STUDY OF THE THERMAL STRESS IN CERAMIC MATERIALS DURING COOLING

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Abstract. *The ceramic materials play a relevant growing role in the development of industrial technology. Because of its physical properties, can be used in extreme conditions of temperature, however, its fragile nature makes the material highly susceptible to fissure flaws. In this work the hybrid numeric-analytical method denominated Generalized Integral Transformed Technique (GITT) is used with the Galerkin method to determine the thermal tensions in the material through the simultaneous solution of a transient heat transfer problem for the coupling conduction-radiation in participant medium.*

Keywords: *Participant medium, Thermal tensions, GITT, Ceramic materials, Coupling conduction-radiation***.**

1. Introduction

According to Johns (1965), most of the substances used in engineering expand when heated and contract during cooling. This dimension change is proportional to the temperature variation, and this proportionality is expressed by the linear thermal expansion coefficient of the material. If an expansion or contraction occurred freely for all the fibers of the body, the temperature alteration would not cause tensions. However, this modification does not occur in an uniform way; the elements that compose the body tend to expand or to contract in different quantities, establishing the thermal tensions, that are responsible for the emergence of cracks in brittle materials, as the ceramic ones. Therefore, this study is important in many aspects of an engineering project.

The analysis of these tensions and the resistance to thermal shock in ceramic materials, for a long time were just accomplished in a qualitative way. However, the industrial competitiveness made the interests turn to evaluations with a quantitative base. That change of focus occurred because analysis and experiments have been demonstrating that certain materials can present better quality indexes during one type of test but accomplish contrary results in other tests. The reasons for this behavior are related to the complexity of knowledge in the thermal field, specially when the body is submitted to high temperatures.

This work investigates the thermal tensions that appear in fragile materials, as ceramic, during the cooling after elevated temperatures were applied. In that way, a parameter of resistance to thermal shock can be established in order to support one definitive thermal gradient. This is accompished through a study of the heat transference for the determination of the transient internal temperature distribution, where the effect of radiation and conduction occur simultaneously.

Problems of heat transference for the coupling conduction-radiation have been investigated by several researchers; Lii and Özişik (1972), Tsai and Lin (1990), Lima et al (2002) and Leite (2004) have analyzed the unidimensional problem, considering the gray participant medium and the boundary surfaces kept in constant temperatures.

2. Problem Formulation

The physical problem consists of ceramic material, in the form of semitransparent infinite parallel plates that isotropically emits, absorbs and scatters the incident radiation. The body with initial temperature, T_i, is placed in an isothermal environment with uniform temperature, T_f.

The figure 1 shows a simplified form of the physical situation of the problem, the coordinate system, the heat transference terms that occur in each boundary surface and its interior, as well as the radiative properties in both external and internal surfaces of the body.

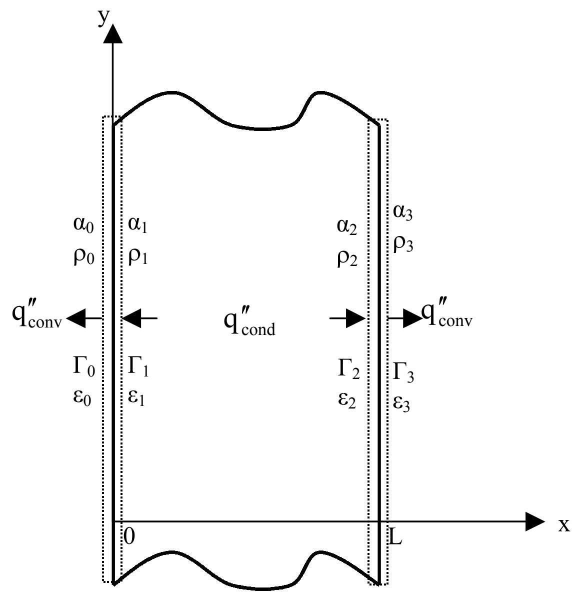


Figure 1: Geometric configuration of the problem.

The temperature distribution in the medium satisfies the energy equation below:

$$\hat{\rho}C_P \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[K(T)\frac{\partial T(x,t)}{\partial x}\right] - \frac{\partial q^r(x,t)}{\partial x}, \quad 0 < x < L, \quad t > 0 \tag{1}$$

and the divergent of the radiative heat flow is given by:

$$\frac{\partial q^r(x,t)}{\partial x} = (1-\omega)\beta\left[4n^2\overline{\sigma}T^4(x,t) - 2\pi\int_{-1}^{1} I(x,\mu)d\mu\right] \tag{2}$$

where $\hat{\rho}$ and C_P are the density and the specific heat for the medium, K(T) is the thermal conductivity of the body in function of the temperature, ϖ is the single scattering albedo, β is the extinction coefficient for the media (m^{-1}), n is the refraction index and σ is the Stefan-Boltzmann constant.

According to Siegel (1998) and Sadooghi (2005), for a semitransparent surface, radiative energy can be transferred from the surroundings to the interior of the layer so that:

$$K(T)\frac{\partial T(x,t)}{\partial x} - h_0\left[T(x,t) - T_f\right] = 0, \quad x = 0, \quad t > 0 \tag{3}$$

$$K(T)\frac{\partial T(x,t)}{\partial x} - h_3\left[T(x,t) - T_f\right] = 0, \quad x = L, \quad t > 0 \tag{4}$$

The initial condition for this problem is:

$$T(x,t) = T_i, \quad 0 \le x \le L, \quad t = 0 \tag{5}$$

3. Analysis

Dimensionless form:

The problem is rewritten in the dimensionless form by using the following dimensionless groups:

$$\tau = \beta y \qquad \xi = \frac{K_0 \beta^2 t}{\hat{\rho} C_P} \qquad \Theta(\tau,\xi) = \frac{T(x,t) - T_f}{T_i - T_f} \tag{6}$$

$$Q_r = \frac{q_r}{4n^2 \sigma T_i^4} \qquad N = \frac{K_0 \beta}{4n_i^2 \bar{\sigma} T_i^3} \qquad \varphi(\tau,\mu) = \frac{\pi I(\tau,\mu)}{n^2 \sigma T_i^4} \tag{7}$$

By making use of the groups above, the energy equation, the related boundary and initial conditions can be written as:

$$\frac{\partial \Theta(\tau,\xi)}{\partial \xi} = \frac{\partial}{\partial \tau}\left[\alpha^* \frac{\partial \Theta(\tau,\xi)}{\partial \tau}\right] - \frac{1}{N}\frac{\partial Q^R(\tau,\xi)}{\partial \tau}, \quad 0 < \tau < \tau_0, \quad \xi > 0 \tag{8}$$

$$K^* \frac{\partial \Theta(\tau,\xi)}{\partial \tau} - Bi_0 \cdot \Theta(\tau,\xi) = 0, \quad \tau = 0, \quad \xi > 0 \tag{9}$$

$$K^* \frac{\partial \Theta(\tau,\xi)}{\partial \tau} + Bi_3 \cdot \Theta(\tau,\xi) = 0, \quad \tau = \tau_0, \quad \xi > 0 \tag{10}$$

$$\Theta(\tau,\xi) = 1, \quad 0 \le \tau \le \tau_0, \quad \xi = 0 \tag{11}$$

where the divergent of the radiative heat flow in the dimensionless form is given by:

$$\frac{\partial Q^R(\tau,\xi)}{\partial \tau} = (1-\omega)\left\{\left[\Theta^*(\tau,\xi)\right]^4 - G^*(\tau)\right\} \tag{12}$$

and:

$$\Theta^*(\tau,\xi) = \Theta_{r1} \cdot \Theta(\tau,\xi) + \Theta_{r2} \qquad \Theta_{r1} = \frac{T_i - T_f}{T_i} \qquad \Theta_{r2} = \frac{T_f}{T_i} \tag{13}$$

Once the médium is considered homogeneous and isotropic, the form in which the conductivity and the diffusivity vary is thru a change in the body temperature. The type of material defines the behavior of these properties, being able to assume several models of mathematical equations. In this study, will be considered a linear variation proposed by Nishikawa et al (1995) in the following form:

$$K^*(\Theta) = 1 + A\Theta(\tau,\xi) \tag{14}$$

$$\alpha^*(\Theta) = 1 + B\Theta(\tau,\xi) \tag{15}$$

where A and B are the temperature constants.

The term $G^*(\tau) = \frac{1}{2}\int_{-1}^{1} \varphi(\tau,\mu')d\mu'$ that appears in equation (12) is the dimensionless incident radiation which is solved by the Garlekin method and $\varphi(\tau,\mu')$ must satisfy the dimensionless radiative transfer equation below, considering the isotropic scattering of the radiation.

$$\mu \frac{\partial \varphi(\tau,\mu)}{\partial \tau} + \varphi(\tau,\mu) = (1-\varpi)\theta^4(\tau,\xi) + \frac{\varpi}{2}\int_{-1}^{1} \varphi(\tau,\mu')d\mu', \quad 0 < \tau < \tau_0, \quad -1 \le \mu \le 1 \tag{16}$$

$$\varphi^+(0,\mu) = \varepsilon_1 \theta_1^4 + 2\rho_1 \int_0^1 \varphi^-(0,-\mu')\mu' d\mu', \quad \mu > 0 \tag{17}$$

$$\varphi^-(\tau_0,-\mu) = \varepsilon_2 \theta_2^4 + 2\rho_2 \int_0^1 \varphi^+(\tau_0,\mu')\mu' d\mu', \quad \mu > 0 \tag{18}$$

where μ is the cosine of the angle formed between the positive axis of y and the direction of the intensity of the radiation ρ_i and ε_i (i=1,2) represents, respectively, the reflectivity and the emissivity of the boundary surface.

The energy and radiative transfer equations are coupled through the high temperature term to the fourth potency and the determination of the temperature distribution and other greatnesses of interest in engineering depend on the solution of these two equations. The resolution procedure accomplished in this work allows, simultaneously, the determination of the thermal field, the radiative heat flow, the incident radiation and the radiative intensity in any point of the medium.

Following the formalism in the generalized integral transform technique, we now select the corresponding auxiliary eigenvalue problems. The present choice corresponds to the classical Sturm-Liouville problem below:

$$\frac{\partial^2 \psi_i(\tau,\gamma_i)}{\partial \tau^2} + \gamma_i^2 \psi_i(\tau,\gamma_i) = 0 , \qquad \text{in } 0 < \tau < \tau_0 \tag{19}$$

with boundary conditions:

$$\frac{\partial \Psi_i(\tau,\gamma_i)}{\partial \tau} - Bi_0 \psi_i(\tau,\gamma_i) = 0 , \qquad \text{at } \tau = 0 \tag{20}$$

$$\frac{\partial \Psi_i(\tau,\gamma_i)}{\partial \tau} + Bi_3 \psi_i(\tau,\gamma_i) = 0 , \qquad \text{at } \tau = \tau_0 \tag{21}$$

The problem described by equations (19-21) was solved by the classic method of separation of variables, where the obtained eigenfunctions are written in the following way:

$$\psi_i(\tau,\gamma_i) = cos(\gamma_i \tau) + \frac{Bi_{ef0}}{\gamma_i} \cdot sin(\gamma_i \tau) \tag{22}$$

satisfying the following ortogonality property:

$$\int_0^{\tau_0} \psi_i(\tau)\psi_j(\tau) d\tau = \begin{cases} 0, & if\ i \neq j \\ N_i(\gamma_i), & if\ i = j \end{cases} \tag{23}$$

The eigenvalues γ_i that appear in the equation (22) and the norms, $N_i(\gamma_i)$, of the eigenfunctions are given respectively as:

$$tg(\gamma_i \tau_0) = \frac{\gamma_i (Bi_0 + Bi_3)}{\gamma_i^2 - Bi_0 \cdot Bi_3} \qquad N_i(\gamma_i) = \frac{1}{2}\left\{ \frac{Bi_0^2}{\gamma_i^2}\left[\tau_0 + \frac{Bi_3}{\gamma_i^2 + Bi_3^2} \right] + \frac{Bi_0}{\gamma_i^2} \right\} \tag{24}$$

Then, through the associated orthogonal property of the eigenfunctions, definition of the integral transform pairs is given as below:

$$\Theta(\tau,\xi) = \sum_{i=1}^{Nc} \frac{\psi_i(\tau,\gamma_i)\overline{\Theta}_i(\xi)}{N_i^{1/2}} \qquad \text{Inversion Formula} \tag{25}$$

$$\overline{\Theta}_i(\xi) = \frac{1}{N_i^{1/2}} \int_0^{\tau_0} \psi_i(\tau,\gamma_i)\Theta(\tau,\xi) d\tau \qquad \text{Integral Transform} \tag{26}$$

The integral transformation of (8) is now accomplished by applying the operator $\frac{1}{N_i^{1/2}} \int_0^{\tau_0} \psi_i(\tau) d\tau$ to yield, after using boundary conditions (9 – 10) and (20 – 21):

$$\frac{d\overline{\Theta}_i(\xi)}{d\xi} = H_i - \sum_{j=1}^{\infty} \left[A_{ij} + B \sum_{k=1}^{\infty} W_{ijk} \overline{\Theta}_k(\xi) \right] \overline{\Theta}_j(\xi) - \frac{1}{N} \overline{G}_i [\overline{\Theta}_i(\xi)] \tag{27}$$

Where $\overline{G}_i[\overline{\Theta}_i(\xi)]$ represents the integral transform of the radiative heat flow divergent, given as:

$$\overline{G}_i[\overline{\Theta}_i(\xi)] = \frac{(1-\omega)}{N_i^{1/2}}\int_0^{\tau_0}\psi_i(\tau,\gamma_i)\left\{\left[\Theta^*(\tau,\xi)\right]^4 - G^*(\tau)\right\}d\tau \tag{28}$$

and:

$$H_i = -\frac{Bi_{ef3}\psi_i(\tau_0)}{N_i^{1/2}}\frac{\alpha^*(\tau_0)}{K^*(\tau_0)}\Theta(\tau_0,\xi) - \frac{Bi_{ef0}\psi_i(0)}{N_i^{1/2}}\frac{\alpha^*(0)}{K^*(0)}\Theta(0,\xi) \tag{29}$$

$$A_{ij} = -\frac{Bi_3\psi_i(\tau_0)\psi_j(\tau_0)}{(N_iN_j)^{1/2}} - \frac{Bi_0\psi_i(0)\psi_j(0)}{(N_iN_j)^{1/2}} + \gamma_i^2\delta_{ij} \tag{30}$$

$$W_{ijk} = \frac{1}{(N_iN_jN_k)^{1/2}}\int_0^{\tau_0}\frac{\partial\psi_i(\tau,\gamma_i)}{\partial\tau}\frac{\partial\psi_j(\tau,\gamma_j)}{\partial\tau}\psi_k(\tau,\gamma_k)d\tau \tag{31}$$

The ordinary differential equation system, given by equation (31) must satisfy the following transformed inlet condition:

$$\overline{\Theta}_i(0) = \frac{1}{\gamma_i^2 N_i^{1/2}}\left[Bi_3\psi_i(\tau_0,\gamma_i) + Bi_0\psi_i(0,\gamma_i)\right] \tag{32}$$

The incident radiation, $G(\tau)$, that appears in equation (28) is obtained by the Garlekin method, and equals:

$$\begin{aligned} G^*(\tau) = \frac{1}{2}\Bigg\{&\left[\varepsilon_1\theta_1^4 + 2\rho_1K_1\right]E_2(\tau) + \left[\varepsilon_2\theta_2^4 + 2\rho_2K_2\right]E_2(\tau_0-\tau) + (1-\omega)\int_0^{\tau_0}\theta^4(\tau',\xi)E_1(|\tau-\tau'|)d\tau' + \\ &\omega\sum_{m=0}^{M}c_m\left\{(-1)^{m+1}m!\,E_{m+2}(\tau) + \sum_{j=0}^{m}\frac{m!\left[1+(-1)^j\right]}{(m-j)!(j+1)}\tau^{m-j} - \sum_{j=0}^{M}\frac{m!}{(m-j)!}(\tau_0)^{m-j}E_{j+2}(\tau_0-\tau)\right\}\Bigg\} \end{aligned} \tag{33}$$

where c_m are the expansion coefficients for the representation of the incident radiation, obtained by the Garlekin method:

$$K_1 = \beta^*\left\{\left[\alpha_2\varepsilon_1\theta_1^4 + \varepsilon_2\theta_2^4\right]E_3(\tau_0) + (1-\omega)\int_0^{\tau_0}\theta^4(\tau',\xi)\left[E_2(\tau') + \alpha_2E_2(\tau_0-\tau')\right]d\tau' + \omega\sum_{m=0}^{M}c_m\left(T_m + \alpha_2T_m^*\right)\right\} \tag{34}$$

$$K_2 = \beta^*\left\{\left[\alpha_1\varepsilon_2\theta_2^4 + \varepsilon_1\theta_1^4\right]E_3(\tau_0) + (1-\omega)\int_0^{\tau_0}\theta^4(\tau',\xi)\left[\alpha_1E_2(\tau') + E_2(\tau_0-\tau')\right]d\tau' + \omega\sum_{m=0}^{M}c_m\left(\alpha_1T_m + T_m^*\right)\right\} \tag{35}$$

The terms T_m e T_m^* are presented by Cengel (1984):

$$\alpha_1 = 2\rho_1E_3(\tau_0) \qquad \alpha_2 = 2\rho_2E_3(\tau_0) \qquad \beta^* = \frac{1}{1-\alpha_1\alpha_2} \tag{36}$$

and $E_n(z)$ is the integral of exponential functions of the type:

$$E_n(z) = \int_0^1 \eta^{n-2}e^{-z/\eta}d\eta \tag{37}$$

Once determined the temperature distribution of the body, the thermal tensions of the body are (Timoshenko e Goodier, 1970):

$$\zeta^* = \frac{\zeta}{\lambda E(T_i - T_f)} = \frac{R_{ST}}{(T_i - T_f)} = \frac{1}{\tau_0}\int_0^{\tau_0}\Theta(\tau,\xi)d\tau - \Theta(\tau,\xi) \tag{38}$$

where λ is the linear thermal expansion coefficient(K^{-1}), ζ^* is the dimensionless thermal tension, E is the modulus of the material elasticity (N/m^2) and R_{ST} is the thermal shock resistance parameter.

4 Results and Discussion

The resulting ordinary differential equation system, submitted to the initial transformed condition was solved by means of computational code written in FORTRAN programming language, using the Fortran Powerstation 4.0 software and implemented in a personal microprocessor Pentium III-450Mhz. The results are shown through tables and graphs, where the analysis is made taking in consideration the obtained quality in terms of the convergence and stability of the solution and computational cost.

Tables 1 and 2 show to the convergence of the results taking in consideration the number of eigenvalues (N_C) that define the truncation order of the eigenfunction series and the number of terms (M) of the power series for representation of the incident radiation. Moreover, the analysis takes in account the thermal dependence of the conductivity and the diffusity of the material.

The analysis of the convergence was accomplished considering the strong influence of radiation in the internal process of heat transference (N=0.001), so that the non linearity of the energy equation becomes more significant. Thus, the values of N_C and M used to obtain the convergence of the solution for this problem can be used to obtain the solution in cases where the value of the parameter conduction-radiation is greater. The properties and parameters used in the determination of the distribution of the thermal tensions in the body are specified in each table.

On the basis of the results, the convergence of the solution is established for the following values: N_C=110, M=7.

Table 1 Convergence of the dimensionless thermal tension (ζ^*) for several eigenvalues (N_C) and Temperature constants, A=3, B=5.

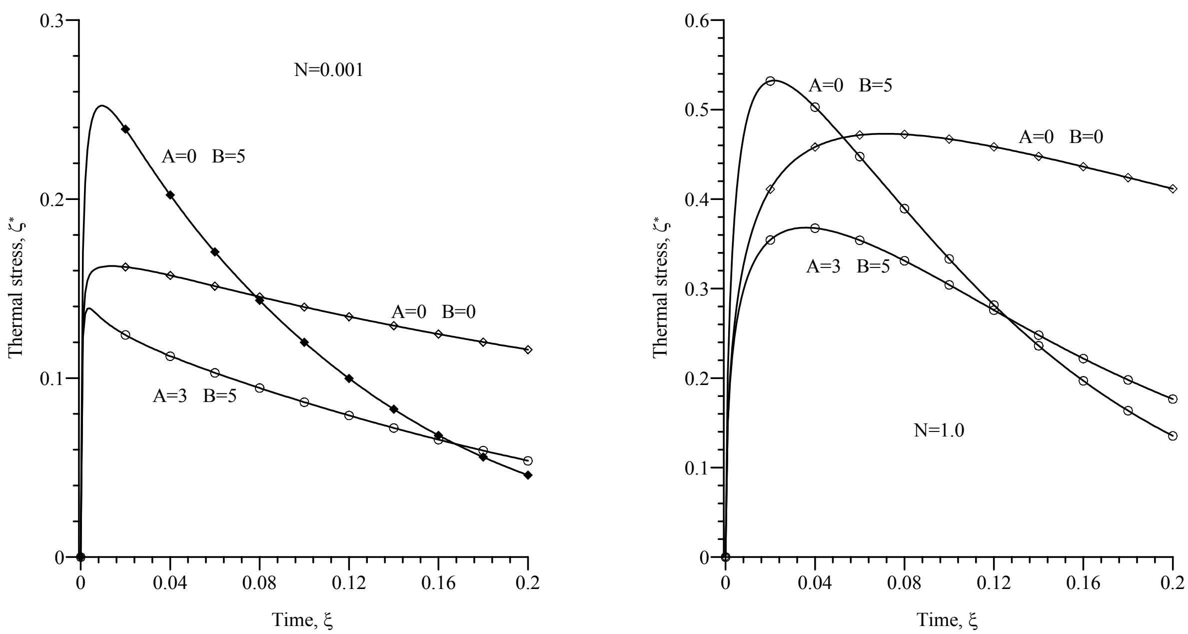
Dimensionless thermal tension - ζ^* N=0.001 ω=0.5 Bi_{ef0}=0.0 Bi_{ef3}=5.0 T_f=0ºC T_i=2000ºC $\rho_1=\rho_2$=0.0 $\varepsilon_1=\varepsilon_2$=1.0									
ξ	**M=6**								
	N_C=20	**N_C=30**	**N_C=40**	**N_C=50**	**N_C=60**	**N_C=70**	**N_C=90**	**N_C=100**	**N_C=110**
0.001	0.1417	0.1349	0.1317	0.1299	0.1286	0.1281	0.1270	0.1266	0.1263
0.020	0.1406	0.1344	0.1313	0.1294	0.1281	0.1271	0.1259	0.1254	0.1251
0.040	0.1250	0.1201	0.1178	0.1163	0.1153	0.1145	0.1136	0.1132	0.1128
0.060	0.1121	0.1093	0.1073	0.1061	0.1052	0.1046	0.1038	0.1035	0.1033
0.080	0.1028	0.0997	0.0980	0.0970	0.0963	0.0958	0.0951	0.0949	0.0947
0.100	0.0935	0.0909	0.0895	0.0887	0.0881	0.0877	0.0871	0.0869	0.0867
0.120	0.0849	0.0827	0.0815	0.0809	0.0804	0.0801	0.0795	0.0794	0.0793
0.140	0.0769	0.0751	0.0741	0.0736	0.0732	0.0729	0.0725	0.0723	0.0723
0.160	0.0695	0.0680	0.0672	0.0668	0.0664	0.0662	0.0659	0.0658	0.0657
0.180	0.0626	0.0615	0.0607	0.0605	0.0602	0.0600	0.0597	0.0596	0.0596
0.200	0.0564	0.0554	0.0549	0.0546	0.0544	0.0543	0.0540	0.0540	0.0539
Computational Time (s)	0448.03	0741.93	1145.03	1708.34	2578.37	3247.91	5981.88	8023.24	10527.29

Figure 2 shows the influence of the conduction-radiation parameter in the thermal tensions that appear in a ceramic body during cooling, for the case of constant and function of the temperature thermal properties. For the simulation, was adopted a body with optical thickness τ_0=1.0, single scattering albedi ω=0.5 and black boundary surfaces with heat transference taxes of respectively Bi_0=0 in τ=0 (isolated surface) and Bi_3=5 in $\tau=\tau_0$. It was also considered that the T_f temperature was kept to zero and the initial temperature of the body was T_i=1200K.

It can be observed through the obtained results that, for small values of N ("a" case), the thermal tensions that occur during cooling - with the indicated Biot numbers – decrease , and the maximum value occurs in a smaller dimensionless time, when compared to the cases where the conduction becomes predominant. The reason for the maximum tension value to occur in a shorter amount of time is the speed in which thermal radiation propagates through the body.

Table 2: Convergence of the dimensionless thermal tension (ζ^*) for several number of terms (N_C) and temperature constants, A=3, B=5.

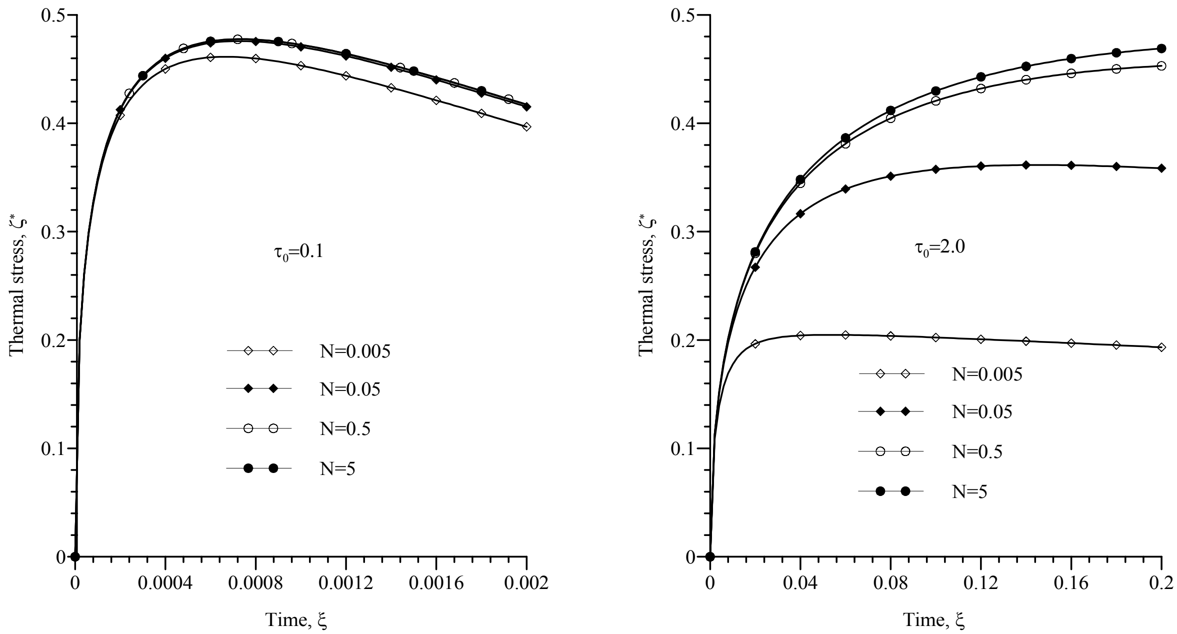
Dimensionless thermal tension - ζ^* N=0.001 ω=0.5 Bi_{ef0}=0.0 Bi_{ef3}=5.0 T_f=0K T_i=2000K $\rho_1=\rho_2$=0.0 $\varepsilon_1=\varepsilon_2$=1.0					
ξ	**N_C=10**				
	N_T=4	**N_T=5**	**N_T=6**	**N_T=7**	**N_T=8**
0.001	0.1693	0.1695	0.1697	0.1696	0.1696
0.020	0.1575	0.1574	0.1574	0.1574	0.1574
0.040	0.1377	0.1377	0.1377	0.1377	0.1377
0.060	0.1233	0.1233	0.1233	0.1233	0.1233
0.080	0.1110	0.1110	0.1110	0.1110	0.1110
0.100	0.1002	0.1002	0.1002	0.1002	0.1002
0.120	0.0903	0.0903	0.0903	0.0903	0.0903
0.140	0.0812	0.0812	0.0812	0.0812	0.0812
0.160	0.0729	0.0729	0.0729	0.0729	0.0729
0.180	0.0653	0.0653	0.0653	0.0653	0.0653
0.200	0.0584	0.0584	0.0584	0.0584	0.0584
Computational Time (s)	091.72	157.81	253.53	389.69	590.94



(a) (b)

Figure 2: Effect of the conduction-radiation parameter in the thermal tensions of the body, with thermal properties varying during cooling. (a) N=0.001, (b) N=1.

Figures 3 shows the influence of the material's optical thickness in the thermal tensions of a body with constant thermal properties, for the following conduction-radiation parameters: N=0.005, 0,05, 0,5 and N=5. In this simulation it was considered that single scattering albedo was ω=0.5, the T_f temperature was kept zero and the initial temperature T_i=1200K. Moreover, the Biot numbers in the boundary surfaces, considered black, for $\tau=0$ and $\tau=\tau_0$ are given respectively by Bi_0=0 and Bi_3=5. The values of the optical thicknesses utilizied were τ_0=0.1 and τ_0=2.0. It must be observed in figure (a) that, for optically thin bodies ($\tau_0 \leq 0.1$), the values of thermal tensions that appear in the material are greater and occur in very small dimensionless times when compared to bodies with greater optical thickness. It can also be observed in this in case that, the obtained tension curves suffer little influence of the conduction-radiation parameter. Thus, the contribution of the radiation in the energy equation can be neglected without affecting significantly the precision of the results, even for very small values N. On the other hand, as the optical thickness increases, the radiating energy affects the temperature distribution in the body and, consequently, the thermal gradients as it can be observed in figure (b).



(a) (b)

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