# AN INTRODUCTION TO THE ACTIVE CONTROL OF VIBRATION IN ROTOR UNBALANCE USING MAGNETIC BEARING

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***Abstract.*** *The magnetic bearings are electromagnetic devices configured to keep a suspend shaft by utilizing magnetic field to support the shaft within a gap. For a magnetic bearing operate, it is necessary having an electronic circuit constituted by sensors, low pass filter, PID controller and power amplifiers for active control of shaft position. The transfer function of the circuit establishes a relationship among the rotor position inside of the bearing and the electric current of control. To obtain the dynamic characteristics of the bearing, a mathematical model has been developed for the rotormagnetic system, which allowed obtaining the expressions to calculate the equivalent stiffness and damping of the bearing. A computational routine has been implemented allowing to describe the stiffness and damping curves as a function of changes in the parameters of the electronic components of the circuit, which can control the dynamic of the rotors to operate with better dynamic stabilities. For theoretical rotor dynimic analysis a computer program for rotor modeling using the Matrix Impedance Method has been also elaborated. By using this program, it was developed a model of a rotor and obtained its modal characteristics (natural frequencies and mode shapes). Reduction in rotor vibration is sometimes obtained by the application of an ''open-loop'' or feedforward control strategy superimposed on the ''closed-loop'' control strategy necessary for rotor support. In this work will be presented the filtered X-LMS algorithm, which is a time domain based adaptive feedforward approach widely used to analyze the performance the strategies of active control of sound and vibration.*

***Keywords***: *Active Control, Rotor dynamic, Magnetic Bearings*

## 1. Introduction

The active magnetic bearings present an important progress and a new concept in bearings technology, such that have been motivating the development of this work. The active magnetic bearings are also efficient as actuators in strategies of active control of vibrations of rotating systems and they present several advantages respect to the conventional bearings for a variety of practical applications. One important advantage is that the system doesn't use lubricating oil, reducing significantly the maintenance operations of the machines. Due to this advantage, the magnetic bearings can be applied in sealed pumps, turbomolecular pumps, turboexpansors and centrifuges pumps, where the lubricating oil cannot be used by the fact of reaching high temperatures, or, where the environment requests the minimal of maintenance. Another advantage is that there is no contact between the stationary part of the bearing and rotor, offering low power loss and large useful life, being especially adapted in applications where the machines operate with higher rotation speeds.

Other important benefit of the technology of active magnetic bearings is its capability of operating as a system of active control of vibrations, once the shaft position inside of the bearing can be corrected thousands of times per second. Schweitzer and Lange (1976) recognized the potential of the active vibration control of rotors using active magnetic bearings technology. Since then, significant works in active vibration control area have been published.

An overview about magnetic bearings applications was presented by Kasarda (2000). Commercial applications examples of pumps and turbomachines are described. In research fields, researchers have been analyzing applications in bearingless motors, biomedical applications (artificial hearts), tool machines, aircraft jet engines, rotating systems for energy storage and miniature systems.

This work uses the transfer functions of each electronic device that constitute the control circuit of these bearings to determine the global transfer function of the circuit. With the purpose of obtaining the dynamic characteristics of the magnetic bearings, a mathematical model was developed for a rotor magnetic bearing system, which allowed obtaining the expressions to calculate the equivalent stiffness and damping of the bearings though its characteristics and the global transfer function of the control circuit. As the global transfer function of the electronic circuit depends on the frequency, then the equivalent stiffness and damping of the magnetic bearing also will depend.

A theoretical model for a rotating system supported by magnetic bearings is presented. The model has been developed by the impedance matrix method. A computational routine has been implemented employing the software

*"MATLAB"* and the dynamics of the rotating system can be analyzed in terms of natural frequencies and mode shapes by taking different values for the parameters of the control circuit.

The free oscillatory motion of any rotor bearing system is defined by its amount of stiffness and damping. In using mechanical bearings, the stiffness and damping characteristics of the system are generally fixed by the bearing project. Differently, in using magnetic bearings the stiffness and damping characteristics can be adjusted choosing a set of control parameters for a given control algorithm. Thus, the magnetic bearings have the capability to change the dynamic of the rotors to operate in better dynamic stabilities.

Reduction in rotor vibrations can be obtained by application of an ‘‘open-loop’’ or feedforward control strategy superimposed on the ‘‘closed-loop’’ control strategy necessary to keep the rotor suspended. In this work the filtered X-LMS algorithm will be presented. It is a time domain based adaptive feedforward approach widely used in the active control of sound and vibration. The X-LMS is a least mean squares approach where a reference signal, typically denoted by “x”, is filtered before the LMS operation is performed.

## 2. Review of Active Vibration Control in Beams.

While there have been considerable work involving control strategies for reducing unbalance responses in rotors, very little work has been presented addressing the effects of actuator and sensor location relative to the force source. There has also been little discussion concerning the number of sensors required to effectively reduce local and global vibration in a rotating machine. There is however a substantial amount of published work on the active control of bending vibration in finite and infinite beams that does address these points Fuller, et al. (1996). The vibration of a beam can be used as a simplified model of a vibrating rotor and is useful for analyzing the potential performance of control systems on rotors.

Fuller et al. (1996) discussed both feedback and feedforward control strategies for controlling the vibration on beams. While Fuller et al. (1996) did not specifically discuss actuator and sensor placement, Nelson and Elliott (1993) gave a general discussion about actuator location in their book on the active control of sound. They showed that control is always most successful when the control actuator is placed close to the source of the disturbance. They also showed that a set of discrete sensors can be used to approximate the total vibrational energy (sound in their case) in a system with a larger number of sensors leading to a better approximation. Fuller et al. (1990) looked at simultaneous control of flexural and extensional waves in a beam and considered the control of noise radiation from beams (Guigou e Fuller, 1993). The most complete discussion of the active control of vibration in beams is given in a publication by Brennan et al. (1995) where the authors compared different wave control and vibrational power minimization strategies for the control of vibration on both finite and infinite beams. They showed that in finite beams, global control (i.e. reduction achieved everywhere on the beam) is relatively easy to achieve at beam resonances (or critical speeds) but difficult when away from resonance conditions. This point has considerable relevance to the control of rotor vibrations since rotors are normally run at speeds away from resonance conditions. The paper also showed that active control using a single secondary actuator resulted in vibration reduction downstream of the secondary actuator but is not very effective upstream of the actuator (often making the vibration larger). Post and Silcox (1990) showed that the minimization of vibration over a section of a beam can actually lead to significant increases in vibration away from the error sensor. This has implications for error sensor location.

## 3. Active Magnetic Bearing

### 3.1. The Bearings Control Circuit

The necessary electric current from the control circuit to the magnetic bearing is determined based on the position of the rotor (it is the electric current magnitude that will determine the position of the shaft inside of the bearing). The position of the rotor is related to the disturbance or control current by the equation (using Laplace transform),

$$i_p(s) = G(s)x(s) \tag{1}$$

where $G(s)$ is the global transfer function. This complex transfer function is composed by the multiplication of the transfer functions of all components of the electronic circuit of control, i.e., the position sensor, low pass filter, PID controller and power amplifier. The most important parameters of the control circuit are the total, derivative, proportional and integral gains of the PID controller ($K_T$, $K_D$, $K_P$ e $K_I$, respectively) and the gain of the power amplifier ($Ka$) (Guiráo and Nascimento, 2004b). They will be later regarded on the rotor modeling.

### 3.2. Mathematical Model, Equivalent Stiffness and Damping of the Rotor Magnetic Bearing System

In taking just one control axis of the magnetic bearing, then there are two opposite magnets acting on it according to Fig. 1. The $F_1$ and $F_2$ are electromagnetic forces that attract the rotor onto the magnets and $F_i$ is external force acting onto the rotor, as instance, an unbalance force.

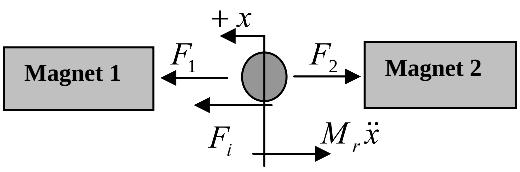


Figure 1. Layout of the loads acting on the control axis

Equating the forces of the mathematical model of Fig. 1, it is possible to obtain the equation of motion of the system. Regarding that the force ($F_i$) is harmonic and introducing the Eq. (1) onto this equation of motion, reminding that the transfer function is composed by real and imaginary parts, the equivalent stiffness and damping expressions for the magnetic bearing can be determined. That procedure can be seen in details in Guiráo and Nascimento (2004a).

$$K_{eq} = K_x + K_i a_G \tag{2}$$

$$C_{eq} = \frac{K_i b_G}{\omega} \tag{3}$$

where $K_x$ is the position stiffness, $K_i$ is the current stiffness, $\omega$ is the rotation frequency and $a_G$ e $b_G$ are the real and imaginary part of the global transfer function, respectively.

Equations (2) and (3) represent the equivalent stiffness and damping of an axis control alone. Those values change as a function of frequency once the real and imaginary parts of the controller transfer function also depend on of the frequency. For successfully modeling a radial magnetic bearing, the function of the transfer function controller should be known. In taking an implemented program on *"MATLAB"* environment, it has been possible obtaining the equivalent stiffness and damping curves as a function of changes on the $K_a$ parameter of the power amplifier and $K_T$, $K_D$, $K_P$ e $K_I$ parameters of the PID controller. Those curves, as well as the characteristics of the analyzed bearing *"MATLAB"* can be seen in Guiráo and Nascimento (2004b).

## 4. Dynamic Simulation of a Rotor
### 4.1. Model of Beam for Rotordynamics Analysis

In this section a simplified model of the rotor vibration (or velocity) in the vertical direction will be described. The rotor will be considered as a free beam (or rod) with gyroscopic neglected with masses attached and supported by bearings modeled as a pair of springs (with damping included) as shown in Fig. 2. An unbalance force has been introduced onto the beam. The velocity of the rotor in the horizontal direction will not be considered in this model and will be assumed to be independent of the vertical velocity. Any velocity in the horizontal direction can be controlled in the same way as the vertical velocity using another uncoupled active control system producing similar results. The bearing supports and masses are attached at three locations along the beam.

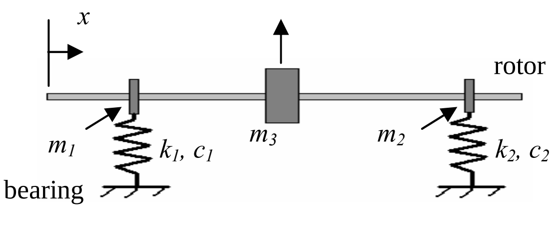


Figure 2. Schematic of rotor system used in the theoretical model.

The velocity, $u$, of a beam or rod can be described as a series or summation of modes (Warburton, 1954).

$$u(x,\omega) = \sum_{n=0}^{\infty} a_n(\omega)\psi_n(x) \tag{4}$$

where the $n^{th}$ mode shape is a function of position $x$ and is given by $\psi_n(x)$ and the complex mode amplitude of the $n^{th}$ mode is given by $a_n(\omega)$. If the frequency range of interest is limited, then it is possible to accurately describe the beam's behavior using a finite set of modes. The mode shapes were determined by Johnson et al. (2003). The mode amplitude $a_n(\omega)$ is a function of angular frequency $\omega$ and can be calculated by considering all of the forces acting on the rotor. The mode amplitude due to a single force $F(\omega)$ acting at a single point $x_j$ on the rotor is given by,

$$a_n(\omega) = \frac{2}{m_r}\left(\frac{j\omega}{\left(\omega_n^2 - \omega^2\right) + 2j\zeta_n\omega_n\omega}\psi(x_j)\right)F(\omega) \tag{5}$$

where the natural frequency and damping ratio for the $n^{th}$ mode are given by $\omega_n$ e $\xi_n$ respectively. The damping ratio is typically low for a steel rotor, on the order of 0.005. Most of the damping in this system will be provided by the bearings. The total mass of the rotor is $m_r = lS\rho$ where $S$ is the cross section area of the rotor, $\rho$ is the density of the rotor material and $l$ is the rotor length. The natural frequencies of the modes shapes were also calculated by Johnson et al. (2003).

By combining Eq. (4) and Eq. (5), point and transfer mobility terms can be calculated. The transfer mobility $T_{ij}$ is defined as the velocity of the beam at position $x_i$ due to an input force at position $x_j$.

$$T_{ij}(\omega) = \frac{2}{m_r}\sum_{n=0}^{\infty}\frac{j\omega}{\left(\omega_n^2 - \omega^2\right) + 2j\zeta_n\omega_n\omega}\psi(x_i)\psi(x_j) \tag{6}$$

At a single frequency the velocities at a number of locations can be described in matrix form as,

$$\boldsymbol{u_i} = \boldsymbol{T_{ij}}\,\boldsymbol{f_j} \tag{7}$$

where the column vector $\boldsymbol{u_i}$ describes the velocities at positions $\boldsymbol{x_i}$ (also a column vector) due to a number of forces $\boldsymbol{f_j}$ acting at locations $\boldsymbol{x_j}$. Each element in the matrix $\boldsymbol{T_{ij}}$ is calculated using Eq. (6).

### 4.2. Matrix Impedance Method for Modeling Bearings

The above theory and Eq. (7) can only be used to model the motion of the rotor itself but does not include any bearing stiffness, damping or mass that may be supporting or supported by the rotor. These elements will be included in the model using a matrix impedance method (Warburton, 1954). In this paper this method will be used to describe the addition of stiffness, damping and mass at three discrete locations along the rotor but can be extended to include any number of mass, stiffness and damping locations. As shown in Fig. 2 the rotor interacts with external loads at three distinct points along the beam, namely, at the two bearing positions where two small masses $m_1$ and $m_2$ are attached and the rotor is supported by two springs of stiffness $k_1$ and $k_2$ and in the middle of the beam where a mass $m_3$ , containing a slight unbalance, is applied. The bearings are also considered to have viscous dampers $c_1$ and $c_2$ included in them. The mass and stiffness attachments create reaction forces when the rotor is moved and these forces can be described using an impedance matrix $\mathbf{Z}$ by,

$$\mathbf{f_r} = -\mathbf{Z}\mathbf{u_m} \Leftrightarrow \begin{bmatrix} f_{r1} \\ f_{r2} \\ f_{r3} \end{bmatrix} = -\begin{bmatrix} j\omega m_1 + c_1 + \frac{k_1}{j\omega} & 0 & 0 \\ 0 & j\omega m_2 + c_2 + \frac{k_2}{j\omega} & 0 \\ 0 & 0 & j\omega m_3 \end{bmatrix}\begin{bmatrix} u_{m1} \\ u_{m2} \\ u_{m3} \end{bmatrix} \tag{8}$$

where $\boldsymbol{u_m}$ is the vector of velocities at the three mass locations and the reaction force is given by the vector $\boldsymbol{f_r}$. The velocity vector $\boldsymbol{u_m}$ can be considered as the combination of the velocity $\boldsymbol{u_{mr}}$ due to the reaction forces $\boldsymbol{f_r}$ and the velocity $\boldsymbol{u_{mj}}$ due to external input forces to the rotor $\boldsymbol{f_j}$. The external input forces in this model are created by the mass unbalance. Using Eq. (7) and Eq. (8) leads to an expression for the reaction force in terms of the external forces,

$$\begin{aligned} &\boldsymbol{u_m} = \boldsymbol{u_{mj}} + \boldsymbol{u_{mr}} = \boldsymbol{T_{mj}}\boldsymbol{f_j} + \boldsymbol{T_{mm}}\boldsymbol{f_r} = \boldsymbol{T_{mj}}\boldsymbol{f_j} - \boldsymbol{T_{mm}}\boldsymbol{Z}\boldsymbol{u_m} \\ &\Rightarrow \boldsymbol{u_m} = [\boldsymbol{I} + \boldsymbol{T_{mm}}\boldsymbol{Z}]^{-1}\boldsymbol{T_{mj}}\boldsymbol{f_j} \end{aligned} \tag{9}$$

$$\Rightarrow \boldsymbol{f_r} = -\boldsymbol{Z}\boldsymbol{u_m} = -\boldsymbol{Z}[\boldsymbol{I} + \boldsymbol{T_{mm}}\,\boldsymbol{Z}]^{-1}\boldsymbol{T_{mj}}\,\boldsymbol{f_j} \tag{10}$$

The two matrices $\boldsymbol{T_{mm}}$ and $\boldsymbol{T_{mj}}$ contain rotor mobilities describing the velocity at the three mass locations due to forces acting at the mass locations and the locations of the external forces respectively. The elements in these matrices can be calculated using Eq. (6). Equations (9) and (10) can be used to calculate a new set of augmented rotor mobilities $\hat{T}_{ij}$, that include the reaction force of the masses, dampers and stiffnesses. From this augmented set of equations the velocity at any point on the rotor due to a force applied at any other point on the rotor can be calculated. The matrix $\hat{T}_{ij}$ can be obtained in the following way,

$$u_i = T_{ij} f_j + T_{im} f_r = \hat{T}_{ij} f_j \tag{11}$$

$$\hat{T}_{ij} = [T_{ij} - T_{im} Z[I + T_{mm} Z]^{-1} T_{mj}] \tag{12}$$

The two matrices $T_{im}$ and $T_{ij}$ contain rotor mobilities describing the velocity at the observation locations $x_i$ due to forces acting at the mass locations and the locations of the external forces. Using this augmented set of equations the performance of an active control system can be evaluated.

### 4.3. Natural Frequencies and Mode Shapes

The transfer matrix $\hat{T}_{ij}$ relates external forces with the velocities, as it is shown in Eq. (11). From this equation it can be obtained the natural frequencies and mode shapes of the system dividing Eq. (11) by the complex frequency to give,

$$x_i = \frac{\hat{T}_{ij} f_j}{j\omega} \tag{13}$$

where $x_i$ is the displacement at a specific point of the rotor, while a force $f_j$ is applied at point $x_j$ of the rotor.

The displacement amplitudes $x_i$ is given as a function of the frequency $\omega$. Thus an average value of $x_i$ considering all nodal points of the rotor can be obtained and a curve of this average value versus $\omega$ can be plotted allowing to identify the regions of frequencies where the amplitudes are higher. Those regions of higher amplitudes point out the natural frequencies. The excitation force acting onto the system is a unitary unbalance. The peaks of higher amplitudes occur when the frequency of unbalance force frequency become close to one of those natural frequencies of the rotor, characterizing the resonance phenomenon.

There is a corresponding mode shape for each natural frequency, which can be determined when an exciting force at natural frequency acts onto the system. As result the displacements of the nodal points of the beam will give the form of mode shape at this natural frequency (Guiráo and Nascimento, 2004a).

### 4.4. Modeling for Modal Parameters Analysis

The theoretical rotor model shown in Fig. 2 has been used to identify the natural frequencies from displacements curve and related mode shapes. The rotor consists of a 457mm steel long shaft with 9.52 mm in diameter and the bearings are located at 114mm and 368mm positions from *x* origin as it can be seen in Fig. 2. An unbalance mass of 0.8 Kg was placed at position 165mm and a material damping ratio of 0.001 was considered. The bearing ferrous magnetic material attached to each shaft has mass of 0.25Kg. The stiffness and damping of the bearings were obtained using the Eqs. (2) and (3) by taking the parameters of the control circuit, whose values are shown in Tab. 1 (Guiráo and Nascimento, 2004a).

Table 1. Stiffness and damping of the bearings and the parameter values used in the control circuit.

| ***Parameters*** | $K_A$ | $K_P$ | $K_I$ | $K_D$ | $K_T$ |
|---|---|---|---|---|---|
| ***Value*** | 790 | 75 | 100 | 0.03 | 0.00006 |
| ***Stiffness*** | 167.26 KN/m | | | | |
| ***Damping*** | 28.8 N.s/m | | | | |

Using the theory of models of a free-free beam together with the impedance matrix method previously presented, and employing the computational routine implemented with the software "MATLAB", it was possible obtaining the curves to identify the natural frequencies and mode shapes of the rotor, as it can be seen in Figs. (3) and (4). The Table 2 presents the values of the natural frequencies for the first three mode shapes analyzed.

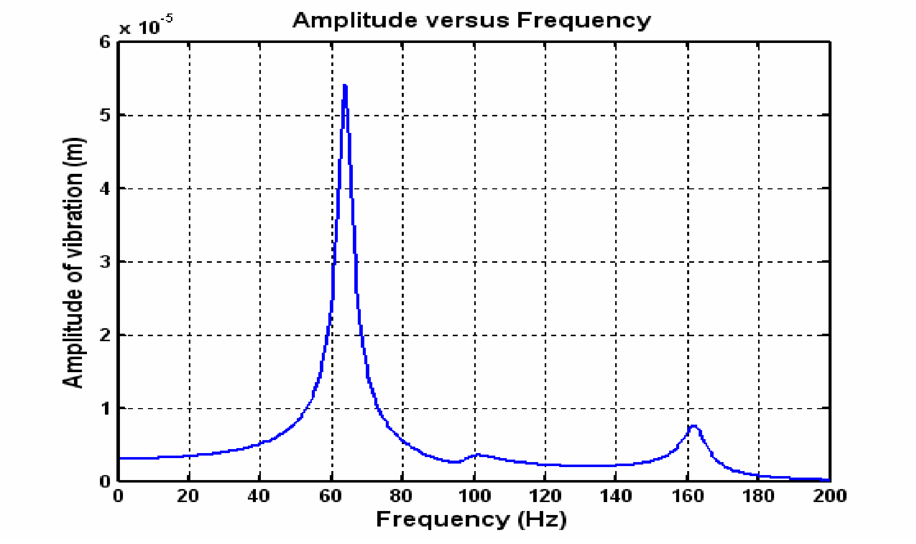


Figure 3. Vibration amplitude as a function of the frequency

Table 2. Natural frequency values of the mode shapes

| ***Mode Shapes*** | **1st mode** | **2nd mode** | **3rd mode** |
|---|---|---|---|
| ***Natural Frequencies*** | 64 Hz | 99.2 Hz | 161.6 Hz |

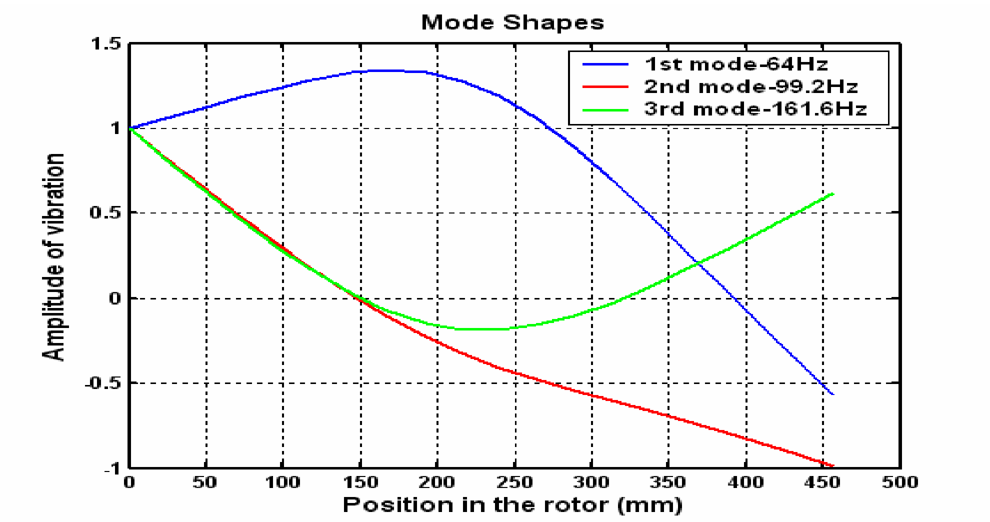


Figure 4. Mode Shapes for the first three natural frequencies

## 5. Active Vibration Control Theory
### 5.1. Filtered X-LMS

In this section the filtered X-LMS algorithm is briefly described, which has been widely used to investigate the performance of active control of rotating system where the magnetic bearings are employing as actuators. For a more depth description of this algorithm the reader should refer to text books such as that by Widrow and Sterns (1985) or by Elliott et al. (1987).

The filtered X-LMS is a time domain feedforward algorithm (see the control flow chart Fig. 5) that uses a reference signal *x*, such as a tachometer signal, to drive a set of secondary actuators in order to affect the system under control. The reference signal is first digitally sampled and then passed through a finite impulse response (FIR) control filter *W* before being converted back into an analog signal *y* and used to drive the control actuators. Another set of sensors, called error sensors, are used to monitor the behavior of the system (error *e*) and are used to automatically adapt the control FIR filter using the LMS algorithm. The ‘‘plant’’ represents the transfer function between the input to the actuators (magnetic bearing currents in this case) and the vibration detected at the error sensors (normally eddy current proximity probes). The disturbance $d$ is the vibration at the error sensors due to the unbalance in the system. This control architecture differs from the LMS in that the reference signal needs to be first filtered by a model of the plant $G$ (i.e. filtered ''$X$'') before being used by the LMS algorithm. The model of the plant is usually stored as an FIR filter (see below) and is measured in a system identification stage before the control system is turned on.

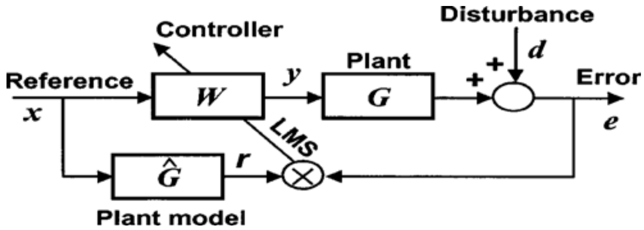


Figure 5. Control flow chart showing a filtered X-LMS adaptive control system

If a signal, such as $x$, has been sampled at discrete time intervals then it can be considered as a sequence $x(n)$ where $n$ can only take integer values. The current output of an FIR filter, such a $W$, is the weighted sum of the previous inputs. For example the current output from the control filter $y(n)$ can be expressed as a series of previous inputs of the reference signal $y(n) = \sum_{i=0}^{I-1} w_i x(n-i)$ where the control filter has $I$ filter weights ($w_i$). The LMS algorithm updates the filter coefficients in $W$ using the most recent error signal $e(n)$ and the past $I$ filtered reference signals $r(n\text{-}i)$.

$$w_i(n+1) = w_i(n) - \alpha e(n) r(n-i) \tag{14}$$

All $I$ filter coefficients can be updated this way. The coefficient $\alpha$ is the convergence coefficient and determines how rapidly the control system converges. $\alpha$ must be large enough such that the convergence time is small but cannot be too large since this can cause instability. Ideally, this algorithm converges to a solution where the time averaged sum of the squared error signals is minimized. In principle, only two coefficients are necessary to achieve good control if the disturbance is at a single frequency. If multiple frequencies need to be controlled, for example harmonics of the rotor speed, then more than two coefficients are necessary.

**5.2. Active Control Performance.**

In this section a method for calculating the control performance of an active control system will be presented. A theoretical model for the rotor using the procedure as presented in section 4 can be developed to analyze the performance of the active vibration control. Thus, an active control system using the filtered X-LMS system will attempt to minimize the sum of the squared outputs from the error sensors placed along of the rotor. If a control system has $J$ control actuators and $L$ error sensors then at a single frequency the $L$ length vector of errors $\boldsymbol{e}$ can be written in terms of the vector of primary disturbance forces $\boldsymbol{f_p}$ (i.e. rotor unbalance) and the $J$ length vector of secondary control forces $\boldsymbol{f_c}$.

$$\boldsymbol{e} = \hat{\boldsymbol{T}}_{\boldsymbol{ep}} \boldsymbol{f}_{\boldsymbol{p}} + \hat{\boldsymbol{T}}_{\boldsymbol{ec}} \boldsymbol{f}_c \tag{15}$$

The two matrices $\hat{\boldsymbol{T}}_{\boldsymbol{ep}}$ and $\hat{\boldsymbol{T}}_{\boldsymbol{ec}}$ contain augmented rotor mobilities describing the velocity at the error locations $\boldsymbol{x_e}$ due to forces acting at the locations $\boldsymbol{x_p}$ of the unbalance forces (primary disturbance) and the locations $\boldsymbol{x_c}$ of the control forces. The sum of the squared error signals can be calculated using the Hermitian transpose (or conjugate transpose denoted by $^{\boldsymbol{H}}$) as $\boldsymbol{e}^{\boldsymbol{H}}\,\boldsymbol{e}$ and is minimized when the secondary control forces $\boldsymbol{f_c}$ are given by Fuller, et al. (1996),

$$\boldsymbol{f}_{\boldsymbol{c\_opt}} = [\hat{\boldsymbol{T}}_{\boldsymbol{ec}}^{\boldsymbol{H}} \hat{\boldsymbol{T}}_{\boldsymbol{ec}}]^{-1} \hat{\boldsymbol{T}}_{\boldsymbol{ec}}^{\boldsymbol{H}} \hat{\boldsymbol{T}}_{\boldsymbol{ep}} \boldsymbol{f}_p \tag{16}$$

Using these optimal forces the velocity at any set of observation locations $\boldsymbol{x_i}$ along the rotor can then be calculated before $\boldsymbol{u_{i\_b}}$ and after $\boldsymbol{u_{i\_a}}$ optimal control as,

$$\boldsymbol{u}_{\boldsymbol{i\_b}} = \hat{\boldsymbol{T}}_{\boldsymbol{ip}} \boldsymbol{f}_{\boldsymbol{p}} \tag{17}$$

$$u_{i\_a} = \hat{T}_{ip} f_p + \hat{T}_{ic} f_{c\_opt} \quad (18)$$

The two matrices $\hat{T}_{ip}$ and $\hat{T}_{ic}$ contain augmented rotor mobilities describing the velocity at the observation locations $x_i$ due to forces acting at the locations $x_p$ of the unbalance forces (primary disturbance) and the locations $x_c$ of the control forces. This process can be repeated for a range of frequencies.

## 6. Conclusions

In this work a procedure for analysis of rotor magnetic bearing systems by using the modal approach of a free-free beam together with the Impedance Matrix Method was presented. A model of a theoretical rotor was developed regarding a specific set of parameters of the electronic components of the feedback control of the magnetic bearings, allowing to determine the natural frequencies and mode shapes of the rotor. Different modal characteristics should be obtained by taking a different set of parameters for the electronic devices of the feedback control. That property of the magnetic bearings allows accomplishing a rotordynamic analysis to find an adequate set of parameters that induce the rotor to a more safe operation (out of resonant conditions). A strategy of active feedforward control of vibration for practical application and theoretical simulation using the algorithm X-LMS also was presented as an alternative to reduce the vibration of rotor magnetic bearing systems. That strategy is employed superimposed to the feedback control of the magnetic bearing. For active control simulation the theoretical model developed in this work should be used.

The next step to this work is developing simulations to analyze the performance of LMS algorithm to reduce rotor vibrations.

## 7. Acknowledgments

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## 9. Responsibility notice

The authors are the only responsible for the printed material included in this paper.