ON THE FATIGUE LIMIT FOR CYLINDRICAL CONTACTS UNDER A PARTIAL SLIP REGIME

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Abstract. *This paper summarises an attempt to propose a methodology suitable for estimating high-cycle fatigue strength of cylindrical contacts under a partial slip regime. In particular, Taylor's point method, usually applied to predict fatigue limits of notched components, was used in conjunction with the Modified Wöhler Curve Method allowing us to formulate a novel fretting fatigue prediction methodology. The devised procedure takes as its starting point the idea that to correctly estimate fatigue damage under fretting fatigue two different aspects must be taken into account: stress gradients and degree of multiaxiality of the stress field damaging the fatigue process zone. The first problem was addressed by using the Theory of Critical Distances, whereas the latter by using an appropriate multiaxial fatigue criterion. In order to check the accuracy of the proposed methodology, twenty-nine tests on cylindrical contacts were selected from the technical literature. The performed analyses showed a sound agreement between estimations and experimental data. In particular, the proposed method correctly predicted failures in the medium-cycle fatigue regime, allowing the high-cycle fatigue estimations to fall within an error interval of about ±20%. This result is very interesting, especially by the light of the fact that such an approach is based on the use of linear-elastic stresses, making it suitable for being used in situations of practical interest by post-processing linear-elastic Finite Element results.*

Keywords*: Fretting Fatigue, notch fatigue, multiaxial fatigue, critical distance, size effect*

1. INTRODUTION

Some recent findings (Araújo et al., 2004, Vallelano et el., 2004) show that the problem of estimating fatigue endurance under fretting conditions can be addressed by taking into account the presence of stress concentration phenomena. To be precise, the material cracking behaviour under fretting fatigue can be assumed to be similar to that occurring in "conventional" notched components under fatigue loading: crack initiation, and its initial growth, depends on the distribution of the entire stress field damaging the fatigue process zone. This analogy is very attractive from a scientific point of view, because it would allow engineers engaged in practical problems to extend the theories already developed to assess notched components under fatigue loading to components damaged by fretting fatigue. The main problem in coherently extending this idea to practical situations is that the early stage of crack propagation under fretting fatigue is mixed mode dominated (Araújo and Nowell, 1999).

In particular, according to the experimental results published by Vallellano et al. (Vallellano et al., 2004) and generated by testing Al 7075-T6 sphere-plane contacts, initiation and early growth of cracks occur at small angles to the surface. Subsequently, cracks change their direction to grow along a line almost perpendicular to the contact zone surface. Finally, the above authors highlighted even that when failures did not occur in the high-cycle fatigue regime, cracks were arrested by the first grain boundary (as it happens to plain metal specimens under conventional fatigue loading). This experimental finding makes it evident that, to correctly model the physical processes leading to crack initiation and its initial propagation under fretting fatigue, two fundamental aspects should be taken into account: the real material morphology in the vicinity of crack initiation sites and the elasto-plastic behaviour of grains. In other words, predicting the material strength in the high-cycle fretting fatigue regime is mainly a short crack problem.

Unfortunately, it is well-known that to correctly model short crack behaviour, linear-elastic approaches may not be the most adequate ones. It is evident that modelling all the phenomena taking place within the fatigue process zone by accounting for grain plasticity is very attractive from a philosophical point of view, but such an approach would be too cumbersome to be applied to assess real components in situations of practical interest. For this reason, the problem must be greatly simplified in order to develop approaches which can be used in the industrial reality.

According to the considerations reported above, this paper attempts to estimate fretting fatigue damage by using a multiaxial fatigue method we recently developed to assess notched components in the high-cycle fatigue regime (Susmel and Taylor, 2003): the aim of this work is to predict fretting fatigue damage accounting for the entire stress field in the vicinity of the contact zone by considering a notch analogue.

2. FATIGUE DAMAGE AND STRUCTURAL VOLUME

The use of the Modified Wöhler Curve Method (MWCM) in conjunction with the Theory of Critical Distances (TCD) is based on the assumption that all the physical processes leading to crack initiation are confined within the so-called structural volume. The size of this volume is assumed not to be dependent on either the stress concentration feature weakening the component or the complexity of the stress field damaging the fatigue process zone (Susmel, 2004).

To define the size of this volume, consider an infinite plate with a central through-crack (Fig. 1a). This plate is subjected to a remote fully-reversed uniaxial fatigue loading (R=-1). Observing that, thanks to the above assumptions, geometry, nominal stress gradient and non-zero mean stress do not affect the crack growth, the above configuration can be assumed to be representative of the "pure" material cracking behaviour.

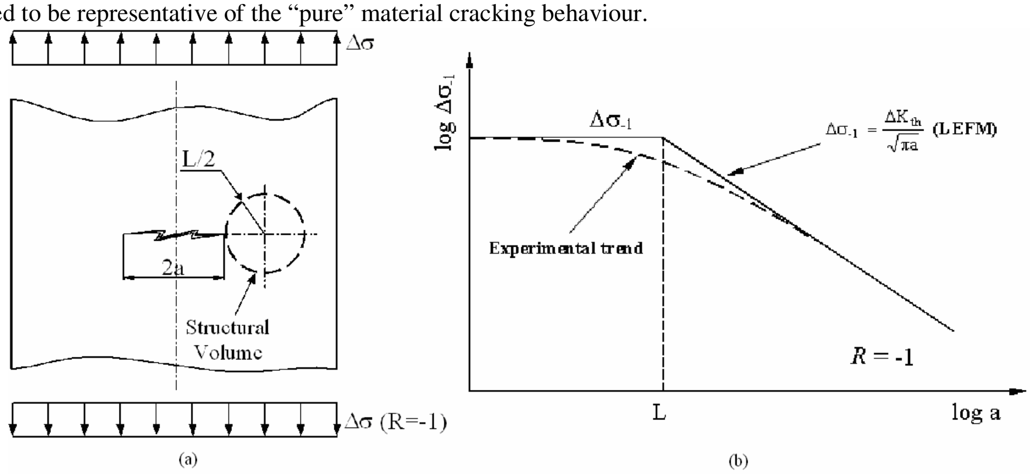

Figure 1: Central through-crack in an infinite plate subjected to a remote uniaxial load (a) and Kitagawa and Takhashi's diagram (b).

Consider now the classical Kitagawa-Takahashi curve by approximating it to the two straight asymptotic lines plotted in Fig. 1b: the horizontal line corresponds to the plain fatigue limit, whereas the sloping one is plotted according to the Linear Elastic Fracture Mechanics (LEFM). The length at which these two lines intersect each other turns out to be:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_{-1}}\right)^2 \tag{1}$$

In the above equation, $\Delta\sigma_{-1}$ is the plain fatigue limit and ΔK_{th} is the range of the threshold value of the stress intensity factor (both determined under load ratios, R, equal to -1). According to the fact that the material characteristic length L is defined by two material properties, it is evident that L turns out to be a material property, which is different for different materials (Taylor, 1999).

Observing now the trend schematised by the two straight asymptotic lines in Kitagawa-Takahashi's diagram, it is possible to assume that as long as the half-length of the crack is lower than L, no reduction of the nominal fatigue limit occurs. Therefore, the size of the structural volume can be considered to be directly related to the material characteristic length, L. To be precise, in order to avoid a reduction of the nominal fatigue limit, all the cracking processes must be confined within this area, which can be supposed to be circular in 2D bodies and spherical in 3D components (Fig. 1a).

It is possible to observe now that when a component is in the fatigue limit condition some micro/meso-cracks are always present within the structural volume, and it holds true independently of the stress concentration feature weakening the component (Akiniwa et al., 2001). In particular, it is important to remember that the sharper the notches are then the longer the length of non-propagating cracks. When plain specimens are in the fatigue limit condition, the crack propagation is arrested either by the first grain boundary or by the first micro-structural barrier (Miller, 1993). On the contrary, in the presence of sharp notches, the maximum length of non-propagating cracks is equal to:

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{F \Delta\sigma_{-1}}\right)^2 \quad (2)$$

where F is the geometrical correction factor for the LEFM stress intensity factor.

If non-propagating cracks emanate from the tip of a notch weakening a real component, F is always larger than unity (Tada et al., 2000). This makes it evident that non-propagating cracks are always confined within the structural volume, even when they reach their maximum length.

All the above arguments seem to strongly support the idea that the fatigue process zone has a size which is directly related to the material characteristic length, L. The question now is "What is the material cracking behaviour within the fatigue process zone?". During the last few years, we extensively investigated crack paths within the structural volume in the high cycle fatigue regime. In particular, we considered specimens of steel weakened by different geometrical features and subjected to both uniaxial and biaxial fatigue loading (Susmel and Taylor, 2004).

Our understanding of the phenomenon is that initiation and initial growth of micro/meso-cracks are always mixed-mode governed and this process can be considered to be similar to the classical Stage 1 taking place in plain specimens (Miller, 1993). In particular, independently of stress concentration feature and degree of multiaxiality of the stress field, crack initiation is mixed-mode dominated and the length of the Stage 1-like crack is equal to about $L/2$. To be precise, the transition from a Stage 1-like to a Stage 2-like process occurs at a distance from the notch tip depending on the material characteristic length, L. When the crack length is larger than about $L/2$, cracks tend to orient themselves in order to experience the maximum Mode I loading (Stage 2-like process). Therefore, if crack initiation is assumed to be the most important stage in determining fatigue limits, it is logical to believe that the critical plane approach is the soundest method to model the physical reality.

3. THE THEORY OF CRITICAL DISTANCES (TCD) AND THE MODIFIED WÖHLER CURVE METHOD (MWCM)

In the recent past, Taylor has proposed a new reinterpretation of the TCD to predict uniaxial fatigue limits of components weakened by any kind of stress concentration feature (Taylor, 1999). This approach postulates that the reference stress to be used to assess notched components can be calculated in different ways. In particular, it can be determined at a certain distance from the apex of the stress concentrator (Point Method, PM), it can be averaged along a line (Line Method, LM), or, finally it can be averaged over an area (Area Method, AM). The TCD was seen to be capable of predictions falling within an error interval of about 20% (Susmel and Taylor, 2003). This method is essentially empirical, but Taylor has proposed that the LM may be related to the conditions for propagation of a notch-root crack of length $2L$. Unfortunately, this would only justify the use of the method for sharp notches, giving no explanation as to the reason why the TCD is also successful in predicting fatigue limits in the presence of blunt notches.

In order to formalise the TCD in terms of the PM, consider a notched specimen subjected to a remote uniaxial fatigue loading. A notched component is in the fatigue limit condition if the range of the maximum principal stress at a distance from the notch tip equal to $L/2$ equals the plain fatigue limit, $\Delta\sigma_{-1}$. In other words:

$$\Delta\sigma_1\left(r = L/2\right) = \Delta\sigma_{-1} \quad (3)$$

where L is given by Eq. (1). It is important to highlight here that, to properly apply the TCD, L must always be determined for the correct load ratio, R (Susmel and Taylor, 2003). According to the PM, the point at which the reference stress must be calculated exactly corresponds to the centre of the structural volume.

The MWCM takes as its starting point the assumption that crack initiation is Mode II dominated, and it holds true independently of both stress concentration feature and degree of multiaxiality of the stress field damaging the fatigue process zone. The MWCM formalised to assess components in the high-cycle fatigue regime is written:

$$\tau_a + m_1 \frac{\sigma_{n,\max}}{\tau_a} - \lambda = 0 \quad (4)$$

In the above equation, τ_a is the shear stress amplitude relative to the material plane experiencing the maximum shear stress amplitude (critical plane), $\sigma_{n,\max}$ is the maximum stress perpendicular to this plane and, finally, λ and m_1 are material constants that can be obtained from two fatigue limits generated under different loading conditions. For instance, if the fatigue limits σ_{-1} and σ_0 generated under fully-reversed (R=-1) and under repeated (R=0) uniaxial load, respectively, are considered, the two relevant constants of the criterion turn out be:

$$m_1 = \frac{\sigma_{-1} - \sigma_0}{2} \qquad e \qquad \lambda = \sigma_{-1} - \frac{\sigma_0}{2}\,. \quad (5)$$

It is interesting to observe that, even though the MWCM can successfully be calibrated using two experimental uniaxial fatigue limits generated under different load ratios, when one of these two fatigue limits is estimated using a proper methodology, the MWCM accuracy becomes sensitive to the detrimental effect of non-zero mean stresses on the assessed material. In other words, it is well-known that fatigue limits under different load ratios can be estimated by using an experimental fatigue limit generated under a reference load ratio and the material static properties, but, unfortunately, the accuracy of well-known methods such as those proposed by Smith-Watson-Topper (Smith et al., 1970) or Goodman (Goodman, 1919) depends on the material fatigue behaviour under superimposed static stresses (Susmel et al., 2004).

Reliability and accuracy of the MWCM were initially checked by considering smooth specimens both in the high-cycle (Susmel and Taylor, 2003) and in the medium-cycle fatigue regime (Susmel and Petrone, 2003), showing that this method was capable of successfully accounting for the presence of both non-zero out of phase angles and non-zero mean stresses (Susmel and Taylor, 2004). Subsequently, the MWCM was applied in conjunction with the TCD to predict high-cycle fatigue strength of notch components under both uniaxial and multiaxial fatigue loading (Susmel and Taylor, 2003).

In particular, according to the arguments summarised in the previous section, it is possible to say that fatigue limits have to be estimated by considering a stress state which is representative of the entire stress field damaging the fatigue process zone. In more detail, using the PM argument, it is possible to assume that the linear-elastic stress state calculated at the centre of the structural volume supplies all of the engineering information needed to perform an accurate high-cycle fatigue assessment (Susmel and Taylor, 2003). Moreover, in fatigue limit conditions, and independently of the stress concentration feature, the crack initiation phenomenon can be assumed to be governed by a Stage 1-like process. According to this, fatigue damage reaches its maximum value on the plane experiencing the maximum shear stress amplitude, and its amount depends on both τ_a and $\sigma_{n,max}$ when calculated at the centre of the structural volume.

4. THE PROCEDURE TO APPLY THE MWCM IN TERMS OF THE TCD TO FRETTING FATIGUE SITUATIONS

A fretted component subjected to a system of external contact forces (P and Q) and also experiencing a bulk fatigue stress (σ_B) gives rise to a subsurface multiaxial stress field within the contact region (Fig. 2). Further, as also happens to components containing geometrical discontinuities, the fretting regime is characterized by the presence of stress concentration phenomena at the contact surface which rapidly decay. This suggests that a threshold condition for crack initiation might be predicted by using methodologies similar to those employed to assess notched components. According to this idea, we seek to use the MWCM re-interpreted in terms of TCD to address the fretting fatigue problem.

In order to apply the procedure proposed in the present paper, it is initially necessary to determine the radius of the structural volume (Fig. 2). It must always be calculated using fatigue properties (that is, $\Delta\sigma_{-1}$ and ΔK_{th}) determined under a load ratio, R, equal to –1 (Susmel and Taylor, 2003). In fact, as said above, the reference configuration is the one given by a through-cracked plate under fully-reversed uniaxial fatigue loading (Fig. 1a) and the presence of non-zero mean stresses as well as of non-zero out-of-phase angles is directly accounted for by Eq. (4).

It is important to remember here that in real joints some localized plasticity may be provoked by the stress concentration phenomenon present at the contact interface. For this reason, a rigorous analysis to determine the stress field in the vicinity of the contact region should consider an appropriate constitutive model capable of accounting for the stress redistribution under the stress raiser. Unfortunately, these kinds of analyses are complex and time-consuming, so that, too often, they are not compatible with the industrial needs. One of the most important features of the TCD is that stress concentration phenomena in fatigue can be assessed just by carrying out linear-elastic analysis, reducing time and costs of the design process. Therefore, and taking advantage from this peculiarity of the TCD, the use of the proposed method is based on linear-elastic solutions.

When the stress tensor is entirely defined during the load cycle at the centre of the structural volume ($y/a=L/2$ in Fig. 2), by using the appropriate algorithms (Papadopoulos, 1998, Weber et al., 1999) it is relatively simple to determine the critical plane orientation as well as the shear stress amplitude and the maximum normal stress relative to such a plane (unfortunately, dealing with complex periodic load histories, this calculation is rather time-consuming). Finally, if the condition expressed by Eq. (4) is assured, then the studied component is predicted to be in the fatigue limit condition. Figure 2 depicts a schematic view of the application of the proposed methodology to a cylinder-on-flat contact configuration under fretting conditions.

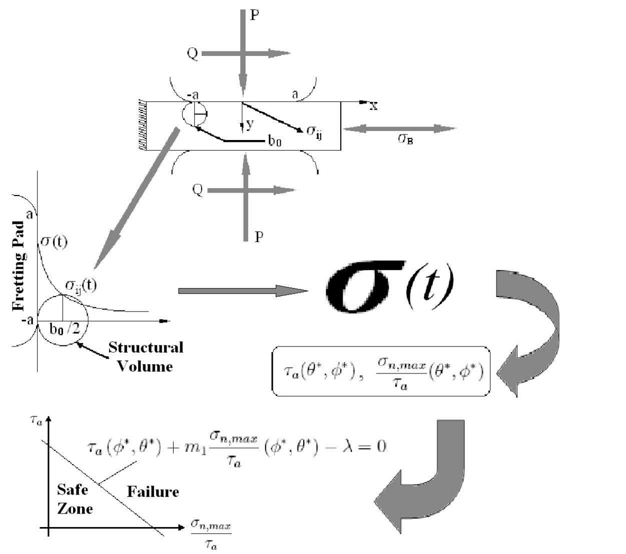

Figure 2: The procedure to apply the MWCM in terms of the TCD for fretting fatigue.

To be precise, our method takes as its starting point the idea that high-cycle fatigue damage in metals depends on both stress gradients and degree of multiaxiality of the stress field in the vicinity of crack initiation sites. Our understanding of the phenomenon is that this fact holds always true independently of the causes these two phenomena are originated from: the MWCM accounts for the multiaxiality of the stress field, whereas the TCD allows the stress gradient effect to be taken into account.

To conclude this section it is worth noting that, in general, analytical approaches are not adequate to determine the stress state at the centre of the structural volume for real mechanical assemblies. For this reason, engineers engaged in practical problems prefer to determine such stress states by post-processing FE results: the proposed methodology is suitable for being used in conjunction with linear-elastic FE results, with the advantage that parameters such as nominal stress, equivalent stress intensity, etc. do not have to be defined.

5. AVAILABLE EXPERIMENTAL DATA FOR CYLINDRICAL CONTACTS

In the next section the proposed procedure will be validated by using experimental data for a cylinder-on-plane contact configuration (Fig 3). These experiments have been reported and discussed in detail elsewhere (Araújo et al., 2004), hence, only the basic pieces of information necessary to carry out the analysis are briefly reported below. In particular, a constant normal load, P, was applied to the fretting pads and held constant. A sinusoidal bulk load B(t) applied to "the dog bone" tensile specimen induced (i) a bulk fatigue stress, $\sigma_B(t) = \sigma_{Bmax} \sin(wt)$, and (ii) a in-phase shear load, $Q(t) = Q_{max} \sin(wt)$, where σ_{Bmax} and Q_{max} were the amplitudes of bulk stress and shear load, respectively, w was the load frequency and t was the time. Tests were designed to run in a partial slip regime, i.e. $Q<fP$, where the friction coefficient for the slip zones, f, was equal to 0.75. Four series of tests were considered. Within each series an average of eight tests using different pad radii varying from 12.5 to 150 mm were performed. Although the pad radius changed, the surface stress field was the same for all tests in a series (but the rate of stress decay varied). This provoked a size effect in the presence of larger contact widths (or pad radii) causing fatigue failures, whereas for the smallest contacts the tests ran up to 10^7 cycles (here considered as infinite life) before being interrupted. The range defined by the largest contact size causing no failure and by the smallest one resulting in fatigue failures was termed critical contact size range, a_{crit}.

Table 1 reports the relevant load parameters and a_{crit} for each data set. Pads and specimens were made of an Al4%Cu alloy (Young's modulus, E = 74GPa, yield stress σ_y = 465MPa and ultimate tensile strength, σ_{UTS} = 500MPa). Finally, it is important to highlight that in the broken specimens cracks initiated within the slip zone, at or close to the trailing edge of the contact.

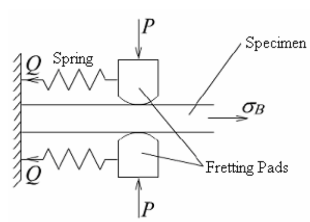

Figure 3: Cylinder on plane contact configuration.

Series	p_0 [MPa]	Q_{max}/P	σ_{Bmax} [MPa]	a_{crit} [mm]
1	157	0.45	92.7	0.28–0.38
2	143	0.45	92.7	0.18–0.27
3	143	0.45	77.2	0.36–0.54
4	120	0.45	61.8	0.57–0.71

Table 1: Experimental parameters and critical contact size range.

Due to the geometry of the tested configuration, the stress field in the fatigue process zone could be directly evaluated by using analytical solutions given in closed form. In particular, the direct and shear tractions could easily be defined according to the solutions proposed by Hertz (Hertz, 1882) and Mindlin (Mindlin, 1949), respectively. These could subsequently be used with Muskhelishvili's potentials (Muskhelishvili, 1953) to determine the stress components associated with the normal and the shear load. By superposing these different contributions, it was possible to directly evaluate the resulting stress tensor. In particular, at the instants of maximum and minimum bulk/shear loads it turned out to be:

$$\frac{\boldsymbol{\sigma}(y,x,t)}{p_o}=\left(\frac{\boldsymbol{\sigma}^n\left(\frac{x}{a},\frac{y}{a}\right)}{p_0}\right)\pm f\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x}{a},\frac{y}{a}\right)}{fp_0}\right)\mp f\frac{c}{a}\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x-e}{c},\frac{y}{c}\right)}{fp_0}\right)+\frac{\boldsymbol{\sigma}_B(t)}{p_0}, \tag{6}$$

being the combination of signs + and – for the maximum load step. On the contrary, during loading or unloading, the appropriate analytical solution resulted in the following form:

$$\frac{\boldsymbol{\sigma}\;(y,x,t)}{p_o}=\left(\frac{\boldsymbol{\sigma}^n\left(\frac{x}{a},\frac{y}{a}\right)}{p_0}\right)\pm f\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x}{a},\frac{y}{a}\right)}{fp_0}\right)\mp 2f\frac{d(t)}{a}\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x-e'(t)}{d(t)},\frac{y}{d(t)}\right)}{fp_0}\right)\pm f\frac{c}{a}\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x-e}{c},\frac{y}{c}\right)}{fp_0}\right)+\frac{\boldsymbol{\sigma}_B(t)}{p_o} \tag{7}$$

For unloading conditions the correct sequence of signs to be considered in Eq. (7) is: –, + and –. In the above equation, p_o is the peak pressure, c and e are the stick zone half width and its offset from the center of the contact at the instant of maximum or minimum shear load. At any other time instant d and e' correspond to the stick zone half width and its offset from the center of the contact. The superscripts n and t stand for the stress components due to the normal and the tangential load, respectively. Finally, $\boldsymbol{\sigma}_B(t)$ is the stress tensor associated with the bulk fatigue load, hence σ_{xx} is its unique stress component different from zero. Plane strain conditions are assumed. Explicit expressions to compute c, e, d, e', $\boldsymbol{\sigma}^n$ and $\boldsymbol{\sigma}^t$ are given in a convenient form by Hills and Nowell (Hills and Nowell, 1994).

6. MWCM ACCURACY IN PREDICTING HIGH-CYCLE FATIGUE STRENGTH UNDER FRETTING FATIGUE

This section summarises the results obtained when applying the MWCM to estimate the high-cycle fatigue strength of the experimental configurations considered in this work and briefly discussed above. In order to evaluate the accuracy of the proposed methodology, the following error index (as defined by Papadopoulos (Papadopoulos, 1995)) was adopted:

$$SU[\%]=\frac{\tau_a+m_l\cdot\rho-\lambda}{\lambda} \tag{8}$$

A negative value of the above error index indicates that fatigue failure should not occur up to a number of cycles to failure theoretically equal to infinity. On the contrary, when SU>0%, the component is not in the fatigue limit condition, but, fatigue lifetime cannot be estimated (Fig. 4). It is interesting to observe also that, from a engineering point of view, a negative value of the SU index is an indication of the fact that the component dimensions could be reduced down to the limiting condition given by SU=0%.

As briefly explained above, the first step in applying the proposed procedure is to define the material characteristic length, L. Unfortunately, this experimental information was not directly available in Refs (Araújo and Nowell, 2004), for this reason it was taken from another source. In particular, Atzori et al. reported in Ref. (Susmell et al., 2005) a value for L equal to 0.1mm for an Al 4%Cu alloy having the same high-cycle fatigue strength ($\Delta\sigma_{-1}$=248MPa) as the one tested by Nowell under fretting (Araújo and Nowell, 2004). According to this value the radius of the structural volume, $L/2$, was assumed to be equal to 0.05mm (Susmel et al., 2005). At this depth and at the trailing edge of the contact zone (*hot spot*), i.e. x/a=-1, the cyclic stress tensor was then analytically calculated at twelve different load steps by using Eqs (6) and (7). Finally, to predict high-cycle fatigue strength under fretting, the SU index, Eq. (8), needed to be computed from the stress history defined at the centre of the fatigue process zone. Its calculation also required two fatigue parameters generated under different loading conditions, such as the fatigue limits under fully-reversed bending and torsion, respectively, or two uniaxial fatigue limits generated under two different load ratios, *R*. In Ref. (Susmel et all., 2005) for the material similar to the one considered in the present study and having L=0.1mm (*R*=-1) two uniaxial high-cycle fatigue strengths, determined at 10^7 cycles to failure, were reported: $\Delta\sigma_{-1}$=248MPa (*R*=-1) and $\Delta\sigma_0$=172MPa (*R*=0). These reference strengths were then used to calculate constants m_l and λ in Eq. (4), resulting in the following values: m_l = 19 MPa and λ = 81 MPa.

Because the above relevant high-cycle fatigue strengths were extrapolated at 10^7 cycles to failure, the experimental results were subdivided into two different categories: for $N_f<10^7$ cycles to failure, the specimens were considered as broken in the medium-cycle fatigue regime, whereas, for $N_f\geq10^7$ cycles to failure, tests were classified as run outs. It is interesting to highlight that, as reported in Refs (Araújo et al., 2004), a direct inspection of the run out specimens ($N_f\geq10^7$ cycles to failure) did not reveal the presence of cracks either within the fretted zones or elsewhere within the contact region.

For every considered test, the maximum shear stress amplitude and the maximum normal stress relative to the critical plane were calculated at the centre of the structural volume and plotted in Figure 4, which summarises the accuracy of the proposed method in predicting fatigue damage under fretting fatigue. The above diagrams make it evident that our method was successful in predicting high-cycle fatigue strength under fretting, allowing run-out data to fall within an error interval of about ±20%. This level of accuracy is very good considering the likely errors in the experimental data and stress-field solutions: similar accuracy was obtained when the TCD was employed to predict fatigue limits of notched specimens (Susmel and Taylor, 2004). Finally, Figure 4 shows also that those data characterised by $N_f<10^7$ cycles to failure were always above the threshold condition predicted by the method (straight lines characterised by SU=0% in Figure 4), confirming again the soundness of the proposed approach.

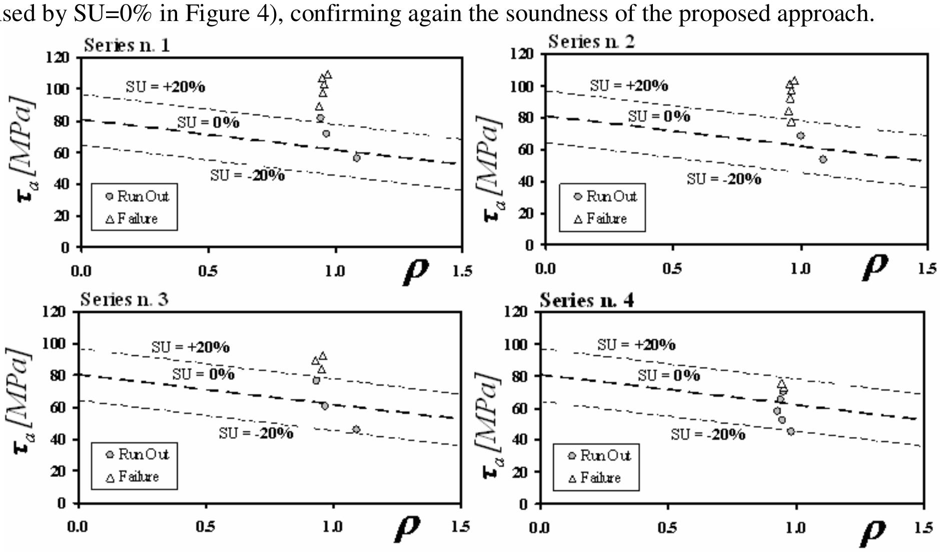

Figure 4: High-cycle fretting fatigue predictions plotted in τ_a vs. ρ diagrams.

7. DISCUSSION

This paper summarises an attempt to propose a novel approach to the prediction of fretting fatigue, based on the use of the MWCM used in conjunction with the TCD (Susmel and Taylor, 2003). The accuracy of the proposed method was systematically checked by considering several experimental results taken from the literature and generated testing cylinder-on-plane contact configurations. The proposed method was seen to be always capable of correctly predicting failures in the medium-cycle fatigue regime providing high-cycle fatigue predictions falling within an error interval of about ±20%. This result strongly supports the validity of this method, even though it is evident that more work has to be done in this area to more deeply investigate its accuracy and reliability when used to assess different materials and different contact configurations.

In the present experimental work, it was noted that there were no non-propagating cracks below the fatigue limit, i.e. the limit for fatigue crack initiation was also the limit for complete failure of the specimen. In fretting fatigue, relatively large non-propagating cracks can occur if the body forces are low compared to the local contact forces, so that the crack runs into a region of low stress as it grows away from the contact point. Since the TCD considers only stresses in the immediate vicinity of the contact point (i.e. within the material's critical volume, which is usually of the order of the grain size) then it is unlikely that it would be able to predict this cessation of crack growth after relatively large extensions. It is thus essentially a criterion for fatigue crack initiation, but including crack growth in what is normally referred to as the 'short crack' regime.

Finally, it can be highlighted even that, compared to other notch analogue methodologies proposed by the authors (Araújo et al., 2004) for fretting fatigue, the method discussed in the present paper has the advantage of defining the critical distance as a material parameter. Hence, if the basic fatigue parameters are appropriately defined for a specific material, the crack initiation risk can be directly computed without the need for carrying out further fretting fatigue calibration tests to define the size of the structural volume.

8. CONCLUSIONS

1) The Theory of Critical Distances, when used in conjunction with the multiaxial critical-plane theory MWCM, is capable of predicting the results of fretting fatigue experiments with a high degree of accuracy ($\pm 20\%$).
2) Fretting fatigue displays significant scaling effects, whereby the fatigue limit increases with increasing pad size when all other parameters are kept constant. This effect was accurately predicted by the TCD/MWCM approach.
3) The approach works because it characterises the stress level in the critical volume of material close to the point of maximum stress, in which short fatigue cracks develop; this critical volume is a material parameter which can be determined from conventional fatigue tests on notched specimens.
4) The methodology is relatively simple to implement because it involves only a linear-elastic analysis; thus complex components can be analysed using finite element methods.
5) More work has to be done in this area to more deeply investigate accuracy and reliability of the proposed methodology when used to assess different materials and different contact configurations.

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