AN ALTERNATIVE APPROACH FOR THE TREATMENT OF THE DYNAMICS OF GROUND VEHICLES ON SUSPENSION.

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Abstract. *The first results reached in the development of a specific modeling procedure for the dynamics of ground vehicles on suspension, based on the coupling of preexhistents modules and on the concept of power flow are presented The model aims at the creation of integrated vehicle systems' models for application in control, and vehicles design analysis and simulation. The simplicity of implementation and the modularity of the alternative approach justify its creation.*

Keywords: *Power Flow, Power Variables, Coupled Models, Vehicles Dynamics, Suspensions Models*

1. Introduction

Currently, there are several specific techniques of Multibody Systems modeling, as presented in Haug (1989), that use pairs of joint formulations, or even Kinematical Transformers of Hiller *et al*. (1997), as described and applied in Costa (2001) and Neves (2002) to vehicle dynamics problems. Also, Bond Graphs, presented in Karnopp et al. (1990), and its more generic form in multibonds, can be used for multibody systems in general, and, in particular, to the vehicle dynamics. In this work, is shown a procedure based on the power flow (Speranza Neto, 1999) and on creation of specific component models coupled by means of the verification of the causality compatibility between power variable inputs and outputs. None of these known techniques is explicitly adopted, but an adequate combination of each one of them. The goal is to obtain individual component models in a simple form, as much analytically as possible, and coupled them with the system's other element models, by the verification of cause and effect relations, in order to find a open global model, not analytical, but totally consistent. As example, the procedure for a 2-DOF quarter-car model is applied, whose degrees of freedom are the center of gravity's vertical displacements of both chassis and the wheel, including the suspension mechanism, and its compliance elements, coil spring and shock absorber, both nonlinear, treated according to functions found in the current literature. The systems are organized in a block diagram, where each one of the blocks contains the equations that represent the respective element. The connection among the blocks is done through power variables, it forces and velocities. The kinematic restrictions imposed by the suspension mechanism are interpreted through closing equations, which are used to generate polynomials, which represent the bonds for the relative position of the degrees of freedom. This condition is used to evaluate the kinematic behavior of the mechanism in a pré-processor. Together with the restrictions imposed to the degrees of freedom of rolling and lateral displacement of the chassis, it is shown as it is obtained a transformer of efforts, which represents the dynamics of the suspension mechanism. Five types of mechanisms are set out, where is used the suspension type known as parallel control arm to obtain the equations of the kinematics and the expression of the transformer of efforts.

2. The Vehicle System

The procedure of modular modeling is initiated with the organization of a system subsystems, in this case a ground vehicle, in a graphic representation form (Fig. 1), where each block represents a subsystem or component with the respective equations that describe its kinematical and/or dynamic behavior. The coupling between blocks is made by power variables, that are, for mechanical translational forces and linear speeds and for rotational systems, torques and angular speeds. Each block can be substituted by another one, given that input-output power variables are of the same kind, characterizing the modeling procedure modularity.

The 2-DOF quarter car model can be divided in four subsystems: chassis, suspension, wheel and tire, as shown in Fig. 2. The chassis and wheel are considered rigid bodies, while the tire is treated as spring-damper system, where the compliant elements are linear. In order to satisfactorily represent the interdependence of chassis and wheel's degrees of freedom, the suspension geometry has to be taken into account. The suspension course is limited by the length of the shock absorber and by the suspension upper bumper. This bumper is mounted coaxially with the shock absorber piston rod. In automotive suspensions, the coil spring always works in compression, and the assembly can receive a pre-load. These conditions are also included in the model. Here five different types of independent suspensions, among the most common used in passenger vehicles, are used. A set of equations representing the different suspension geometries have been created for each of them. The five types are: Swing Axle, Parallel Control Arms (equal length control arms), McPherson, Double A-arm (unequal length control arms) and one named Conventional. This last one restricts the wheel movement in order to keep it parallel to the vertical line described by the chassis' center of gravity. In the four first suspension models, the wheel's CG describes an arc in relation to the chassis. In all cases are, coil spring and the shock absorber are considered to be coaxial, and their constitutive relations are represented by polynomials.

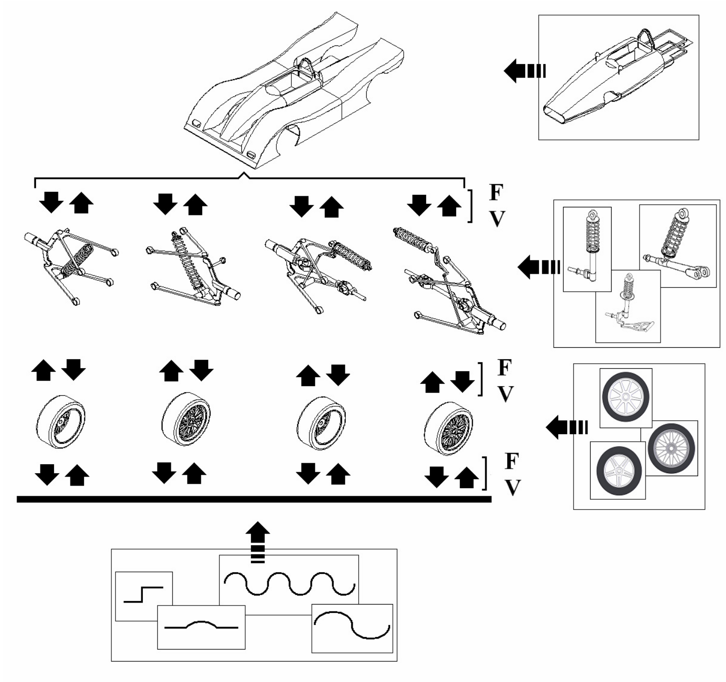

Figure 1. Modularity in the treatment of a ground vehicle with suspension.

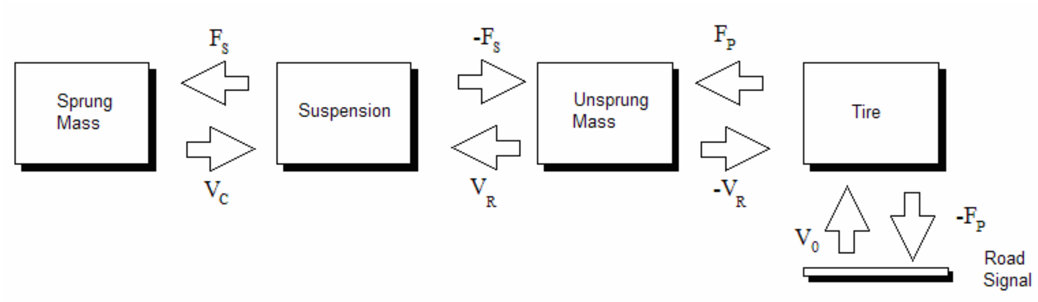

Figure 2. Power flow of a ground vehicle model for vertical dynamics analysis.

The shock absorber and the coil spring generate forces that are functions, respectively, of the chassi-wheel relative speed and relative positions of wheel's CG – chassis' CG. The value of the deformation of the coil spring is needed to calculate its force, given by a set of equations that represent the suspension geometry; and to calculate the force generated by the shock absorber, the relative speed between the chassis and wheel is calculated. The suspension is seen as an element that receives relative speeds and produces a force.

The geometry, however, can be simplified in a way that it is represented by a function that describes its behavior. Given that it is a closed-loop mechanism with just one degree of freedom, any geometric configuration can be determined in function of the position of just one of its parts in relation to the chassis. Thus, the coil spring length is calculated in function of the wheel position in relation to the chassis' CG. Because it is made out of linked elements that have movements relative to others, relations between the element positions are generally nonlinear. The geometric relations between the mechanism elements are initially given by the formularization of Kinematical Transformers (Hiller *et al.*1997), where mechanism links are represented by variables that describes their degrees of freedom, trying to relate them by scalar equations. From this point on, the procedure does not follow the Kinematical Transformers method.

To determine the position of the CGs of wheel and chassis, three Cartesians reference frames, F, Q and R (Fig. 3). The frame F is fixed on the ground; the frame Q follows the vehicle, and its origin is the projection of point C on the ground; the frame R is the chassis body-frame, and its origin, the point C, coincides with the chassis CG. The z_F and z_Q axis are always parallel, and x_Q and y_Q are in the same plan as x_F and y_F and, finally, x_Q lies in the $x_R z_R$ plan (the longitudinal chassis plan).

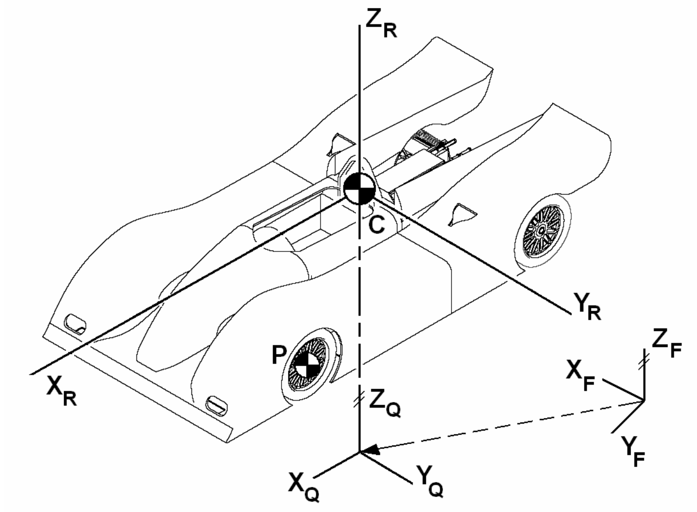

Figure 3. F, Q and R frames and point P where CG-wheel is located.

3. Kinematical Transformer and Effort Transformer

The suspension is analyzed separately, with their proper variables, called internal variables, that represent distances between points located on its parts, or angles formed between them, always in accordance with the suspension geometry. One of these internal variables is the instantaneous length of the coil spring, that is adopted to represent the suspension degree of freedom, because it can be directly measured. The chosen coordinate is called DOF-variable. Writing the set of n internal variables in the vector form, we have:

$$\{\beta\}^T = [\beta_1 \quad \beta_2 \quad \beta_3 \quad \cdots \quad \beta_n] \tag{1}$$

According to the Kinematical Transformers method (Hiller *et al.*1997), internal variables can be calculated sequentially using the set of equations with closed solution. These equations are scalars, and are written based on the mechanism's geometry. The set of these equations is called closure-loop vector:

$$\{g(\{\beta\},\lambda)\} = \begin{bmatrix} \beta_1 - h_1(\lambda\ ) \\ \beta_2 - h_2(\beta_1) \\ \beta_3 - h_3(\beta_1,\beta_2) \\ \vdots \\ \beta_n - h_n(\beta_1,\beta_2,\beta_3,\cdots,\beta_{n-1}) \end{bmatrix} = \underline{0} \tag{2}$$

where:
λ: degree of freedom;
h_i: expression that relates an internal variable β_i to λ or at least to one of the remaining *n-1* internal variables β_j where $i \neq j$ are written in the form of a function.

The variable λ can assume m distinct values, and, as it represents the current length of the coil spring, it is restricted to vary within a finite interval. Any determined variable β_Σ is not necessarily calculated in function of all other previous $s - 1$ variables, it can be the function of only one. The position of the suspension mechanism parts are limited by values that lies in an interval of *1* to m; therefore, it can be assumed that, when attributing one defined value to the DOF-variable, all the other internal variables are then defined. For each distinct coil spring length λ_k there are a corresponding vector $\{\beta_k\}$ calculated by the closure-loop vector:

$$\{\beta_k\}^T = [\beta_{1,k} \quad \beta_{2,k} \quad \beta_{3,k} \quad \cdots \quad \beta_{n,k}],\ 1 \le k \le m \tag{3}$$

The closure-loop vector defines the internal variables for one determined position of the mechanism, the vertical coordinate of the vector-position of the wheel's CG in relation to the chassis, called $^{R}\boldsymbol{q}_{P,k}$, can also be included, where:
R: chassis-frame, with origin C in the chassis' CG;
P: point where the wheel's CG is located.

Such inclusion can be made because the position of the wheel's CG in relation to chassis' frame keeps the link created by the suspension, independently of the relative movement of the chassis-wheel. It can be understood that the vertical coordinate of the wheel, when written in the referencial R, is also an internal variable. In general, any variable related to the wheel that can be written in the R-frame, without depending on the chassis' absolute coordinates, or on the wheel's, can be interpreted as an internal variable. However, since they do not directly belong to the set $\{\beta\}$, they are called *augmented internal variables*. When including such variables, the closure-loop vector of the Eq. (3) is written in the form:

$$\{G_k(\{\beta\},\lambda)\} = \begin{bmatrix} \{g(\{\beta_k\},\lambda)\} \\ --- \\ \{g'(\{\beta_k\})\} \end{bmatrix} = \begin{bmatrix} \{g(\{\beta_k\},\lambda)\} \\ -------------- \\ {}^{R}q_{P,k} - f_n(\beta_{1,k},\beta_{2,k},\beta_{3,k},\cdots,\beta_{n,k}) \end{bmatrix} = \{0\} \tag{4}$$

called *augmented closure-loop vector*. It is important to mention that one can include any augmented internal variable that he wants to calculate, for example, α_i as far as it is linked to at least one internal variable:

$$\{G_k(\{\beta\},\lambda)\} = \begin{bmatrix} \{g(\{\beta_k\},\lambda)\} \\ --- \\ \{g'(\{\beta_k\})\} \end{bmatrix} = \begin{bmatrix} \{g(\{\beta_k\},\lambda)\} \\ ------------------ \\ {}^{R}q_{P,k} - f_n(\beta_{1,k},\beta_{2,k},\beta_{3,k},\cdots,\beta_{n,k}) \\ \vdots \\ \alpha_{i,k} - \phi_i(\beta_{1,k},\beta_{2,k},\beta_{3,k},\cdots,\beta_{n,k}) \end{bmatrix} = \{0\} \tag{5}$$

where:
ϕ_i : expression that relates the augmented internal variable α_i to at least one internal variable β_i writed as a function;
μ: number of augmented internal variables; $1 \le k \le m$; and $1 \le i \le \mu$.

It is included, for example, in the augmented closure-loop vector, the camber angle and the angle of the direction of the shock absorber in relation to the chassis, measured in the latter's referencial. The values of augmented internal variables and internal variables are ordained according the vector:

$$\{\Psi_k\}^T = [\beta_{1,k} \quad \beta_{2,k} \quad \beta_{3,k} \quad \cdots \quad \beta_{n,k} \quad | \quad {}^{R}q_{P,k} \quad \cdots \quad {}^{R}\alpha_{i,k}] \tag{6}$$

and this vector is *the vector-solution of internal and augmented internal varibles.* That is, for each value λ_k attributed to the DOF-variable λ, there is a vector $\{\beta_k\}$, a closure-loop vector $\{g\}$, and a vector $\{\Psi_k\}$. Once the wanted variables are organized, we have the matrix M, defined as:

$$\{M\} = \begin{bmatrix} \{\Psi_1\}^T \\ \vdots \\ \{\Psi_m\}^T \end{bmatrix} = \begin{bmatrix} \beta_{1,1} & \beta_{2,1} & & \beta_{n,1} & {}^{R}q_{P,1} & \cdots & {}^{R}\alpha_{\mu,1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ \beta_{1,m} & \beta_{2,m} & \cdots & \beta_{n,m} & {}^{R}q_{P,m} & \cdots & {}^{R}\alpha_{\mu,m} \end{bmatrix} \tag{7}$$

where each line represents a configuration of the mechanism (line 1 in function of the value λ_1, line 2 in function of the value λ_2, and so on, until the m-line) and each column represents an internal variable or an augmented internal variable.

The matrix M has the dimension $m \times (n+\mu)$, where m is the number that represents how many values the variable λ can assume, n is the number of internal variables, and μ the number of augmented internal variables. From this matrix polynomials can be generated having a β_i as the only independent variable, or even an augmented internal variable like $^{R}\boldsymbol{q}_{Pk}$, can be chosen too. The choice of which variables, internal or augmented internal, will be the independent variable is made in accordance with convenience, and for all suspensions treated in the planar case, the independent variable is exactly $^{R}\boldsymbol{q}_{Pk}$. All other variables can be calculated with a polynomial of degree z:

$$\beta_j = p(\beta_i) = \sum_{s=0}^{z} a_s (\beta_i)^s,\ i \ne j \tag{8}$$

where a_s are the polynomial coefficients. Given that the suspension block inputs are the chassis and wheel's absolute vertical speed, one can obtain by integration the absolute vertical positions of each one of these parts, and then calculate the wheel position in the chassis frame:

$$\int_{t=0}^{t=T} {}^{Q}V_C dt - \int_{t=0}^{t=T} {}^{Q}V_P dt = {}^{Q}q_C - {}^{Q}q_P = {}^{R}q_P \tag{9}$$

For this reason, it is that the augmented internal variable $\boldsymbol{{}^{R}q_P}$ is chosen as the independent variable of the polynomial of Eq. (8). In the augmented closure-loop vector, shown in Eq. (5), each value assumed by $\boldsymbol{{}^{R}q_P}$ is calculated in function of the internal variables. The input and output are now inverted, and each β_i is calculated in function of $\boldsymbol{{}^{R}q_{Pk}}$ using the polynomial generated from the matrix $\{M\}$:

$$\begin{aligned} \beta_1 &= p_1({}^{R}q_P) = \sum_{s=0}^{z} a_{1s}\left({}^{R}q_P\right)^s \\ &\dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ \beta_n &= p_n({}^{R}q_P) = \sum_{s=0}^{z} a_{ns}\left({}^{R}q_P\right)^s \end{aligned} \tag{10}$$

It is not necessary, however, to calculate all internal variables, only the ones that are relevant. The degree of each polynomial is 3, because it was verified that this degree is the best for the different suspensions.

Since each suspension has its geometry represented by polynomials that define the length of the spring, the angle that the telescopic column makes with z_R axis, and the camber angle, an analogous procedure is established for the calculation of the forces that the suspension mechanism as a whole produces on the chassis and wheel. It is not created a force polynomial, but scalar functions, one for each direction (x_R, y_R and z_R), such as they can represent the effect of geometry in the resultant force (forces in the links and forces of the compliance elements) that acts on the chassis and the wheel. The resultant suspension force on the chassis and wheel in one determined direction is written, when possible, as a product of two factors, where one of them is the absolute value of the sum of compliance elements force, and the other is the function that contains the pertinent information of the suspension mechanism geometry. This function, in turn, is called Effort Transformer:

$$F_{i,j} = \Theta_j \cdot F_S \tag{11}$$

where:
$F_{i,j}$: absolute value of the suspension force on body i (chassis or wheel) in j-direction (x_R, y_R ou z_R);
Θ_j: Effort transformer in j-direction;
F_S: absolute value of the sum of compliance elements force.

The effort transformer is calculated in function of the internal variables and the augmented internal variables in the chassis' R-frame:

$$\Theta_j = \Theta_j\left(\{\beta\},\{\Gamma\}\right) \tag{12}$$

where:
$\{\beta\}$: vector of the n internal variable β_i
$\{\Gamma\}$: vector of augmented internal variables μ;
j: direction associated to the effort transformer (x, y and z axis).

To get the expressions of the effort transformers, the restrictions that are imposed to the chassis, that in case of a quarter-vehicle can only be move along the z_Q axis, must be considered. The wheel has one degree of freedom, since the geometry of the suspension imposes it.

4. Example: Parallel Control Arms Suspension Model (Equal Control A-Arm Model)

As an example, the procedure is applied to a suspension of the type shows in Fig. 4, called Parallel Control A-arms, or Equal A-Arms Control, that is totally independent, being a Double A-arm variation. Its characteristic is to have the control arms with equal lengths. For the analysis, the suspension is divided in two mechanisms. One is the suspension itself, with the telescopic column, represented in Fig. 4 by the ABD triangle, where side BD has a variable length. The other is the wheel positioning, represented in Fig. 4 by AEFG rectangle. The link ADG is common to the two mechanisms.

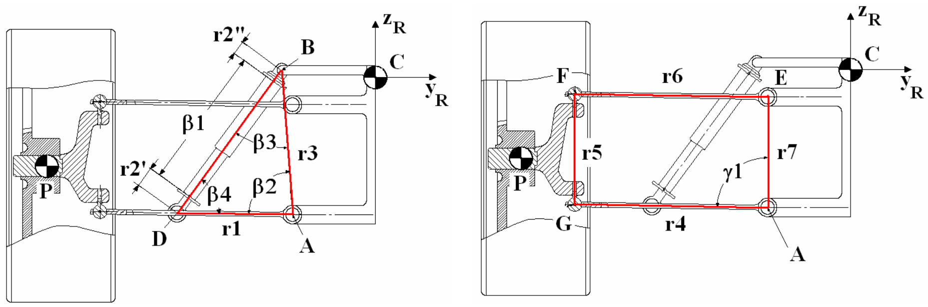

Figura 4. Equal (parallel) control A-arms geometry suspension in a quarter car 2-DOF model.

The wheel's CG is located in point P. The internal variables β_i define the first mechanisms, while the internal variable γ_1 defines the second. Each link has its length defined by a constant called r_i. The equations representing the geometry are:

$$\{g\}(\{\beta\},\{\gamma\},\lambda)=\begin{bmatrix}\{g_1(\{\beta\},\lambda)\}\\ \hline \{g_2(\{\beta\},\{\gamma\})\}\end{bmatrix}=\begin{bmatrix}\beta_1-\lambda\\ \beta_2-\cos^{-1}\left[\dfrac{r_1^2+r_3^2-(\beta_1+r_2)^2}{2r_1r_3}\right]\\ \beta_3-\mathrm{tg}^{-1}\left[\dfrac{r_1\mathrm{sen}\beta_2}{r_3-r_1\cos\beta_2}\right]\\ \beta_4-[\pi-(\beta_2+\beta_3)]\\ \hline \gamma_1-(\beta_2-\rho_2)\end{bmatrix}=\{0\} \tag{13}$$

where:
λ: coil spring length (degree of freedom);
β_i: ABD mechanism internal variables;
γ_i: AEFG mechanism internal variables;
ρ_2: BAE angle.

The expression of the effort transformer is obtained with aid of the suspension's free body diagram. The links masses are neglected (Eq. 14):

$$\Theta_Z(\gamma_1,\gamma_S)=\left[\frac{\mathrm{r}_1}{\mathrm{r}_4}\mathrm{ctg}\gamma_1\mathrm{sen}\gamma_\mathrm{S}-\left(2\frac{y_B}{y_A}-\frac{\mathrm{r}_1}{\mathrm{r}_4}\right)\cos\gamma_S\right] \tag{14}$$

where:
γ_S: angle formed by DB and z_R.
y_A: y coordinate of point A in the R-frame;
y_B: y coordinate of point B in the R-frame.

Thus, the resultant of the vertical forces produced by the suspension acting on the chassis is given substituting Eq. 14 into Eq. 11:

$$F_{CH,Z}=\Theta_Z(\gamma_1,\gamma_S)\cdot F_S \tag{15}$$

where F_S is the absolute value of the sum of the shock absorber and coil spring forces. The resultant vertical suspension forces applied on the wheel are given by:

$$F_{RD,Z}=-F_{CH,Z}=-\Theta_Z(\gamma_1,\gamma_S)\cdot F_S \tag{16}$$

5. Some Results

Block diagrams for each one of the main subsystems is implemented in the language *Simulink/MatLab* (Fig. 5), and then coupled. In the chassis and wheel blocks are their respective Newton-Euler equations of motion. In the suspension block are the equations that represent the geometry, its described in part 4, and the constitutive relations for each compliance element.

In the cases presented, the model allows for the loss of contact between the ground and the wheel. The results represent the vertical displacements of the chassis and wheel's center of gravity that are compared to a classical mass-spring-absorber system with two degrees of freedom. There are two kinds of chassis: on for the Conventional, Swing-Axle, and McPherson suspensions, and other for the Parallel Control Arms and Double A-arm suspensions. The reason for this division is that the hardpoints for the Double A-arm and Parallel Control Arms cannot be the same as for the other three. However, the mass for the two kinds of chassis are the same. The simulation parameters are shown in Tab 1 and in Fig. 6 are shown the suspension elements characteristics. Fig. 7 shows the vertical displacement of the sprung mass *versus* time for the five different suspension types and for the classical 2-DOF mass-springs-shock absorber linear model. The length of the suspension links were taken from real measurements of original parts.

Table 1. Parameters used in simulations. Values in **bold** are common to all models.

Parameter	Value	Parameter	Value	Parameter	Value
Chassi	250kg	r_1	250mm	$y_A = y_E$	-180mm
Wheel	42,5kg	$r_2 = r_2' + r_2''$	75mm	y_B	-203,13mm
Tire stiffness K_p	200kN/m	r_3	315,85mm	z_A	-300mm
Tire damper b_p	100Ns/m	$r_4 = r_6$	417mm	z_B	15mm
BAE	4,20°	$r_5 = r_7$	240	z_E	-60mm

6. Conclusions

The results show the functionality of the procedure applied to the quarter-car model, where suspensions and some tire properties, like height-width ratio and tire width, have been changed without necessary having to modify the system inputs, or to get the equations of motion for the system as a whole, characterizing it as modular. The use of polynomials obtained by interpolation methods and closed-loop equations make it possible to elect as output any variables of interest. Models built and based on Rigid Multibodies and Kinematical Transformers techniques generate systems of equations that contain all the relations between the considered bodies and subsystems, with advantage for the Kinematical Transformers technique, where the number of equations of motion is equal to system's degrees of freedom. In the work presented, each block has its proper set of equations, and can be analyzed, or modified and refined separately, without loss of clarity and the necessity to rewrite the system of equations of motion. The *Simulink/MatLab* language allows the building of subsystems assembly of clear understanding, keeping also clear the vision of the global system. Evaluating the results obtained for each model we can see the influence of the suspension configuration in the vertical displacement of the chassis for the two analyzed groups. In the first group, with the suspensions Swing-Axle, McPherson and Conventional, the displacement of the chassis is larger when the second configuration is used. In the second group, with the suspensions Double To-Arm and Parallel Control Arms, the displacement doesn't present significant differences. The linear classic model is the one that presents the less conservative result, because of not considering the interdependence of the two degrees of freedom related by the geometry of a mechanism.

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8. Responsibility notice

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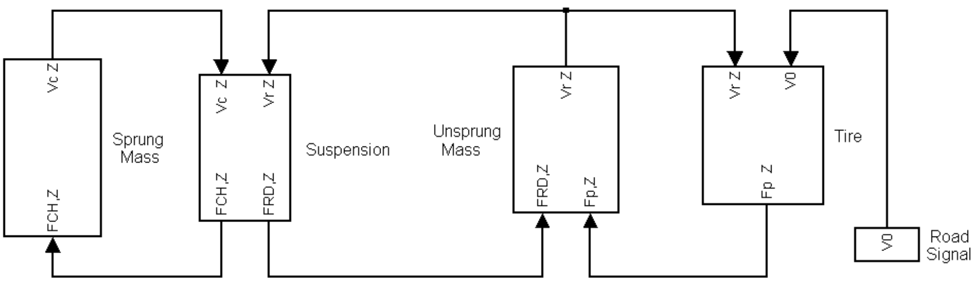

Figura 5. Block diagram for the quarter car model.

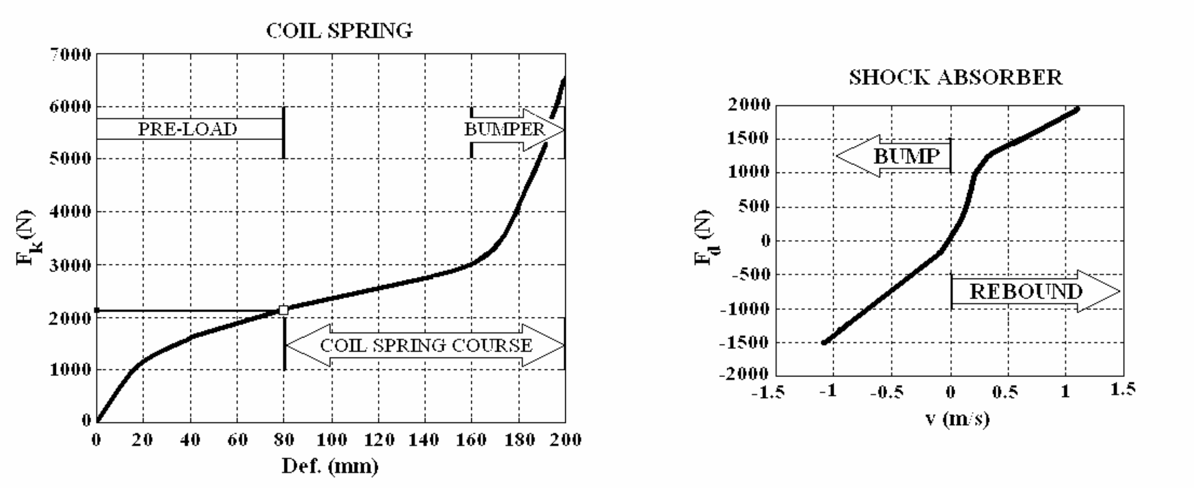

Figure 6. Spring and shock absorber characteristics.

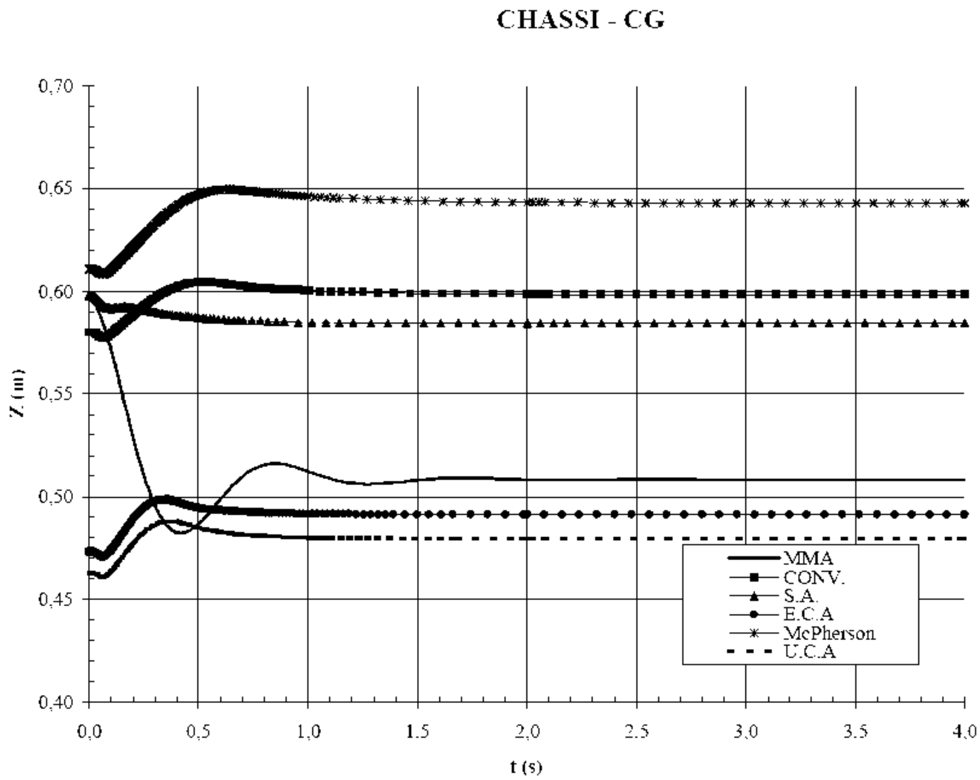

Figure 7. Sprung mass CG *z*-coordinate behavior *versus* time for five different types of suspensions.