considered at this first analysis. It corresponds to a distinct behavior that modifies vehicle characteristics. Likewise when at brake mode other characteristics, especially those of tire behavior, its steers and forces, may change model global behavior.

A second observation is that there are different models for different actions, like accelerations and braking, or constant speed movement. There is then a problem of defining validity regions for the distinct types and behaviors a vehicle can assume during its movement. The most natural idea is to split the main model into specific parts, each one able to correctly representing its behavior and allowing for the design of a controller that can track a desired path, no matter what kind of behavior the vehicle is subjected to.

That is what can be done as initial research work, comprising linear and piecewise nonlinear systems theory. As there is also a need to check if each part of the modeling is valid and if all pieces, once examined, can be reconnected, it is necessary to research linearization methods and stability analysis. Now work is being done to develop techniques capable of spreading stability regions, by specific actions and well-defined conditioning rules, which could take the controller back to stability and enlarge its validity. This line of thought will lead this work and stability analysis will be left for further works on the subject.

2. Modeling and Control Design

From the results of Spinola (2003), it was taken the decision to evaluate one of the applications found, over a well known vehicle dynamic, and try to begin the design of a controller able to checking its behavior and promoting correctional actions, whenever necessary. The closer of the proposed control application was found in Takata (1996) and corresponds to linearize the model, using 1st order Taylor series expansion around a well-known initial point of the domain, a system such as described in equation (1).

$$\dot{x} = f(x) + g(x)u \therefore x \in D \tag{1}$$

where $D \subset R^n$ is the domain of the system.

This domain can be divided into known pieces and make the design of accurate controllers simpler. The idea consists in building distinct controllers able to acting in each subdomain of the system, without discontinuities that may cause changes on its behavior. To do so, Takata (1996) uses optimal linear control theory to design the controller in each of the subdomains, together with a smoothing function to avoid leaps among different subdomains. Although simulations prove this broach to be effective, Takata (1996) still introduces a choosing function to decide which controller to use, depending on which subdomain in use. This new concept has allowed a bigger enlargement of the validity regions for piecewise controllers, enhancing their performance and the system itself.

Based on Takata (1996), some modifications are introduced to adequate the kind of problem found in vehicle dynamics. The first modification on the work done in Takata (1996) deals with the type of controller used. It is a state regulator that aims only system stabilization, according to the definitions of the domain of the phase angle, one of the states of the system. When working with vehicle dynamics, it is necessary to change the type of controller from a state regulator to a tracking controller of a well-known path. To do so it was decided to use the optimal linear control theory presented in Sage (1977) and discussed here, as follows.

Beginning with equation (1), we linearize the system obtaining a linear representation, described in equation (2).

$$\begin{aligned} \dot{x} &= A_i x + B_i u + w_i \\ y &= Cx \end{aligned} \tag{2}$$

where

$$A_i = \frac{\partial f(\hat{X}_i)}{\partial \hat{X}_i} \tag{3}$$

$$B_i = g(\hat{X}_i) \tag{4}$$

$$w_i = f(\hat{X}_i) - A_i \hat{X}_i \tag{5}$$

Following the optimal control theory presented in Sage (1977), it is proposed a cost function to minimize state variables and the error among desired path and output of the system, as the one described in equation (6) bellow.

$$J = \frac{1}{2}\int_0^\infty (e^T Q_i e + u^T R_i u)dt \tag{6}$$

$$e(t) = \eta(t) - y(t) \tag{7}$$

where $\eta(t)$ is the desired path to be followed. As in Takata (1996), Q_i and R_i are positive definite matrices with their values defined according to some experience and engineering sensibility. Applying optimal control theory to equations (2) and (6), we find as control function the relation presented in (8).

$$\hat{u}_i = -R_i^{-1} B_i^T \left[P_i x_i - \xi_i \right] \tag{8}$$

The matrix P_i is satisfied with the Riccati equation as shown in (9).

$$A_i^T P_i + P_i A_i - P_i B_i R_i^{-1} B_i^T P_i + C_i^T Q_i C_i = 0 \tag{9}$$

The vector ξ_i is given by equation (10).

$$\begin{cases} \xi_0 = 0 \\ \xi_1 = \left(A_i^T - P_i B_i R_i^{-1} B_i^T \right)^{-1} \left[P_i w_i - C_i^T Q_i \eta \right] \end{cases} \tag{10}$$

Following the very same concept adopted in Takata (1996), we can establish a weighting function to avoid shifts at the equilibrium point, as shown in (11).

$$\begin{cases} u_0(x) = \hat{u}_0(x) \\ u_i(x) = \hat{u}_i(x) \beta_i(x) \end{cases} \tag{11}$$

where $\beta_i(x)$ stands for

$$\beta_i(x) = 1 - exp(-x^T S_i x),$$

and S_i is a positive semi-definite symmetric matrix.

As the intention here is to verify the applicability of the work developed in Takata (1996) to vehicle dynamics, as a start point for the homeostatic control, we reintroduce the choosing functions used in Takata (1996).

$$\begin{cases} I_{oN}(x) = 1 - \sum_{i=1}^{M} I_{iN}(x) \\ I_{iN}(x) = \prod_{j=1}^{L} I_{iN}(x; j) \quad (i \neq 0) \end{cases} \tag{12}$$

where I_{iN} can be of two types: trapezoid or sigmoid. These equations are presented as follows, in (13) and (14).

$$I_{iN}(x; j) = \frac{1}{1 + \left(\left(C_j(x) - m_{ij} \right) / h_j \right)^{2N}} \tag{13}$$

where $m_{ij} = (a_{ij} + b_{ij})/2$, $h_j = (b_{ij} - a_{ij})/2$.

$$I_{iN}(x; j) = \left\{ 1 - \frac{1}{1 + exp\left(2N \left(C_j(x) - a_{ij} \right) / h_j \right)} - \frac{1}{1 + exp\left(-2N \left(C_j(x) - b_{ij} \right) / h_j \right)} \right\} \tag{14}$$

where $0 < h_j < \propto$, $a_{ij} = -\propto$ or $b_{ij} = +\propto$ is allowed.

By joining equations (11) and (12) we find equation (15), the feedback control, at which the choosing functions, represented by (13) and (14), choose the most suitable controller to each subdomain.

$$u(x) = \sum_{i=0}^{M} u_i(x) I_{iN}(x) \tag{15}$$

All this development to compute a piecewise linear controller will be then applied over a simplified model of vehicle dynamics, as introduced in Spinola (2003), where the interest is focused at the lateral movement with nonlinearities applied to the coordinate transformation from a local to a global referential, fixed in space. This model is presented in equation (16).

$$\begin{bmatrix} \dot{v} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} -U\dot{\theta} + \dfrac{F_y}{m_{tot}} \\ \dot{\theta} \\ \dfrac{\Gamma}{I_z} \\ -U\ sen\ \theta - v \cos\theta \end{bmatrix} \tag{16}$$

where

$$F_y = 2C_f\left(u - \frac{v + a\dot{\theta}}{U}\right) + 2C_r\left(\frac{b\dot{\theta} - v}{U}\right) \tag{17}$$

$$\Gamma = 2aC_f\left(u - \frac{v + a\dot{\theta}}{U}\right) - 2bC_r\left(\frac{b\dot{\theta} - v}{U}\right) \tag{18}$$

At equations (16) to (18), the constants m_{tot}, U and I_z represent the total mass of the vehicle, the longitudinal velocity and moment of inertia around *Z-axis* of the vehicle referential, respectively. C_f stands for the frontal stiffness and C_r for the rear, a is the distance between the frontal axle and the CG and b is the distance between the rear axle and the CG. The control input u corresponds to the steering angle provided by the controller. The numerical values used to simulate the model with controller action are as follows:

$C_f = C_r = 20000\ N/rad$;

$m_{tot} = m_s + m_{uf} + m_{ur} = 1280+119+96 = 1495\ kg$;

$I_z = 2500\ kgm^2$;

$U = 18{,}3\ m/s \approx 65{,}88\ km/h$;

$a = 1{,}203\ m$;

$b = 1{,}217\ m$;

As the state variable associated to the model nonlinearity is the yaw angle of the curve movement, this will be the separative variable *C(x)*, whose domain shall be split into different amounts of subdomains. This quantity shall be determined during the test phase, as a device to improve control action. One possible choice is:

$$
\begin{aligned}
D_1 &= \left[-\pi, \frac{-2\pi}{3}\right], \hat{X}_1 = \frac{-5\pi}{6} \\
D_2 &= \left[\frac{-2\pi}{3}, \frac{-\pi}{3}\right], \hat{X}_2 = \frac{-\pi}{2} \\
D_3 &= \left[\frac{-\pi}{3}, \frac{\pi}{3}\right], \hat{X}_3 = 0 \\
D_4 &= \left[\frac{\pi}{3}, \frac{2\pi}{3}\right], \hat{X}_4 = \frac{\pi}{2} \\
D_5 &= \left[\frac{2\pi}{3}, \pi\right], \hat{X}_5 = \frac{5\pi}{6}
\end{aligned}
\tag{19}
$$

3. Tests

After defining model and control to be applied, some simulation tests of trajectory tracking were made, seeking to determine the best amount of specific subdomains. The answers obtained so far have shown good results, considering the very simple model for the vehicle dynamics adopted. Two points that must be brought to attention are the fact that only one control input was used, more specifically the steering angle at the driver wheel, and the division of subdomains associated to only one separative variable.

One of the tests done represent a maneuver for simple lane change, where the driver commands a steering angle, either positive or negative depending on which side he wants to go, then returning the wheel to straight position. To represent the steer command, a senoid function was used to generate the desired trajectory. A change at its amplitude would represent a change at the steer angle passed to the controller. The senoid function was chosen for it best represents a true driver action with all its smoothness. The situations tested comprised variations over the number of subdomains the original system was divided into, variation over the steer angle being passed as input and velocity of the vehicle.

The best results were seen during the change at the number of subdomains. It is possible to see in Figures 1 through 9 some improvements with the growth of divisions of the domain of the yaw angle. One clear benefit of this growth is the decrease at the need for controller action, as seen in Figure 1, Figure 4 and Figure 7. Also its amplitude falls bellow half of the initial value. The improvement at the final result is very mild, but yet noticeable, as seen in Figure 3, Figure 6 and Figure 9. Also, with a greater variety of linear models, each connected to a different subdomain, the controller designed gets simpler and so its work to maintain the vehicle at the desired path.

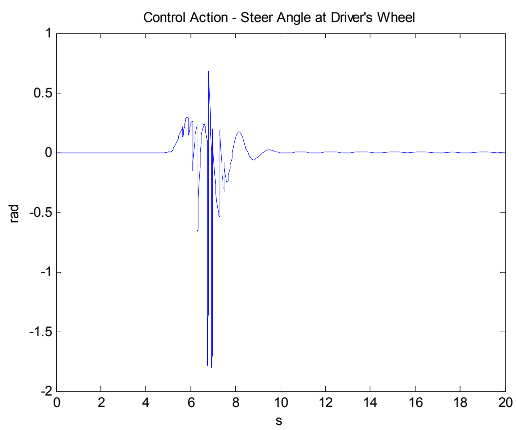

Figure 1: Control Action with 25 subdomains.

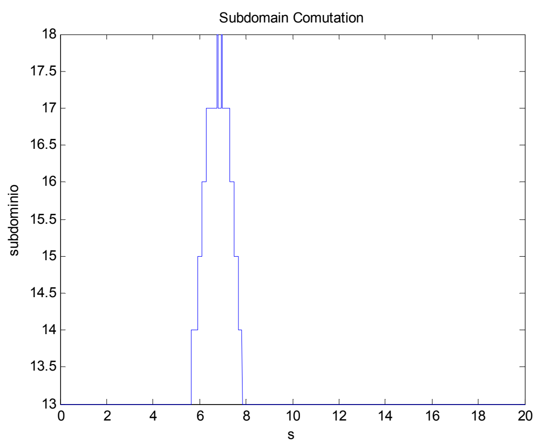

Figure 2: Commutation of subdomains among 25 possible.

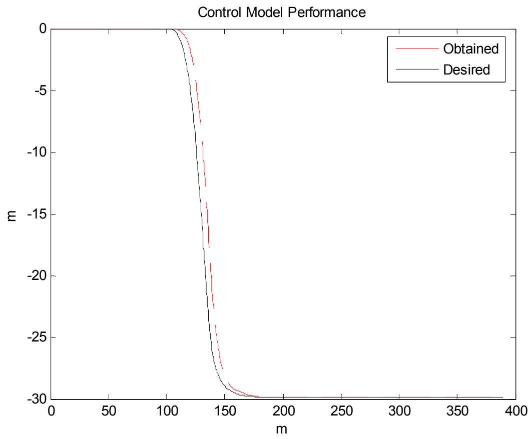

Figure 3: Final result with 25 subdomains.

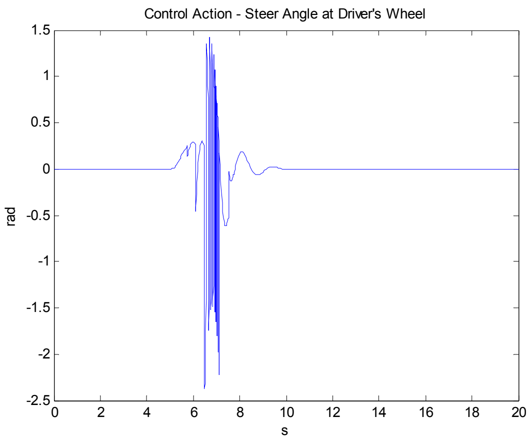

Figure 4: Control Action with 15 subdomains.

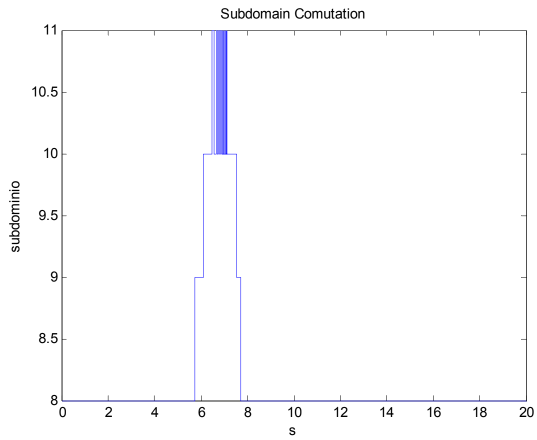

Figure 5: Commutation among 15 possible.

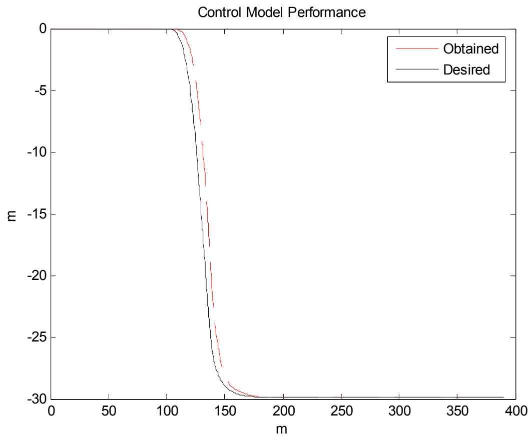

Figure 6: Final result with 15 subdomains.

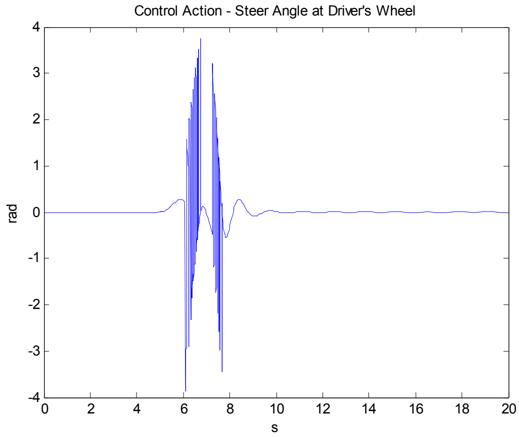

Figure 7: Control action with only 5 subdomains.

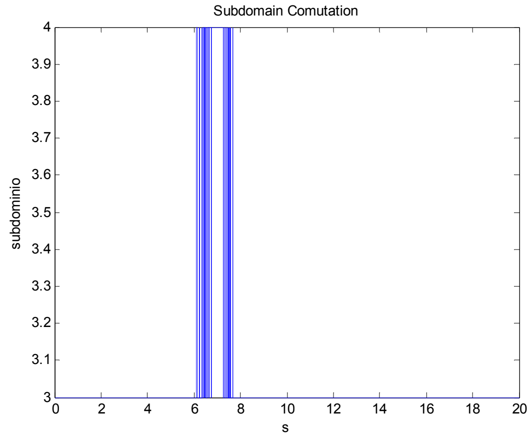

Figure 8: Commutation among 5 possible.

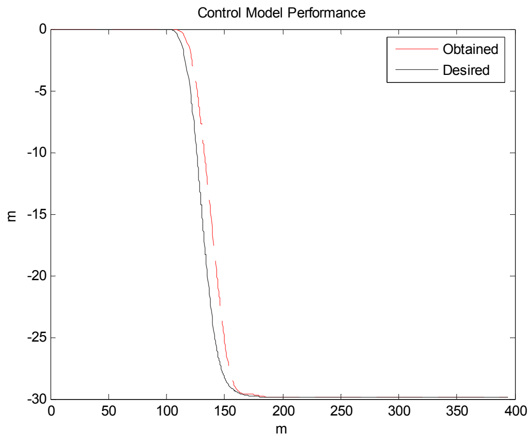

Figure 9: Final result with 5 subdomains.

4. Conclusions

According to the research done it is possible to identify an advance through ground vehicles control problem, such as presented in Spinola (2003). It consisted of promoting a piecewise linearization on a simpler model, around a known initial point, and then to design an optimal linear controller. Apart from testing a piecewise linearization technique, we should have in mind the variety of behaviors the vehicle can assume, which leads to the design of distinct models for each different type of behavior and that must be taken into account. Following this idea Takata (1996) has shown to be the most adequate as a start point to a much deeper study on solutions that can be used as a basis to the trajectory control of ground vehicles problem.

After some initial testing in a simpler model, but yet nonlinear, it was possible to identify that the technique used in Takata (1996) is suitable to vehicle dynamics. It was possible to see an improvement in tracking performance with the growth of the division of the original domain. An immediate consequence observed at the global controller was a smaller amount of action required, because of a greater diversity of controllers and smaller areas for analysis, which simplify the understanding of the system from the control point of view. All this improvement made the system faster and with better answers. Even tough it has worked properly, there are some important points that must be studied on further works.

The first one is to try and determine an optimal subdivision of the domain of the system, avoiding an excessive amount of controllers. A second point would be trying other, more complete and more complex dynamic models. But these models are still being developed and must be tested and validated prior to use with control. A third and important point deals with physical restrictions inherent to vehicles and that do not represent the restrictions of a state variable. Restrictions such as a physical limit to the steering of a wheel, a tire adherence limitation, and brake or acceleration pedals control to avoid slip. Also there exists the question on what action to take once the vehicle has passed its stability limits. At this moment the controller should actuate in a manner to restore any parameter, which is responsible for this outbound characteristic, back to its safe region of operation.

These considerations are a bit far from the technique used throughout this work. In an opposite way, the technique intends to improve behavior of a dynamic system inside its stability regions, by increasing them or improving closed loop system response. The homeostatic controller needs this improvement, but also needs a collection of rules to allow the system to choose which action to fulfill, once outside stability bounds.

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