FOUR-WHEEL VEHICLE SUSPENSION MODELING FOR CONTROL SYSTEM DEVELOPMENT

Cláudio Crivellaro
DANA Structural Solutions International – Brazil Division (Dana Indústrias Ltda)
Av. Presidente Médici, 939, Zip code 06268-000, Osasco, SP, Brazil
e-mail: claudio.crivellaro@dana.com

Edilson Hiroshi Tamai
State University of São Paulo (USP) – Escola Politécnica – DEM
Zip code 05508-900, São Paulo, SP, Brazil
e-mail: edhtamai@usp.br

Décio Crisol Donha
State University of São Paulo (USP) – Escola Politécnica – DEM
Zip code 05508-900, São Paulo, SP, Brazil
e-mail: decdonha@usp.br

Abstract. *In recent years several works have been published regarding active and semi-active suspension system for vehicles in general; however few articles have dealt with an adequate four-wheel suspension model for control development. When a four-wheel model is used, several control problems may appear, such as the presence of non-minimum phase transmissions zeros, the lack of observability and controllability, and ill-conditioned systems. In this work a seven-degree of freedom four-wheel vehicle model is presented, and a balanced state-space realization and model reduction is proposed to get a fully controllable and observable model for vehicle suspension controller design. In addition a robust control (LQG/LTR) is designed and simulations results comparing passive and active systems are presented.*

Keywords: *Vehicle suspension, four-wheel model, balanced realization, space-state, robust control.*

1. Introduction

The vehicle mathematical modeling is the base of several control strategies applied to vehicle suspension systems. In recent years several works have been published regarding active and semi-active suspension system for numerical simulations. Some works dealing with different kind of control approach used just a quarter-car model with two degree of freedom (Rao and Prahlad, 1995; Stutz and Rochinha, 2005). Other works aimed at other vehicles movements such as roll or pitch rather then with the control approach have presented half-car models with four degree of freedom (Hać *et al.*, 1996; Tsao and Chen, 2001; Simon and Ahmadian, 2002). Other works such as Cruz *et al.*, 2003, and Yamamura and Masada, 1979, have used four-wheel modeling for MAGLEV vehicle controller development, but did not use a MIMO (multiple input multiple output) control approach, although Cruz *et al.*, 2003, had to deal with the case of an over actuated system, solved with an optimal distribution of forces among the actuators designed to minimize the maximum force.

The goal of this work is to propose an alternative strategy of vehicle modeling, representative of a full four-wheeled vehicle, which is suitable for MIMO control development. The MIMO control approach can consider the global performance of vehicle movements to determine the best control action, which is, *a priori*, a better choice than the use of several SISO local controllers working independently. In addition, a state-space theory is an elegant way to approach a control problem, mainly regarding MIMO systems which are naturally dealt. This theory has given important concepts such as observability and controllability, and leaded to several control design methods, such as linear quadratic regulator (LQR), pole placement, optimal H_2, and robust control design methods as LQG-LTR (Doyle and Stein, 1981; Cruz, 1996) and H_∞ (Glover and Doyle, 1989).

The choice of MIMO approach for suspension system control development brings up several control issues as for example the presence of non-minimum phase transmission zeros. A right half plane zero gives an upper bound to the achievable bandwidth. The bandwidth decreases with decreasing frequency of the zero. It is thus more difficult to control systems with slow zeros.

On the other hand, when the vehicle body is modeled as a rigid body, the four independent suspension system (one for each wheel) can only control three (heave, pitch and roll) from the six degree of freedom. In this case, it is very common to use four actuators – one for each wheel – and just three linear-independent signals could be measured to determine uniquely the position of vehicle body (heave, roll and pitch movements) or its acceleration state (heave, roll and pitch accelerations). In this way, a four dimensional control signal cannot influence some movements of vehicle body, which cannot be represented in a three dimensional space, for example, when the control signals excite a twist mode of car body structure. This situation is extended to the system output when, for example, the vertical acceleration in four points of the vehicle body (each point over its respective wheel) is measured instead of heave, roll and pitch acceleration measurements. Thus a three dimensional space is represented in a four dimensional space, meaning that the "true" output represents only a hyper-plane in a four-dimensional output. In this situation, a system model produces an ill-conditioned transfer function matrix, which is not adequate to control design.

In addition, it would be very interesting for the controller designer if the controller action could be focused in a part of the system where the energy applied is better used to get a desired system performance. In other words, it is desirable a special realization where the system model is separated into two parts: one where the states are easily controlled by the inputs and easily estimated from the system outputs, and other where the states are not controlled or poorly controlled by the inputs, and also, these states are not estimated or badly estimated from the outputs. For minimal realizations, it is possible to find a so-called *balancing* similarity transformation T_B such that the controllability and observability gramians are both equal $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)$, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$ (for minimal systems $\sigma_n > 0$). σ_i are the *Hankel singular values*, and gives a precise measure of how observable and controllable the states are, in view of the energy interpretations of gramians. This energy interpretation show that each Hankel singular value is a measure of the amount of energy injected into the i-th state x_i, and the amount of energy of x_i in the output energy. Furthermore, the fact that Σ is diagonal indicates that, in this realization, the different state components are uncorrelated. It is reasonable, therefore, to discard the states corresponding to the smallest σ_i, when is important to reduce the order of the model, while preserving as much as possible the input-output behavior (Moore, 1981; Schelfhout, 1996).

Depending on the choice of states to be represented by the mathematical vehicle model and what signals are measured and the position of control forces, the system could not be completely observable and/or controllable, and in this case, it is considered a non-minimum system in the McMillan sense (Moore, 1981). The minimization of such systems, separating the controllable and observable part from the non-controllable and non-observable one, is the necessary condition for applying a Truncated Balanced Realization (TBR) procedure, as described above. Ill-conditioned controllability and/or observability matrices can result in a nearly non-minimum system, which must be minimized even having a full rank, in order to avoid numerical problems in TBR algorithm.

Once a balanced reduced realization of the system is available, it is the time to design the controller. In this work a LQG/LTR design method is used. The controller designed is then applied to the original full-order system and its performance is measured by the RMS values of pitch, roll and heave accelerations, while the system is excited by disturbances generated by road unevenness. These results are afterwards compared with a passive suspension, where dampers with fixed damping coefficients replace the active actuators.

2. Mathematical modeling

2.1. Four-wheel vehicle model

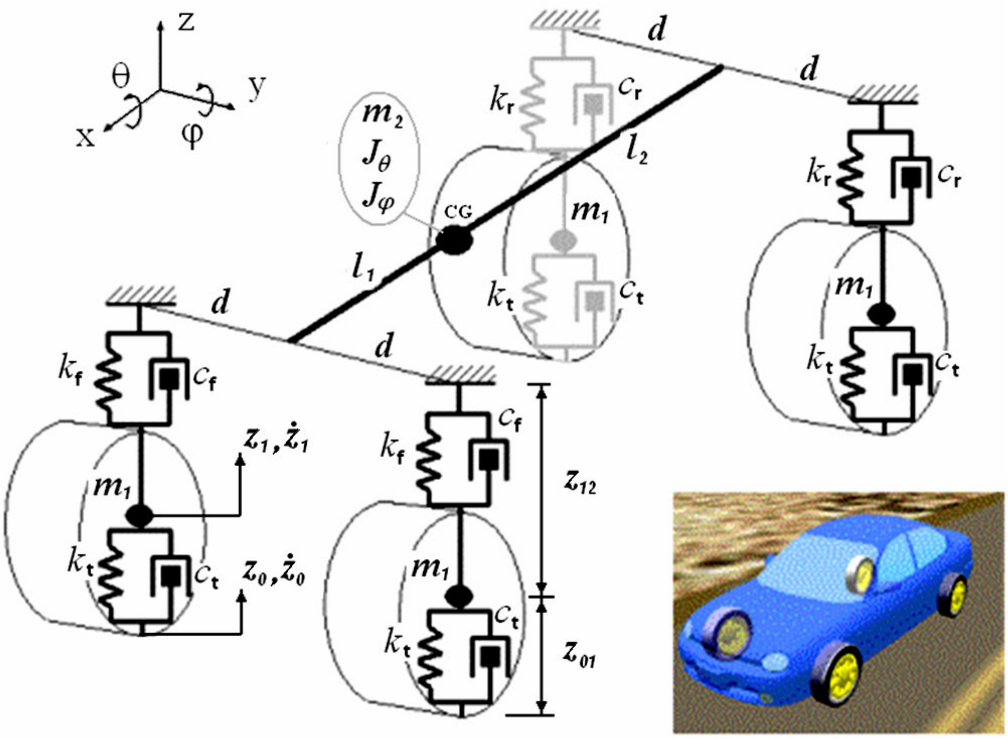

Figure 1. Vehicle physical model

The complete physical vehicle model is shown in Fig.1. The model has a rigid body representing the vehicle sprung body and other four masses representing the four wheels. The vehicle sprung body has three degrees of freedom: heave, roll and pitch. Each wheel has only the vertical movement, resulting in a seven degree of freedom model in a whole.

Variables $\mathbf{z}_{01}(t)$ and $\mathbf{z}_{12}(t)$ define vectors of the four relative displacements between the ground and each wheel and between each wheel and a point in the vehicle body where each suspension is connected. In the same way $\dot{\mathbf{z}}_{01}(t)$ and $\dot{\mathbf{z}}_{12}(t)$ are the relative velocity. $\dot{\mathbf{z}}_{0}(t)$ is the vector of vertical velocity, applied by ground in each wheel and $\dot{\mathbf{z}}_{1}(t)$ is the vector of the vertical velocity of the each wheel.

The variables describing the vehicle body position are $z_{CG}(t)$, $\varphi(t)$ and $\theta(t)$, for heave, pitch and roll movements, respectively. Thus a vector $\xi(t) = [z_{CG}(t) \quad \varphi(t) \quad \theta(t)]^T$ is defined.

The forces that the suspension system applies to the vehicle sprung body is as follows:

$$\mathbf{F}_2(t) = \mathbf{K}_{12}\mathbf{z}_{12}(t) + \mathbf{C}_{12}\dot{\mathbf{z}}_{12}(t) \tag{1}$$

In Eq.(1), $\mathbf{K}_{12} = diag(k_f, k_f, k_r, k_r)$ represents the four spring stiffness, and $\mathbf{C}_{12} = diag(c_f, c_f, c_r, c_r)$ represents the four damping coefficients, where index "*f*" means *front* and "*r*" means *rear*. The resultant forces in wheels are represented as:

$$\mathbf{F}_1(t) = -\mathbf{F}_2(t) + \mathbf{K}_{01}\mathbf{z}_{01}(t) - \mathbf{C}_{01}\dot{\mathbf{z}}_1(t) + \mathbf{C}_{01}\dot{\mathbf{z}}_0(t) \tag{2}$$

In Eq.(2), $\mathbf{K}_{01} = diag(k_t, k_t, k_t, k_t)$ gives the four tire stiffness, and $\mathbf{C}_{01} = diag(c_t, c_t, c_t, c_t)$ gives the four tire damping coefficient. Thus the dynamical vehicle model can be written as follows:

$$\mathbf{M}_2\ddot{\xi}(t) = \mathbf{L} \cdot \mathbf{F}_2(t) \tag{3}$$

$$\mathbf{M}_1\ddot{\mathbf{z}}_1(t) = \mathbf{F}_1(t) \tag{4}$$

In Eq. (3) and Eq. (4), $\mathbf{M}_2 = diag(m_2, J_\varphi, J_\theta)$ represents the mass and moments of inertia of the sprung vehicle body, $\mathbf{M}_1 = diag(m_1, m_1, m_1, m_1)$ represents the four wheel masses, $\mathbf{L}$ is a transformation matrix that relates the four vertical displacements of the sprung vehicle body at the suspension connection points to heave, pitch and roll displacements given by:

$$\mathbf{L} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ l_1 & l_1 & -l_2 & -l_2 \\ d & -d & d & -d \end{bmatrix} \tag{5}$$

Defining the disturbance input as $\mathbf{w}(t) = \dot{\mathbf{z}}_0(t)$ and the state variables as follows:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{z}_{01}(t) \\ \mathbf{z}_{12}(t) \\ \dot{\mathbf{z}}_1(t) \\ \dot{\xi}(t) \end{bmatrix} \tag{6}$$

equations (1) and (2) are rewritten as follows:

$$\mathbf{F}_2(t) = \mathbf{R}_2\mathbf{x}(t) \tag{7}$$

$$\mathbf{F}_1(t) = \mathbf{R}_1\mathbf{x}(t) + \mathbf{C}_{01}\mathbf{w}(t) \tag{8}$$

$$\mathbf{R}_2 = [\mathbf{0}_{4x4} \quad \mathbf{K}_{12} \quad \mathbf{C}_{12} \quad -\mathbf{C}_{12}\mathbf{L}^T] \tag{9}$$

$$\mathbf{R}_1 = [\mathbf{K}_{01} \quad -\mathbf{K}_{12} \quad (\mathbf{C}_{01} - \mathbf{C}_{12}) \quad \mathbf{C}_{12}\mathbf{L}^T] \tag{10}$$

2.2. Actuator model

In this work it is assumed that the actuator is a device able to generate axial forces and that has a residual damping coefficient (c_a). The actuator time response is considered 5ms, so that the actuator dynamics is fast enough to be disregard in the model. The actuator forces (u_i) are applied in the same time in both sprung and unsprung masses, as action and reaction forces; i.e., if the force in the sprung mass is positive than the force in the unsprung mass will have the same amplitude but will be negative.

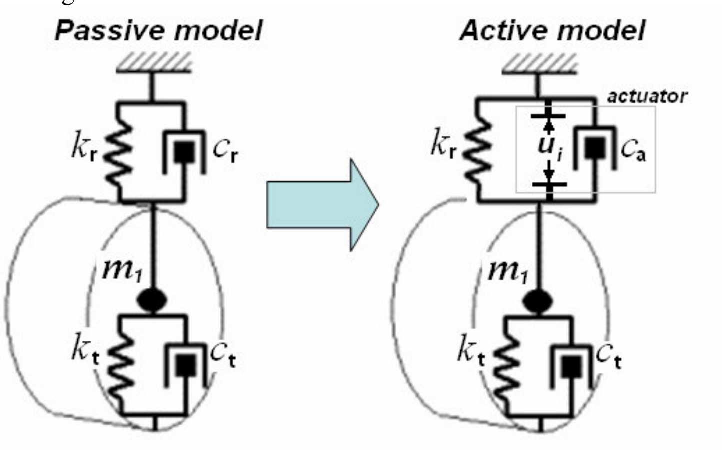

Figure 2. Actuator model

As shown in Fig. 2, to replace a passive suspension system with an active one, an active actuator replaces the shock absorber (damper). Matrix $\mathbf{C}_{12} = diag(c_f, c_f, c_r, c_r)$ is than replaced by $\mathbf{C}_{12} = diag(c_a, c_a, c_a, c_a)$ for active controlled system. In general, c_a is considered at least about four times smaller than c_f and c_r.

2.3. Space State Dynamic System Representation

The space-state representation of the system is given by Eq. (11), Eq. (12) and Eq. (13).

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{w}(t) + \mathbf{B}_2\mathbf{u}(t) \tag{11}$$

$$\mathbf{y}_1(t) = \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_{11}\mathbf{w}(t) + \mathbf{D}_{21}\mathbf{u}(t) \tag{12}$$

$$\mathbf{y}_2(t) = \mathbf{C}_2\mathbf{x}(t) + \mathbf{D}_{12}\mathbf{w}(t) + \mathbf{D}_{22}\mathbf{u}(t) \tag{13}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{4x4} & \mathbf{0}_{4x4} & -\mathbf{I}_{4x4} & \mathbf{0}_{4x3} \\ \mathbf{0}_{4x4} & \mathbf{I}_{4x4} & \mathbf{0}_{4x4} & \mathbf{0}_{4x3} \\ \multicolumn{4}{c}{\mathbf{M}_1^{-1} \cdot \mathbf{R}_1} \\ \multicolumn{4}{c}{\mathbf{M}_2^{-1} \cdot \mathbf{L} \cdot \mathbf{R}_2} \end{bmatrix} \tag{14}$$

$$\mathbf{B}_1 = \begin{bmatrix} \mathbf{I}_{4x4} \\ \mathbf{0}_{4x4} \\ \mathbf{M}_1^{-1} \cdot \mathbf{C}_{01} \\ \mathbf{0}_{3x4} \end{bmatrix} \tag{15}$$

$$\mathbf{B}_2 = \begin{bmatrix} \mathbf{0}_{4x4} \\ \mathbf{0}_{4x4} \\ -\mathbf{M}_1^{-1} \\ \mathbf{M}_2^{-1} \cdot \mathbf{L} \end{bmatrix} \tag{16}$$

$$\mathbf{C}_1 = \begin{bmatrix} \mathbf{I}_{4x4} & \mathbf{0}_{11x3} \end{bmatrix} \tag{17}$$

$$\mathbf{C}_2 = \left[\mathbf{L}^T \cdot \mathbf{M}_2^{-1} \cdot \mathbf{L} \cdot \mathbf{R}_2\right] \tag{18}$$

$$\mathbf{D}_{11} = \mathbf{D}_{12} = \mathbf{D}_{21} = \mathbf{0}_{4x4} \tag{19}$$

$$\mathbf{D}_{22} = \left[\mathbf{L}^T \cdot \mathbf{M}_2^{-1} \cdot \mathbf{L}\right] \tag{20}$$

In these equations, the input $\mathbf{u}$(t) is a vector of the four actuator forces u_i(t), i= 1,2,3 and 4, one for each wheel. The output $\mathbf{y}_1$(t)=$\mathbf{z}_{01}$(t) is the relative tire displacements between the ground and the center of wheel. The output $\mathbf{y}_2$(t) is a vector of accelerations at four points in vehicle body over each wheel. As a measure of $\mathbf{y}_1$(t) is very difficult, it was not considered in the control feedback, it was just considered here to show the tire behavior of the system under control compared to the passive one. Whereas $\mathbf{y}_2$(t) is easily measured by a simple accelerator device used to measure the vertical acceleration only.

2.4. Frequency Domain Approach

Since the control design is based on a frequency domain approach, the dynamical system described above must be represented as a Transfer Function Matrix (TFM). Thus the vehicular system is represented by the TFM $\mathbf{G}$(s) (Eq. (21)) and the controller will be represented by the TFM $\mathbf{K}$(s) as follows:

$$\mathbf{G}(s) = \begin{bmatrix} \mathbf{G}_{11}(s) & \mathbf{G}_{12}(s) \\ \mathbf{G}_{21}(s) & \mathbf{G}_{22}(s) \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} \mathbf{y}_1(s) \\ \mathbf{y}_2(s) \end{bmatrix} = \mathbf{G}(s) \cdot \begin{bmatrix} \mathbf{w}(s) \\ \mathbf{u}(s) \end{bmatrix} \tag{22}$$

$$\mathbf{u}(s) = \mathbf{K}(s) \cdot \mathbf{y}_2(s) \tag{23}$$

3. Model Reduction

The main goal of model reduction is to get a reduced system able to generate a output very similar to $\mathbf{y}_2$(t) from a respectively input $\mathbf{u}$(t), and that have good properties for multivariable control development. The procedure begins with the original form of $\mathbf{G}_{22}$(s), which is analised by its Frequecy Response Function (FRF).

It emerged from this analysis that $\mathbf{G}_{22}$(s) is ill-conditioned, since one of its singular values is almost zero along all frequency spectrum of interest. The main cause of this ill condition is because the system has three variables (heave, roll and pitch) to be controlled by four actuators and measured by four accelerometers. So the first step is to eliminate this system miscondition.

3.1. Elimination of Twist Mode

If the vehicle body was flexible, the four actuator would be able to control the twist mode of vehicle body vibration. But since the vehicle body is rigid, it have leaded to the ill-conditioned form of $\mathbf{G}_{22}$(s). To avoid this undesirable situation the capacity of control to act over the twist mode must be eliminated. To do this, a new coordinate system is used to represent the TFM. $\mathbf{S}_r$ is a base of a new subspace, where each column represent a movement of the vehicle body (roll, heave, pitch and twist respectively), and is a orthonormal matrix given by:

$$\mathbf{S}_r = \frac{1}{2} \cdot \begin{bmatrix} 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ -1 & 1 & -1 & -1 \end{bmatrix} \tag{24}$$

The transformation $\mathbf{S}_r$ must be applied as follows:

$$\mathbf{B}_1' = \mathbf{B}_1 \cdot \mathbf{S}_r \quad ; \quad \mathbf{B}_2' = \mathbf{B}_2 \cdot \mathbf{S}_r \quad ; \quad \mathbf{C}_2' = \mathbf{S}_r^T \cdot \mathbf{C}_2 \quad ; \quad \mathbf{D}_{22}' = \mathbf{S}_r^T \cdot \mathbf{D}_{22} \cdot \mathbf{S}_r \quad ; \quad \mathbf{D} = \mathbf{D}_{22}' \tag{25}$$

Afterwards, the last column of matrixes “B”, the last line of matrix “C”, and both last column and last line of matrix “D” must be eliminated. The last column of matrix S must also be eliminated in order to maintain the matrix multiplication match.

3.2. System Order Minimization

The rank of controlability matrix $[\mathbf{B}_2 \mid \mathbf{AB}_2 \mid \mathbf{A}^2\mathbf{B}_2 \mid \ldots \mid \mathbf{A}^{n-1}\mathbf{B}_2]$ is calculated and if the system turns to be not completely controlable, a minimization procedure must be applied. This processes consists to get a similarity transformation **T** such that:

$$\overline{\mathbf{A}} = \mathbf{T}\cdot\mathbf{A}\cdot\mathbf{T}^T \quad ; \quad \overline{\mathbf{B}}_1 = \mathbf{T}\cdot\mathbf{B}_1^{'} \quad ; \quad \overline{\mathbf{B}}_2 = \mathbf{T}\cdot\mathbf{B}_2^{'} \quad ; \quad \overline{\mathbf{C}}_2 = \mathbf{C}_2^{'}\cdot\mathbf{T}^T \tag{26}$$

And the transformed system has the form presented in the sequel:

$$\overline{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_{nc} & \mathbf{0} \\ \mathbf{A}_{21} & \mathbf{A}_c \end{bmatrix} \quad ; \quad \overline{\mathbf{B}}_1 = \begin{bmatrix} \mathbf{B}_x \\ \mathbf{B}_{1c} \end{bmatrix} \quad ; \quad \overline{\mathbf{B}}_2 = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_{2c} \end{bmatrix} \quad ; \quad \overline{\mathbf{C}}_2 = \begin{bmatrix} \mathbf{C}_{nc} & \mathbf{C}_c \end{bmatrix} \tag{27}$$

The algorithm used to get the matrix **T** was the “Staircase Algorithm of Rosenbrock” (1968). The last operation consists in separate the controllable part to be used in controller design. The resulting system is:

$$\overline{\mathbf{G}}_{22}(s) = \mathbf{C}_c\cdot(s\mathbf{I}-\mathbf{A}_c)^{-1}\cdot\mathbf{B}_{2c} \tag{28}$$

A similar, but dual, process must be applied in case of lack of observability.

3.3. Truncated Balanced Realization

The TBR procedure as first presented in Moore (1981) is centered around information obtained from the controlability Grammian $\mathbf{W}_c$ and the observability Grammian $\mathbf{W}_o$, which can be obtained from solving the two Lyapunov equations below:

$$\mathbf{A}_c\mathbf{W}_c + \mathbf{W}_c\mathbf{A}_c^T = -\mathbf{B}_{2c}\mathbf{B}_{2c}^T \quad ; \quad \mathbf{A}_c\mathbf{W}_o + \mathbf{W}_o\mathbf{A}_c^T = -\mathbf{C}_c^T\mathbf{C}_c \tag{29}$$

Once solved the Grammians, the algorithm consists in six more steps:

1. compute Cholesky factors $\mathbf{W_c} = \mathbf{L_c}\mathbf{L_c}^T$, $\mathbf{W}_o = \mathbf{L}_o\mathbf{L}_o^T$;
2. compute the singular value decomposition (SVD) of Cholesky product $\mathbf{U\Sigma V} = \mathbf{L}_o^T\mathbf{L_c}$, where $\mathbf{\Sigma}$ is diagonal positive and **U**, **V** have orthonormal columns;
3. compute de balancing transformation as presented next:

$$\mathbf{T} = \mathbf{L}_c\cdot\mathbf{V}\cdot\mathbf{\Sigma}^{-1/2} \quad ; \quad \mathbf{T}^{-1} = \mathbf{\Sigma}^{-1/2}\cdot\mathbf{U}^T\cdot\mathbf{L}_o^T \tag{30}$$

4. form the balanced realization as in following equations:

$$\mathbf{A}_b = \mathbf{T}\cdot\mathbf{A}_c\cdot\mathbf{T}^{-1} \quad ; \quad \mathbf{L} = \mathbf{T}\cdot\mathbf{B}_{1c} \quad ; \quad \mathbf{B}_b = \mathbf{T}\cdot\mathbf{B}_{2c} \quad ; \quad \mathbf{C}_b = \mathbf{C}_c\cdot\mathbf{T}^{-1} \tag{31}$$

5. sort $\mathbf{\Sigma}$ in descending order of *Hankel singular values* and organize $\mathbf{A}_b$, $\mathbf{B}_b$ and $\mathbf{C}_b$ in the same order;
6. truncate $\mathbf{A}_b$, $\mathbf{B}_b$ and $\mathbf{C}_b$ to form the reduced realization by elimination of row and columns related to neglected *Hankel singular values*.

In this work, the original 15th order system was reduced to a completely controllable and observable balanced 12th order system. The matrix **L** in Eq. (31) was used to design the Kalman Filter in the LQG/LTR procedure, which is succinctly described in the next section.

4. Control Design

The aim of controller design is to find a TFM **K**(s), which improves the comfort of the vehicle, i. e., reduces the acceleration amplitude in vehicle body movements. Thus, as can be seen in Fig. 3, the controller design consists to calculate the matrixes **H** and **G**. **K**(s) can than be written as follows:

$$\mathbf{K}(s) = -\mathbf{S}_r \cdot \mathbf{S}_u \cdot \mathbf{G} \cdot \left(s\mathbf{I} - \mathbf{A}_b + \mathbf{B}_n \cdot \mathbf{G} + \mathbf{H} \cdot \mathbf{C}_b - \mathbf{H} \cdot \mathbf{D}_n \cdot \mathbf{G}\right)^{-1} \cdot \mathbf{H} \cdot \mathbf{S}_r^T \quad (32)$$

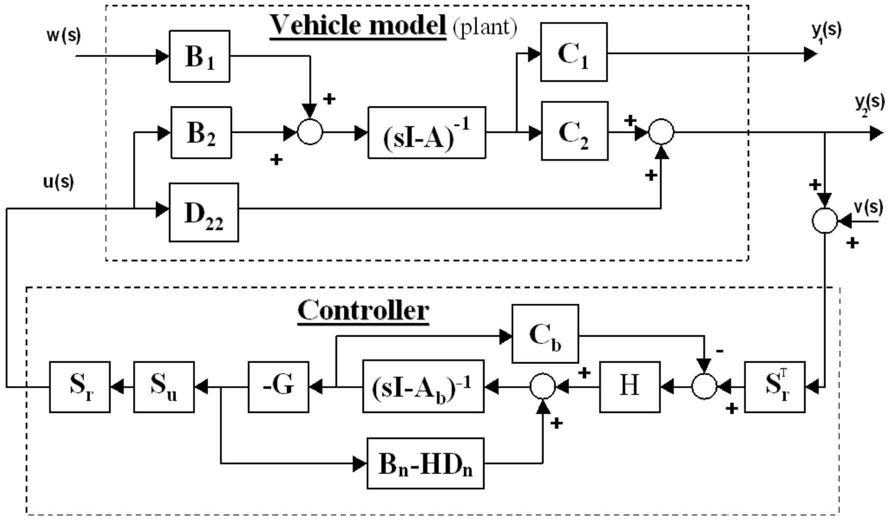

Figure 3. Block diagram of controlled suspension system.

To calculate the matrices **H** and **G**, it is applied a LQG/LTR methodology as described in Crivellaro and Tamai (2003). Also in that work, a FRF normalization is proposed by the use of the matrix $\mathbf{S}_u=((\mathbf{D}^T \cdot \mathbf{D})^{½})^{-1}$, which was used to get matrix $\mathbf{B}_n=\mathbf{B}_b \cdot \mathbf{S}_u$ and $\mathbf{D}_n=\mathbf{D} \cdot \mathbf{S}_u$. The result of this controller design is shown in Fig. 4 through the FRF of $\mathbf{G}_{11}(j\omega)$ and $\mathbf{G}_{21}(j\omega)$ for passive suspension system and active suspension system in closed loop.

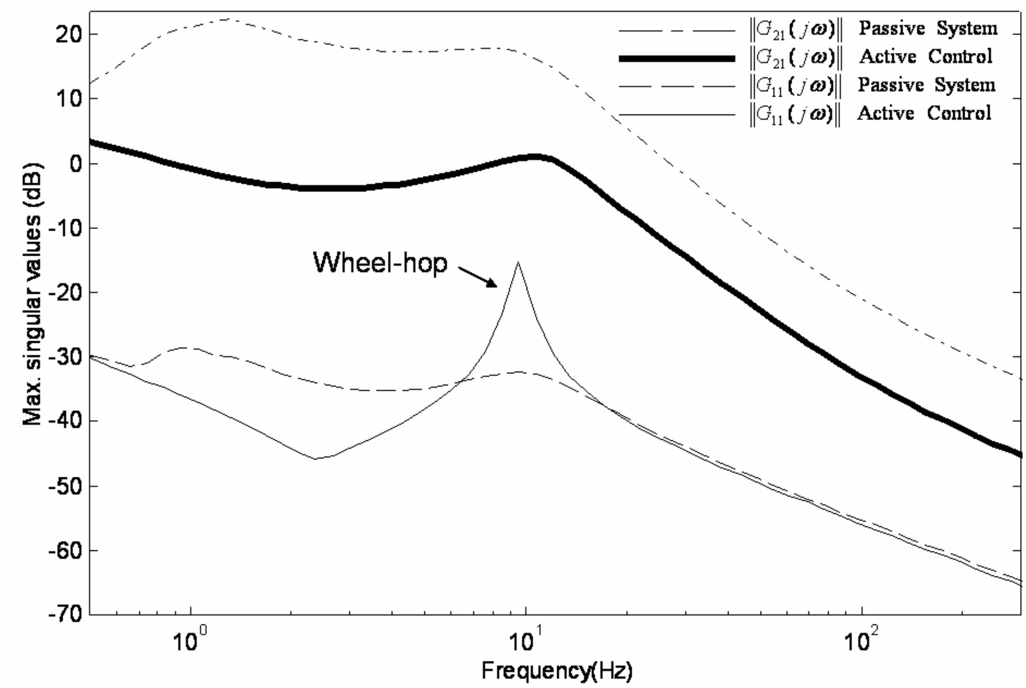

Figure 4. FRF of $\mathbf{y}_2(s) = \mathbf{G}_{21}(s) \cdot \mathbf{w}(s)$ and $\mathbf{y}_1(s) = \mathbf{G}_{11}(s) \cdot \mathbf{w}(s)$ for "passive system" and "active closed loop system".

As showed in Fig. 4, the acceleration amplitude in vehicle body movements is significantly decreased along all frequency range analyzed, which means a good improvement in vehicle comfort. On the other hand, the active closed loop system performance result in an improvement of wheel hop around 10 Hz, what can cause lost of adherence between wheel and ground, impacting negatively in vehicle handling performance. This issue must be the focus for suspension controller design improvements.

5. Original Model Controller Simulation

Simulations were accomplished using Runge-Kuta 4th and 5th order methods. A white noise processed by a suitable "shaping filter" was used to simulate the disturbance signal generated by the road. The values of acceleration of the output of the plant were evaluated as its RMS values. In the simulations, the vehicle speed was 60 km/h, during 10.8 seconds, which is equivalent to 180 meters run. It is important to highlight that the controller was applied in an original plant, instead of the balanced model used as the base of controller design.

Table 1 shows simulation results comparing passive system and active closed loop system performance.

Table 1. Simulation results – RMS values.

	Heave acc. (m/s^2)	**Pitch acc. (m/s^2)**	**Roll acc. (m/s^2)**
Passive system	**0.3195**	**0.2211**	**0.4518**
Active system	**0.0348**	**0.0246**	**0.0682**

It is than clear that the proposed control methodology achieved its goals.

6. Conclusions

The modeling strategy presented in this work is a good approach for suspension systems controller development, which has brought the following advantages:

- a simple model with only four sensors (accelerometers) and four actuators;
- a reduced order model, and consequently a reduced controller order;
- the TBR approach provides a better use of the system energy, since it is based on Grammian approach.

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8. Responsibility notice

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