ANALYSIS OF BIFURCATION AND CHAOS IN A COUPLED RAYLEIGH OSCILLATORS SYSTEM APPLIED IN THE LOCOMOTION OF A BIPEDAL ROBOT

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Abstract. *Coupled oscillators can be used in control systems of locomotion as pattern generators similar to the pattern of human gait, providing approached trajectories of the legs. In this work, the central pattern generator (CPG), responsible for the production of rhythmic movements, is formed by a set of mutually coupled Rayleigh oscillators. From a model of two-dimensional robot, oscillators were used for simulating the behavior of the hip and knees. Each oscillator has its own parameters and the link to the other oscillators is made through coupling terms. The objective of this work is to analyze the nonlinear dynamics of this system using bifurcation diagrams and Poincaré maps. By means of the analysis and graphs generated in MATLAB, it was possible to evaluate some characteristics of the system, such as: sensitivity to the initial conditions and presence of strange attractors. Based on the results of this study, we conclude that although the use of Rayleigh oscillators represents an excellent way for generating pattern signals of locomotion, its application in the control of a bipedal robot will only be possible with the correct choice of parameters, which must be done from the data provided by the analysis of bifurcation and chaos.*

Keywords*: bipedal locomotion, central pattern generator, chaos, nonlinear dynamics, oscillators.*

1. Introduction

In the course of many years the human being has been trying, in all manners, to recreate the complex mechanisms that form the human body. Such task is extremely complicated and the results are frequently unsatisfactory. However, with the technological advances, based on theoretical and experimental researches, the man achieves, in a way, to copy or to imitate some systems of the human body. It is the case, for example, of the central pattern generator (CPG), responsible for the production of rhythmic movements, such as to swim, to walk, and to jump, that it can be modeled by means of mutually coupled nonlinear oscillators. There are some significant works about the locomotion of vertebrates controlled by central pattern generators: Grillner (1985), Collins and Stewart (1993), and Pearson (1993).

The human locomotion is partially controlled by a CPG, what can be evidenced in works such as Calancie *et al*. (1994) and Dimitrijevic *et al*. (1998). A correctly projected CPG can generate trajectories of reference for locomotion and can be used in the control of bipedal robots. In this work the CPG is formed by a set of mutually coupled nonlinear oscillators, in which each oscillator generates angular signals of reference for the movement of the legs. Each oscillator has its proper amplitude, frequency and parameters, and the coupling to the other oscillators is made through the choice of coupling terms. We intend to evaluate the use of Rayleigh oscillators.

Many works about the study and application of nonlinear oscillators in the locomotion were been made previously, in particular, about the van der Pol oscillators (Bay and Hemami (1987), Dutra (1995), Zielinska (1996) and Dutra *et al*. (2003)). However, the study of Rayleigh oscillators was less explored, as well as its application in the locomotion (Pina Filho *et al*. (2005) and Pina Filho (2005)). The work presented here contributes to demonstrate that not only the coupling of van der Pol oscillators can generate patterns of locomotion, configuring the so-called CPG, but also the Rayleigh oscillators can be used.

Thus, the objective of the this work is to analyze the dynamics of the coupled Rayleigh oscillators system using bifurcation diagrams and Poincaré maps. By means of the analysis and graphs generated in MATLAB®, it was possible to evaluate some characteristics of the system, such as: sensitivity to the initial conditions and presence of strange attractors.

2. Rayleigh oscillator

In the course of century XIX some works about nonlinear oscillators were been made, particularly in conjunction with models of musical instruments. At this time, the British mathematical physicist Lord Rayleigh (John William Strutt, 1842-1919) introduced an equation of the form:

$$m\ddot{x} + kx = a\dot{x} - b(\dot{x})^3 \tag{1}$$

(with nonlinear damping velocity) to model the oscillations of a clarinet.

Rearranging the Eq. (1) and dividing the same one for m, we have:

$$\ddot{x} - \frac{a}{m}\left(1 - \frac{b}{a}(\dot{x})^2\right)\dot{x} + \frac{k}{m}x = 0 \tag{2}$$

With this, considering that $\frac{a}{m} \equiv \delta$, $\frac{b}{a} \equiv q$, and $\frac{k}{m} \equiv \Omega^2$, we found the form of the Rayleigh equation that will be used in the analyses:

$$\ddot{x} - \delta\left(1 - q\dot{x}^2\right)\dot{x} + \Omega^2\left(x - x_0\right) = 0 \qquad \delta, q \geq 0 \tag{3}$$

where: δ, q e Ω correspond to the parameters of the oscillator, and x_0 is an initial value of displacement.

3. Coupled oscillators

Systems of coupled oscillators have been used extensively in studies of physiological and biochemical modeling. Since the years of 1960, many researchers have studied the case of coupling between two oscillators, because this study is the basis to understand the phenomenon in a great number of coupled oscillators. In their works, French (1971) and Gaitskell (2002) present analyses of the coupling between two linear mass-spring systems. Other works about this subject can be seen in Kozlowski *et al*. (1995) and more recently Wirkus and Rand (2002).

One of the types of oscillators that can be used in coupled systems is the auto-excited ones, which have a stable limit cycle without external forces. These will be the oscillators used in the analyses presented here. In relation to the type of coupling, considering a set of n oscillators, there are three basic schemes of coupling (Low and Reinhall, 2001). Figure 1 presents these schemes, knowing that the last configuration of coupling will be used in the analyses, since we want that each oscillator has influence on the others.

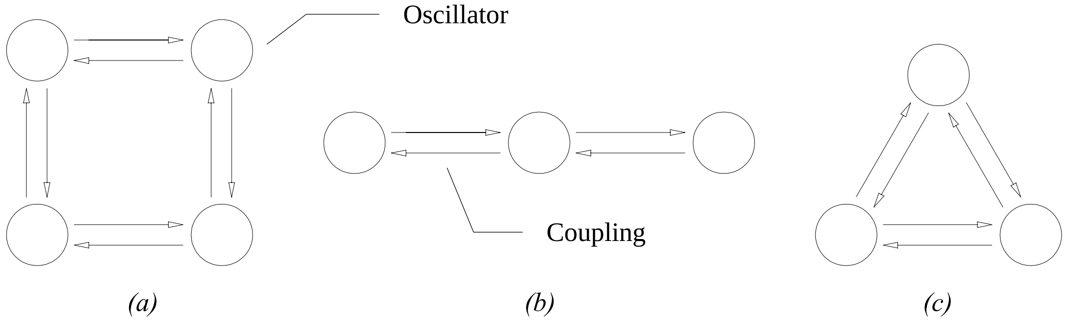


Figure 1. Basic schemes of coupling of oscillators: in ring *(a)*, in chain *(b)* and mutually coupled *(c)*.

Considering a net of n coupled Rayleigh oscillators, from Eq. (3) and adding coupling terms that relate the velocities of the oscillators, we have:

$$\ddot{\theta}_i - \delta_i\left(1 - q_i\dot{\theta}_i^2\right)\dot{\theta}_i + \Omega_i^2\left(\theta_i - \theta_{io}\right) - \sum_{j=1}^{n} c_{i,j}\left(\dot{\theta}_i - \dot{\theta}_j\right) = 0 \qquad i = 1, 2, ..., n \tag{4}$$

which represents coupling between oscillators with the same frequency, where θ corresponds to the degrees of freedom of the system. In the case of coupling between oscillators with integer relation of frequency, the equation would be:

$$\ddot{\theta}_h - \delta_h\left(1 - q_h\dot{\theta}_h^2\right)\dot{\theta}_h + \Omega_h^2\left(\theta_h - \theta_{ho}\right) - \sum_{i=1}^{m} c_{h,i}\left[\dot{\theta}_i\left(\theta_i - \theta_{io}\right)\right] - \sum_{k=1}^{n} c_{h,k}\left(\dot{\theta}_h - \dot{\theta}_k\right) = 0 \tag{5}$$

where the nonlinear term $c_{h,i}[\dot{\theta}_i(\theta_i - \theta_{io})]$ is responsible for the coupling between oscillators with different frequencies, while the term $c_{h,k}(\dot{\theta}_h - \dot{\theta}_k)$, also seen in Eq. (4), makes coupling between oscillators with the same frequency. Both terms were defined by Dutra (1995).

4. Application of the coupled oscillators in the bipedal robot

Consider the model presented in Fig. 2. The angle of the hip θ_4 and the angles of the knees θ_3 and θ_5 will be determined by the system of coupled oscillators. The other angles are calculated by equations determined by the kinematical analysis of the mechanism. In this work we will not present details of this analysis, which can be seen in Pina Filho (2005). Besides the angles we have: x_t and y_t, which are the coordinates of the tip of the foot; ℓ_s is the length of the part of the foot responsible for the support (toes); ℓ_p is the length of the part of the foot that raises from the ground (sole); ℓ_t is the length of the tibia; and ℓ_f is the length of the femur.

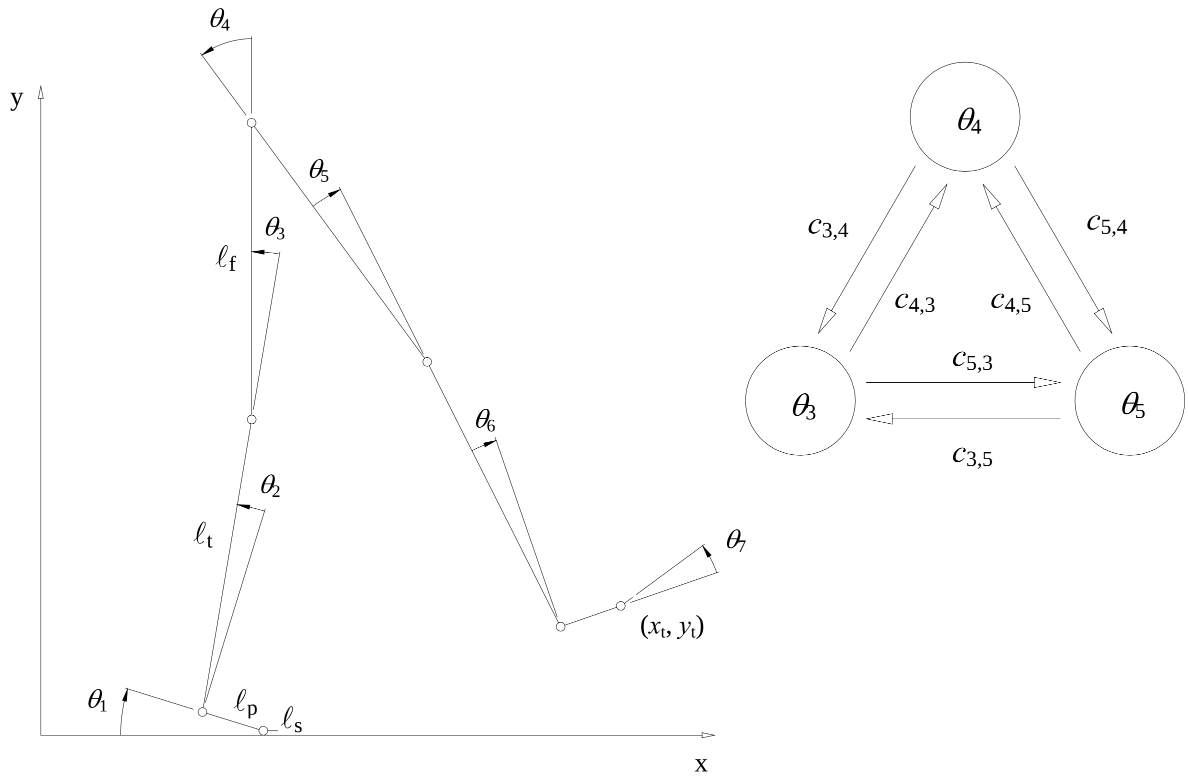


Figure 2. Model of the bipedal robot to be analyzed and structure of coupling between the oscillators.

Experimental studies of human locomotion (Braune and Fischer, 1987, and Raptopoulos, 2003) and of Fourier analysis of these data (Dutra, 1995) show that the movements of the angles θ_3, θ_4 and θ_5 can be described very precisely by their fundamental harmonic, with the biped in double support phase (two feet on the ground) or in single support phase (only one foot touching the ground).

A set of three coupled oscillators have been used to generate the angles θ_3, θ_4 and θ_5 as a periodic attractor of a nonlinear net. These oscillators are mutually coupled by terms that determine the influence of an oscillator on the other oscillators, as seen in the Fig. (2). With the reduction of the value of these coupling terms, the weaker is the relation between the oscillators.

Applying Eq. (4) and (5) to the proposed problem, knowing that the frequency of θ_3 and θ_5 (angles of the knees) is the double of θ_4 (angle of the hip), we have the following equations:

$$\ddot{\theta}_3 - \delta_3\left(1 - q_3\dot{\theta}_3^2\right)\dot{\theta}_3 + \Omega_3^2(\theta_3 - \theta_{3o}) - c_{3,4}[\dot{\theta}_4(\theta_4 - \theta_{4o})] - c_{3,5}\left(\dot{\theta}_3 - \dot{\theta}_5\right) = 0 \quad (6)$$

$$\ddot{\theta}_4 - \delta_4\left(1 - q_4\dot{\theta}_4^2\right)\dot{\theta}_4 + \Omega_4^2(\theta_4 - \theta_{4o}) - c_{4,3}[\dot{\theta}_3(\theta_3 - \theta_{3o})] - c_{4,5}[\dot{\theta}_5(\theta_5 - \theta_{5o})] = 0 \quad (7)$$

$$\ddot{\theta}_5 - \delta_5\left(1 - q_5\dot{\theta}_5^2\right)\dot{\theta}_5 + \Omega_5^2(\theta_5 - \theta_{5o}) - c_{5,4}[\dot{\theta}_4(\theta_4 - \theta_{4o})] - c_{5,3}\left(\dot{\theta}_5 - \dot{\theta}_3\right) = 0 \quad (8)$$

From Eq. (6)-(8), using the parameters shown in Tab. 1 together with values supplied by Pina Filho (2005), were been generated the graphs in MATLAB® shown in Fig. 3, which present, respectively, the behavior of the angles as a function of time and the stable limit cycles of the oscillators.

Table 1. Parameters of Rayleigh oscillators.

$c_{3,4}$	$c_{4,3}$	$c_{3,5}$	$c_{5,3}$	$c_{4,5}$	$c_{5,4}$	δ_3	δ_4	δ_5
0.001	0.001	0.1	0.1	0.001	0.001	0.01	0.1	0.01

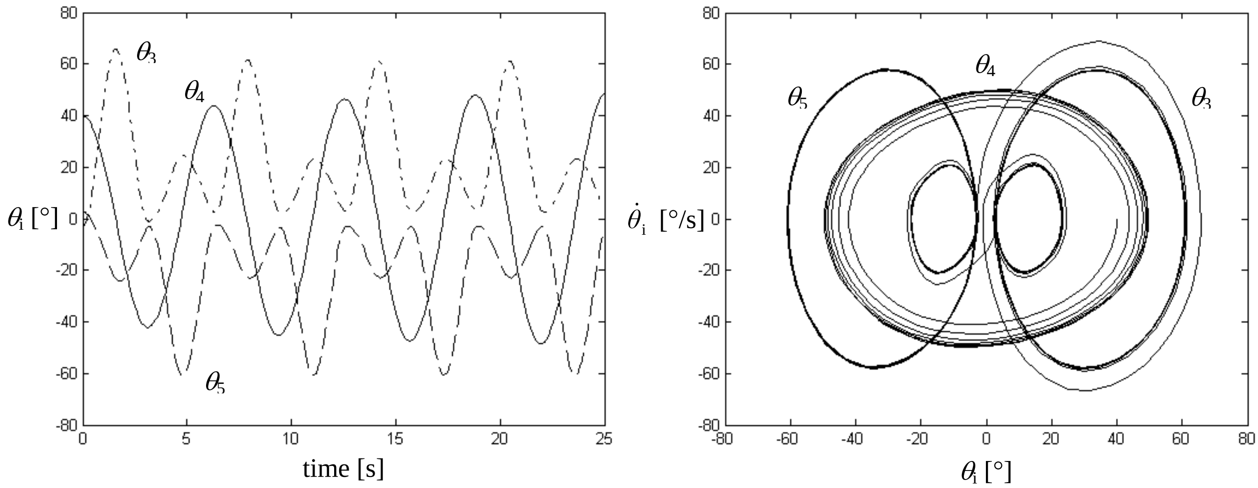


Figure 3. Graphs of θ_3, θ_4 and θ_5 as a function of time and trajectories in the phase space.

In Fig. 3 the great merit of the system can be observed, i.e. if an impact occurs and the angle of a joint is not the correct or desired one, it returns to the desired trajectory in a small number of periods. Considering, for example, a frequency equal to 1 s^{-1}, with the locomotor in repose with arbitrary initial values: $\theta_3 = -3°$, $\theta_4 = 40°$ and $\theta_5 = 3°$, after some cycles we have: $\theta_3 = 3°$, $\theta_4 = 50°$ and $\theta_5 = -3°$.

5. Nonlinear dynamical analysis of the coupled oscillators

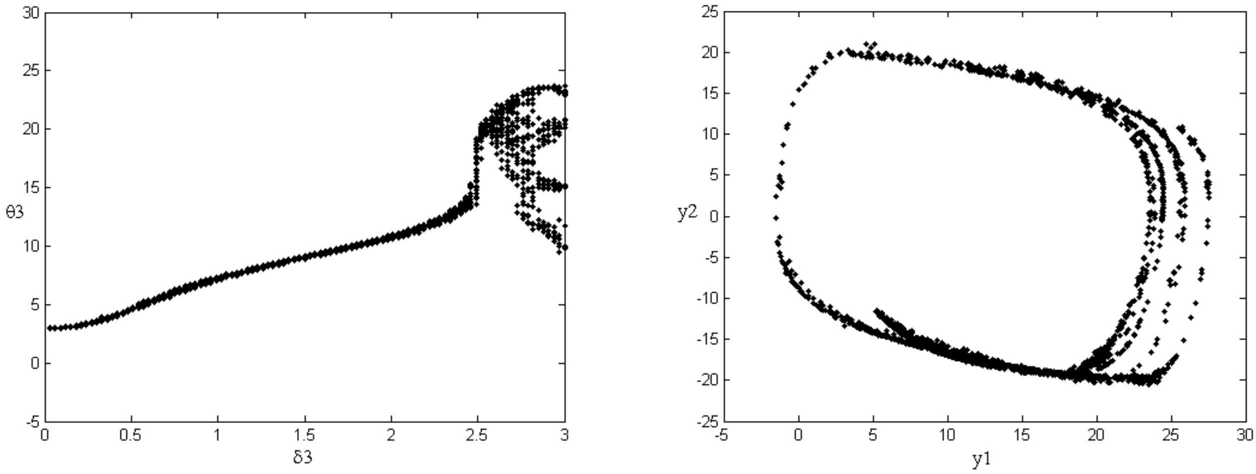
The nonlinear dynamical analysis of the system presented here requires the definition of some usual concepts. The first one of them is the definition of the characteristics of a chaotic system. Usually, for some values of parameters, the behavior of the system is periodic, and for other values the behavior is chaotic. According to Moon (1998), a periodic system is that one that returns to its state after a finite time *t*. The trajectory in the phase space is represented by a closed curve. The chaotic system presents trajectories of not-closed orbits that are generated by the solution of a deterministic set of ordinary differential equations..

Two conditions must be satisfied to make possible that a system presents chaotic behavior: the equations of motion must include a nonlinear term; and the system must have at least three independent dynamic variables. The main consequence associated with the chaos is the so-called “sensitivity to the initial conditions”. In chaotic systems, a small change in the initial conditions results in a drastic change in the behavior of the system. This phenomenon was first recognized by Henri Poincaré (1854-1912), in the end of century XIX. More details about the Chaos theory and its characteristics can be found in many works, such as Strogatz (1994) and Moon (1998).

5.1. Analysis and results

Considering different values for the parameters δ_3, δ_4 and δ_5, the tests have been performed using the MATLAB® to generate the bifurcation diagrams and Poincaré maps. In principle, keeping values of $\delta_4 = 0.1$ and $\delta_5 = 0.01$, the value of δ_3 was varied from 0 up to 3. All the other values of the system have been kept. Figure 4(a) presents the bifurcation diagram showing the behavior of the oscillator of the knee (θ_3) with variation of the parameter δ_3, which represents the damping term related with this oscillator. Observe that a chaotic regime is already configured when $\delta_3 > 3$.

Another interesting point of the analysis of chaos is the presence of the so-called “strange attractor”, which can be observed through the Poincaré map. In dissipative systems, the Poincaré map presents a set of points disposed in an organized way, with a preferential region in phase space that attracts the states of the dynamic system. Figure 4(b) presents the strange attractor generated in the analysis of the knee oscillator (θ_3).



(a) (b)

Figure 4. Bifurcation diagram for θ_3 with variation of δ_3, and strange attractor for θ_3.

Considering the coupling between the oscillators, the degree of influence of one on the others is defined by the coupling term. Then, a change of parameters of one oscillator must influence in the behavior of others.

Figure 5 presents the bifurcation diagram showing the behavior of the knee oscillator (θ_5) with variation of the parameter δ_3. The influence of the knee oscillator (θ_3) on the hip (θ_4) is small, since the behavior of θ_4 does not suffer many alterations. This result was already expected due to the small value of the coupling term between the oscillators ($c_{34} = c_{43} = 0.001$). In relation to the knees, the coupling term is greater ($c_{35} = c_{53} = 0.1$), configuring a more significant influence.

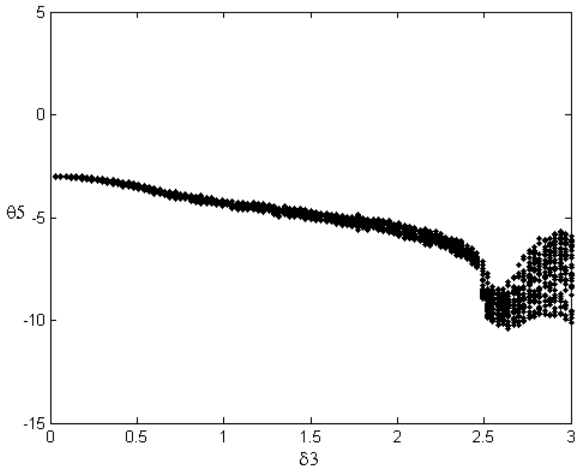


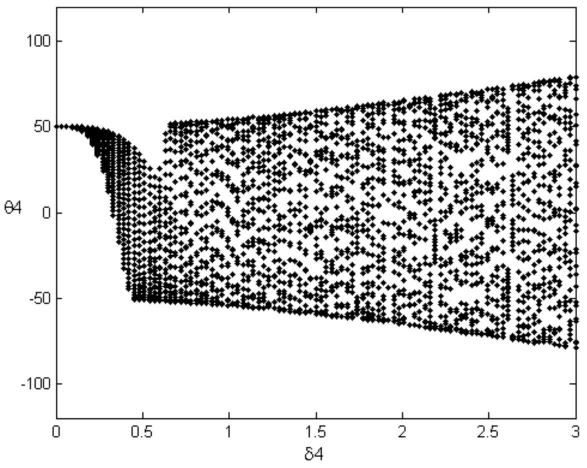
Figure 5. Bifurcation diagram for θ_5 with variation of δ_3.

In analogous way to what was made for δ_3, the response of the system can be analyzed by varying the values of δ_4 (from 0 up to 3) and keeping the other values fixed. Figure 6(a) presents the bifurcation diagram showing the behavior of the hip oscillator (θ_4) with variation of the parameter δ_4, which represents the damping term related with this oscillator. Figure 6(b) presents the strange attractor generated in the analysis of this oscillator.

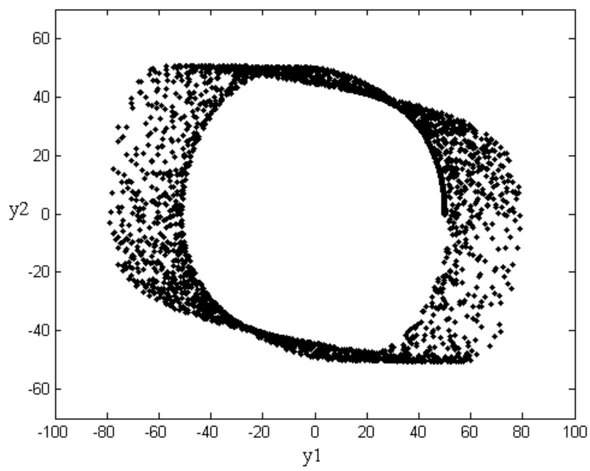
As seen previously in the analysis of δ_3, the influence of the hip on the knees is small, then a variation of δ_4 does not bring about great changes in θ_3 and θ_5.

Finally, the response of the system can be analyzed by varying the values of δ_5 (form 0 up to 3) and keeping the other system values fixed. Figure 7(a) presents the bifurcation diagram showing the behavior of the oscillator of the knee (θ_5) with variation of the parameter δ_5, which represents the damping term related with this oscillator. Figure 7(b) presents the strange attractor generated in the analysis of this oscillator.

Figure 8 presents the bifurcation diagram showing the behavior of the knee oscillator (θ_3) with variation of the parameter δ_5.

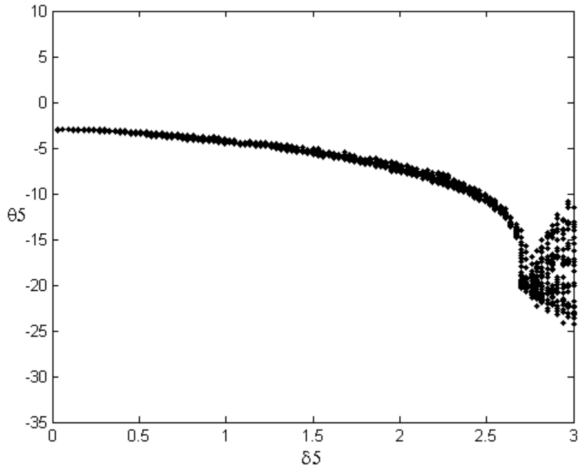


(a)

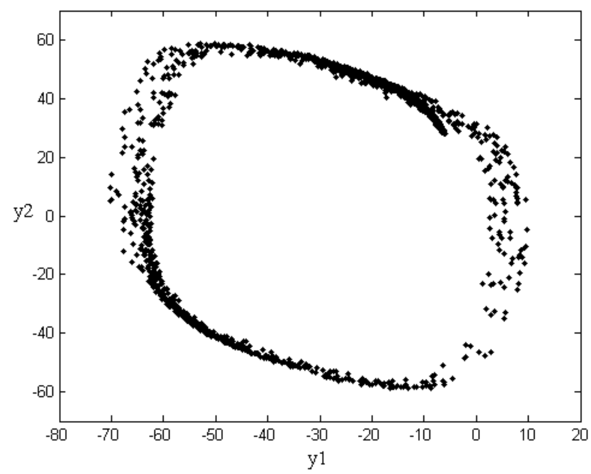


(b)

Figure 6. Bifurcation diagram for θ_4 with variation of δ_4, and strange attractor for θ_4.



(a)



(b)

Figure 7. Bifurcation diagram for θ_5 with variation of δ_5, and strange attractor for θ_5.

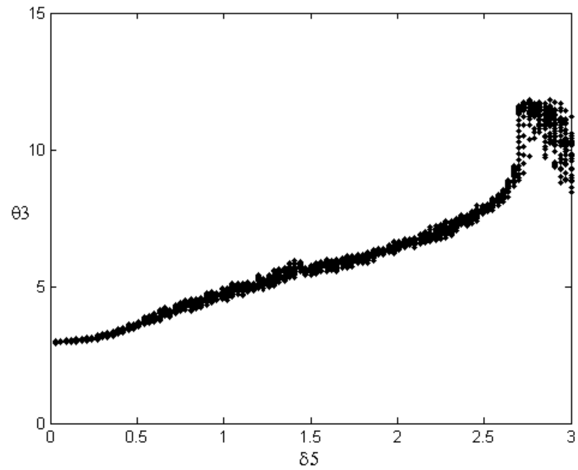


Figure 8. Bifurcation diagram for θ_3 with variation of δ_5.

6. Conclusion

From presented results and their analysis and discussion, we come to the following conclusions: the use of mutually coupled Rayleigh oscillators can represent an excellent way to generate locomotion pattern signals, allowing its application for the control of a bipedal robot by the synchronization and coordination of the legs, once the choice of parameters is correct, which must be made from the data supplied for the analysis of bifurcation and chaos. By means of the nonlinear dynamic analysis it was possible to evidence a weak point of coupling systems. The influence of the oscillators of the knees on the hip, and vice versa, is very small, what can harm the functionality of the system, i.e. if one of the knees suffers some disturbance, it will be automatically felt by the other knee, but it is possible that no reaction occurs in the hip. The solution for this problem seems immediate: to increase the value of the coupling term between the hip and the knees. However, a fast test proves that this can make the system unstable. Then, it is necessary a more refined study of the problem.

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