VIRTUAL PROTOTYPING: A VEHICLE MODEL FOR INTEGRATED MOTION CONTROL STUDIES

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Abstract. Future applications of control in automotive vehicles will follow a trend towards system integration, leading ultimately to the development of integrated vehicle control systems capable of coordinating the action of the various subsystems. The coordination and integration of automotive vehicle subsystems require the interaction amongst the various subsystems to be taken into consideration at the control design stages, resulting in full vehicle models. Therefore, a nonlinear 10 degree of freedom model is obtained through MBS modeling techniques present in ADAMS package software. Then, a linear model is obtained by linearization of the system equations through the Jacobian facility also present in ADAMS. The resulting linearized models are simulated and their response are compared to the previous non-linear one in order to validate the linear approximations. This work also presents a suspension control system based in optimal control theory: a controller designed at ADAMS (with the non-linear vehicle model) and a LQR (Linear Quadratic Regulator) with output feedback based on the state space linear vehicle model representation. This was designed through ADAMS/Simulink co-simulation facilities. This designed suspension control is a first attempt to future developments of integrated vehicle control systems .

Keywords: *MBS modeling, linear/non-linear full vehicle model, LQR with output feedback, simulation, integrated motion control.*

1. Introduction

Seeking for safety, stability, comfort, and driveability, the automotive industry has been developing systems that control the vehicle motion. The present electronic and computational developing stage made great improvements in the vehicle modeling and simulating field, with the use of virtual prototypes. Difficult math manipulation situations were made easier and complex model systems could be generated and analyzed in a more accurate way. As a result, it was possible to generate models that are able to represent the interaction amongst the various vehicle subsystem dynamics (i.e., traction, suspension, brakes and steering systems), being of great importance to the project and development of systems capable of coordinating and controlling the overall vehicle dynamics.

As time went by, the volume of control systems increased and the present tendency is the implementation of a central system that manages all the vehicle self-controlled subsystems (Wallentowitz & Roppenecker, 1993; Zanten et al., 1996). This tendency is a world-wide perspective that increases the market dispute by the industries, and promotes more satisfaction and practicality to driving.

In this sense, this work presents the development of math models capable of representing the vehicle and its main subsystems in a way that it is possible to use them in projects of integrated motion control systems. Many aspects of automotive vehicle math modeling are discussed, with emphasis on the computational modeling techniques of multibody systems and LQR control techniques. The vehicle model is implemented with the usage of the generic modeling present in ADAMS package software. This model is developed in such a way to incorporate the main characteristics of suspension, steering-wheel and pneumatics systems, but the braking and traction systems are mentioned in a simplified way. The obtained model is complex enough to describe the complete vehicle movement and yet is simple enough to be used in a project of movement integrated control system.

To validate the obtained model, many simulations were made in which the main aspects of longitudinal, lateral and vertical dynamics were analyzed observing the behavior of the model which represents the vehicle in different situations according to driver's commands (steering wheel turning and acceleration), to external disturbances (road irregularities and aerodynamics disturbances), as well as the used tire model influences . The complete description of the model and simulations can be read in Barbieri (2002).

The obtained models of the vehicle are used in the design of suspension control systems based on optimal control techniques, which has two different outlooks: First, a skyhook controller was designed directly at ADAMS (Barbieri, 2002) with the non-linear vehicle model. Then, another strategy was adopted using a suspension controller based on a LQR (Linear Quadratic Regulator) with output feedback, which utilizes linear state-space representation of the same model, obtained directly from ADAMS facilities through linearization of the full model in various operation conditions. It was designed through ADAMS/Simulink co-simulation facilities which use simultaneously non-linear and linear models for the project of the output feedback suspension controller. This last approach is the work shown in this paper. In this way, we could obtain good results from a non-linear system using controllers based on linear models. The possibility of this verification is a powerful tool in terms of virtual prototyping in which the simulation results are much closer to the actual system. In addiction, data post-processing and animation tools improved significantly the result visualization. However, though this approach is able to design better controllers using more complete models, the complexity of the control system project, in computational terms, is also considerably increased by the quantity of utilized information and by the size of the generated files during computation.

The used vehicle models and the controller design are presented in sequence.

2. Vehicle modeling

Apart from aerodynamic resistances and gravitational effects, all external forces acting on a vehicle are applied through the tires. Consequently, motion management will entail the application of control action at the wheels, and will be based on a combination of propulsion, steering and suspension control. Therefore, it is necessary to develop vehicle models which are capable of representing all these actions simultaneously.

The application of a MBS modeling approach to vehicle dynamics involves two essential considerations: the way in which the MBS model is formulated and the particular MBS program which is to be employed. The MBS model formulation determines how the equations of motion are derived. The MBS program determines the ultimate form of the dynamic equations, as implemented within the computer simulation program. There are different MBS programs based on Kane´s or Lagrange´s methods, for example. In this work, we have utilized the ADAMS software, based on Lagrange formulation of the equations of motion.

MBS vehicle models usually include a complete representation of the suspension, in which individual suspension elements are detailed. The modeling of this work is based on an alternative approach, in which suspension geometry effects are incorporated in a way that does not involve a detailed representation of the suspension system. The basis of the approach is an empirical black-box representation data describing the suspension trajectory (fore-aft and lateral displacement as a function of vertical displacement) at each wheel hub. From these data, an equivalent swing axle, or trailing arm model is derived using techniques from differential geometry. Other quantities, such as camber and toe angles are considered in an approximate fashion because the wheel hub is rigidly attached to the swing axle. However, from a control point of view and for the initial purposes of this work, the present representation has proved adequate (Costa, 1992; Hiller et al, 1991). The swing axle model provides the simplest possible representation of the suspension system which is consistent with a non-linear chassis model intended for motion control studies.

A schematic view of the non-linear chassis model is presented in Figure 1. The model represents the kinematics of the sprung mass and the four wheels, and also incorporates the effects of geometrical constraints associated with the suspension. At each wheel, the suspension geometry effects are represented as a swing axle connecting to a wheel hub H to the sprung mass via a single degree of freedom rotational joint (pin joint) at the points P. The pin position and the arm length of the swing axle are derived empirically from data describing the suspension trajectory at each wheel hub. The vectors describing the positions and orientations are given in the Tab. 1. The first of the three vectors **op** describes the position of the pin joint relative to the centre of gravity of the base body (sprung mass) and this vector is fixed in the base body. The second vector **hp** describes the position of the pin joint relative to the centre of gravity of the unsprung mass and is fixed in it. The third vector **b** describes the orientation of the pin joint axis.

Table 1.Geometrical description of MBS chassis model.

		Left	Right
Front	**op**	(1.441, 0.303, -0.242)	(1.441, -0.303, -0.242)
	hp	(0.121, -0.447, 0.058)	(0.121, 0.447, 0.058)
	b	(0.966, 0.257, -0.029)	(0.966, -0.257, -0.029)
Rear	**op**	(-1.516, 0.105, -0.157)	(-1.516, -0.105, -0.157)
	hp	(-0.016, -0.645, 0.143)	(-0.016, 0.645, 0.143)
	b	(0.994, 0.0, 0.113)	(0.994, 0.0, 0.113)

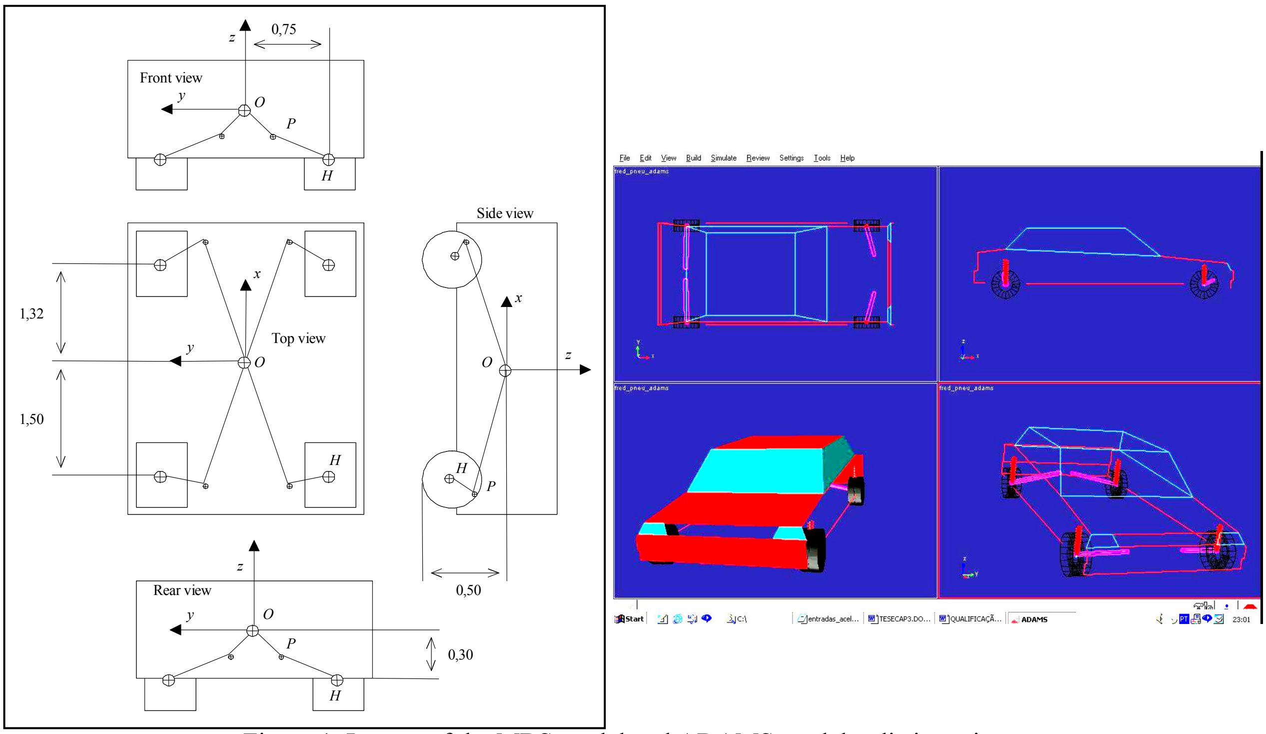


Figure 1. Layout of the MBS model and ADAMS model – distinct views

The suspension forces at each wheel are represented as a spring and damper which are active in the vertical direction (the suspensions are represented in red color in the figure above). In addition, the front suspension forces are supplemented by a further force to represent the effects of an anti-roll torsion bar. Also, to represent the effects of a suspension actuator, another force element parallel to vehicle passive suspension is added. An equivalent lumped mass, concentrated at point *H*, represents the combined wheel and suspension inertia effects for each wheel. The central or base body corresponds to the sprung mass.

Two types of tire models are used. First, a simplified tire model is used in which vertical tire forces are represented at each wheel as a linear stiffness term. Longitudinal and lateral tire forces are also represented linearly, and it is assumed that the wheel does not spin or lock. Then, a non-linear tire model (Fiala tire model) was used where tire spin or lock is made possible. This new model, with a more complete tire model, will make integrated motion control management studies possible, including suspension and brake/traction control systems (ABS/ASR) in future works.

The model also includes propulsion (rear drive), braking and aerodynamic forces acting on the vehicle so that it is possible to access, for example, the dynamic effects of steering and propulsion maneuvers on suspension control. Propulsion and brake forces are applied at the wheels while the aerodynamic forces are applied at the sprung mass, in all directions.

The simulation model is constructed using the MBS package ADAMS. The obtained model has 10 degrees of freedom, six for the chassis and one for each suspension arm.

2.1. Linearized model

For the purposes of this work, the model as it has been presented has to be linearized in state space form so the linear control based on LQR output feedback can be utilized. The non-linear model has been linearized in several operation conditions and the linearized models were exercised by various types of simulations as can be seen in Barbieri (2002). In this work, it is utilized a reference configuration in steady state condition corresponding to a constant vehicle forward velocity of 20 m/s when the vehicle is traveling in a straight line. This configuration results in a plant matrix with 20 states as shown later. The linearizing procedures adopted by the software ADAMS are described in Sohoni & Whitesell (1986) and are briefly discussed in sequence. The obtained equations for any model in ADAMS are based on Lagrange´s method, resulting in:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}-\left[\frac{\partial \Phi}{\partial q}\right]^T \lambda = F \qquad (1)$$

$$\Phi(q,t)=0$$

Where T is the system kinetic energy, q are the generalized coordinates of the systems, Φ are the constraints set of the system, λ are the reaction forces of the constraints and F are the generalized forces acting on the system.

If the kinetic energy are described in terms of generalized coordinates q and their derivatives, we have a system with algebraic and differential equations of second order resulting in a vector of implicit equations in the form $G=0$, where $G(q , \dot{q} , \ddot{q} , \lambda , f, t) = 0$

These equations may be transformed in first order equations:

$$Y = \begin{bmatrix} q \\ \dot{q} \\ \lambda \end{bmatrix} \text{, resulting in } G = (Y, \dot{Y}, f, t) = 0 \qquad (2)$$

the equations in the form (2) can be linearized in an operation point $Y^* = (Y_0 , \dot{Y}_0 , t)$, resulting in:

$$\left[\frac{\partial G}{\partial Y}\right]_{Y^*} \delta Y + \left[\frac{\partial G}{\partial \dot{Y}}\right]_{Y^*} \delta \dot{Y} = 0 \qquad (3)$$

where δY e $\delta \dot{Y}$ are small perturbations around the operation point Y^*. The two matrices of the above equation constitute the Jacobian that is obtained for the linearized model, and represents the plant matrix of the system:

$$[J] = \left[\frac{\partial G}{\partial q} + \beta \frac{\partial G}{\partial \dot{q}}\right] = \mathbf{A} \qquad (4)$$

The resulting **A** matrix is of dimension 20x20 and the state variables are automatically chosen by ADAMS. The tab.2 shows the state variables of the linearized model. Note that q_n are the generalized coordinates and u_n are the derivatives of q_n, corresponding to the generalized velocities:

Table 2. State variables of the linearized model

x1=u1: roll angular velocity of chassis (dφ/dt);	*x11=u6*: vertical velocity (dz/dt) of rear left susp.;
x2=q1: roll angular displacement of chassis (φ)	*x12=q6*: vertical displacement (z) of rear left susp;
x3=u2: lateral velocity (dy/dt) of front left susp.;	*x13=u7*: angular velocity of rear left susp.;
x4=q2: lateral displacement (y) of front left susp.;	*x14=q7*: angular displacement of rear left susp.;
x5=u3: vertical velocity (dz/dt) of front left susp.;	*x15=u8*: longitudinal velocity (dx/dt) of rear right susp.;
x6=q3: vertical displacement (z) of front left susp;	*x16=q8:* longitudinal displacement (x) of rear right susp.;
x7=u4: angular velocity of front left susp.;	*x17=u9*: lateral velocity (dy/dt) of rear right susp.;
x8=q4: angular displacement of front left susp.;	*x18=q9*: lateral displacement (y) of rear right susp.;
x9=u5: angular velocity of front right susp.;	*x19=u10*:angular velocity of rear right susp.;
x10=q5 angular displacement of front right susp.;	*x20=q10*: angular displacement of rear right susp.;

The **B** matrix is of dimension equal to the number of states by the number of inputs. If all the inputs are considered, that is, steering angles on all wheels, suspension actuator forces on all four suspension arms, road vertical disturbances at the wheels and aerodynamic disturbances at the sprung mass, the total number of inputs are 17 and **B** is of dimension 20x17.

The **C** matrix depends on the observability of the states and which variables are defined as outputs. In our case, the variables defined as outputs are chosen depending upon which quantities are of interest. For the first studies the C matrix was the dimension 20x20. Another output matrix, named performance output matrix, utilized for the controller design is presented in item 4 of this work.

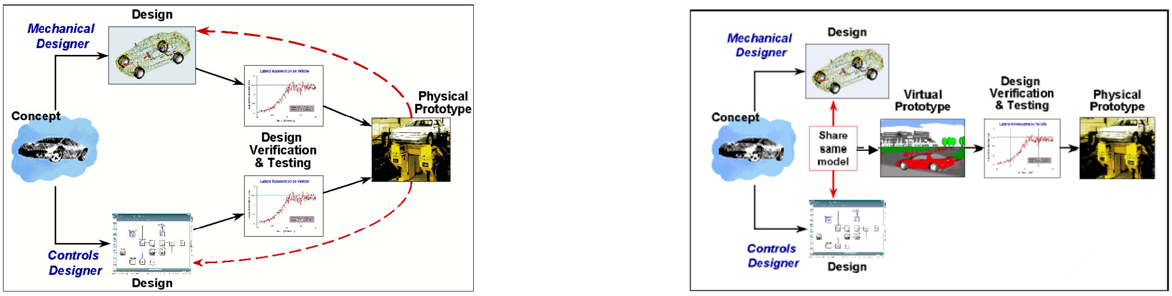
The **D** matrix is null because no output was defined to depend on the inputs.

3. Active suspension design

Totally active suspensions are defined as those which do not react to a load unless they receive a command to do so. In this kind of suspension the control functions are basically exerted by actuator commanded by feedback loop control systems. In a totally active system there is no limitation that is present in the passive systems, in which the forces applied by the suspension are always opposite to the relative motion between the sprung mass and unsprung mass. In a passive suspension this restriction is due to the fact that the spring only keeps the energy, and the dampers only dissipate it. To apply forces in the same direction of the relative chassis-tire motion it is necessary to introduce energy to the system through fast actuators. Once the active systems are able to provide this energy, it is possible to implement a wide range of control strategies specifically related to the acquisition of pre-established performance requirements. In doing so, the active systems are able to achieve performance levels that go beyond the possibilities of the passive systems.

Obtaining an adequate control strategy to an active suspension is a complex enough task to discourage an outlook based only in attempts and errors. Therefore, the convenience of using optimal control theories became clear for such theories and offers direct methods to determine the control laws from the minimization of pre-established performance indices. From the above mentioned, this work presents a first attempt to the design of a totally active suspension control system based on optimal control theories with output feedback, as a first step to the development of an integrated motion control system utilizing a complete vehicle model .

The control is obtained with the use of a linearized model in the form of state space and project techniques based on multivariable optimal control, designed at Matlab/Simulink facilities, resulting in a gain matrix which is used in the suspension control feedback. The feedback gain is though used through ADAMS/Simulink co-simulation which uses both softwares simultaneously to reproduce the behavior of the active suspension vehicle. Figure 2 shows the co-simulation environment between ADAMS and Simulink and the conventional modeling and controllers design techniques:



2(a) 2(b)

Figure 2: Two different approaches on control design: 2(a)- typical approach ; 2(b)- co-simulation approach.

In the typical design process of a mechanical system with control, the mechanical designer and the controls designer work from the same concept, but use different sets of software tools. The result is that each designer produces a model for the same problem. Each design is then subject to verification and testing, and the first time the two designs are brought together is during physical prototype testing. If a problem occurs during the interaction between the control design and the mechanical design, the engineers must refine the control design, the mechanical design, or both, and then go through the entire verification process again as shown in Fig. 2(a). With co-simulation, the two designers can share the same mechanical model. They can also verify from one database the combined effects of a control system on a nonlinear, non-rigid model. The physical testing process is greatly simplified, as you can see in Figure 2(b). This was the procedure utilized in the first steps for the suspension control designed in this work. A series of definitions must be established in a way that both softwares (ADAMS and Matlab/Simulink) could exchange information with each other. The four steps listed below shows the main procedures adopted:

1. The vehicle model is implemented at ADAMS.
2. The model is linearized in different operation conditions.
3. The linear state space model is utilized for the calculations of the optimal control feedback gain matrix (**K**), implemented in Matlab. In this stage, inputs and outputs have to be defined so that the softwares can exchange data, as shown in fig. 3.
4. The gain matrix (**K**) is utilized at Simulink to make possible the connection with ADAMS in order to perform the co-simulation, as showed in fig. 2(b).

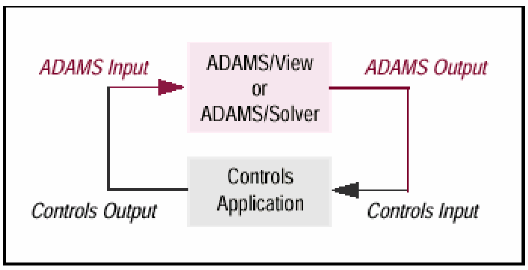


Figure 3. Defining inputs and outputs for data exchange during co-simulation.

3.1. Controller based on linear quadratic regulator with output feedback

The term optimization refers to the science of maximizing or minimizing a certain performance which needs to be measured or estimated. When mathematically formulated, this measurement is called Performance Index (PI) and when used to optimize control systems we have an optimal control problem.

The Linear Quadratic Regulator objective is to regulate states to zero using the output feedback and increasing the *plant* stability. The *plant* is described by the linearized equations of motion written in the state-space form:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{D} \end{aligned} \qquad (5)$$

With $\mathbf{x}(t) \in R^n$ the state, $\mathbf{u}(t) \in R^m$ the control input, and $\mathbf{y}(t) \in R^p$ the measured output. Let us select a feedback control input on the form:

$$\mathbf{u} = -\mathbf{Ky} \qquad (6)$$

where **K** is a *m* x *p gain* matrix with constant coefficients that is determined by the controller design. As the control input $\mathbf{u}(t)$ depends upon the output $\mathbf{y}(t)$ instead off the complete state vector $\mathbf{x}(t)$, the control law is obtained utilizing reduced state information or output feedback.

The objective of state regulation may be achieved by selecting the control input $\mathbf{u}(t)$ to minimize a quadratic cost or performance index (PI) of the type:

$$\mathbf{J} = \frac{1}{2}\int_0^\infty (\mathbf{x}^T\mathbf{Qx} + \mathbf{u}^T\mathbf{Ru})dt \qquad (7)$$

where **Q** and **R** are symmetric positive semidefinite weighting matrices. We shall select the **K** matrix in the control input that minimizes the value of PI.

By substituting the control 6.13 into 6.14 the closed loop system equations is found to be:

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BKC})\mathbf{x} \equiv \mathbf{A}_c\mathbf{x} \qquad (8)$$

and the PI may be expressed in terms of K as:

$$\mathbf{J} = \frac{1}{2}\int_0^\infty \mathbf{x}^T(\mathbf{Q} + \mathbf{C}^T\mathbf{K}^T\mathbf{RKC})\mathbf{x} \quad dt \qquad (9)$$

the design problem is now to select the gain **K** so that **J** is minimized subject to the dynamical constraint (6.15). This dynamical optimization problem can be solved utilizing a set of Lyapunov equations in the form (LEWIS, 1992):

$$\begin{aligned} &0 = \mathbf{A}_c^T\mathbf{P} + \mathbf{PA}_c + \mathbf{C}^T\mathbf{K}^T\mathbf{RKC} + \mathbf{Q} \\ &0 = \mathbf{A}_c\mathbf{S} + \mathbf{SA}_c^T + \mathbf{X} \\ &0 = \mathbf{RKCSC}^T - \mathbf{B}^T\mathbf{PSC}^T \\ &\text{where} \\ &\mathbf{A}_c = \mathbf{A} - \mathbf{BKC}, \quad \text{e} \quad \mathbf{X} = E\{\mathbf{x}(0)\mathbf{x}^T(0)\} \end{aligned} \qquad (10)$$

The first two of these equations are Lyapunov equations and the third is an equation for the gain **K**. If **R** is positive definite (i.e., all the eigenvalues greater than zero) and $\mathbf{CST}^T$ is non singular , then we can obtain **K** from:

$$\mathbf{K} = \mathbf{R}^{-1}\,\mathbf{B}^T\mathbf{PSC}^T(\mathbf{CSC}^T)^{-1} \qquad (11)$$

To obtain the output feedback gain **K** minimizing the PI, we need to solve three coupled equations on (10). To do so, the Matlab software was used to compute the **K** matrix from the linearized model obtained before.

4. Simulation and result analysis

As mentioned before, the procedures to obtain the feedback gain matrix K involve the parameters selection of the matrices **Q** and **R**, in an interactive way. If the results are not satisfactory, new **Q** and **R** matrices are chosen and the procedure is repeated. This means a lot of simulations run until good results are obtained. Besides that, different simplified output matrices were chosen, the so called output performance matrices (Lewis, 1992) in the form

$$\mathbf{z} = \mathbf{H}\mathbf{x} \qquad (12)$$

where **z** is the output performance vector, utilizing reduced state information, so the choice of **Q** can be confronted more easily by considering the performance objectives of the LQR. For example, in the simulation described below the performance output was chosen to be controlled by the output feedback, resulting in a matrix **H** that has only 11 outputs:

Table 3. Performance output chosen for simulation.

y_1 = front left vertical suspension deflection; y_2 = front right vertical suspension deflection; y_3 = rear left vertical suspension deflection; y_4 = rear right vertical suspension deflection; $y_5 = \dot{y}_1$ = front left vertical susp. velocity; $y_6 = \dot{y}_2$ = front right vertical susp. velocity;	$y_7 = \dot{y}_3$ = rear left vertical susp. velocity; $y_8 = \dot{y}_4$ = rear right vertical susp. velocity; y_9 = vehicle roll angle; y_{10} = vehicle pitch angle; y_{11} = vehicle vertical acceleration;

the signal **z***(t)* can be made small by LQR design by selecting the PI so that the **Q** may be computed from **H**. That is, by weighting performance outputs in the PI. A convenient guideline for selecting **Q** and **R** is given by (Lewis, 1992):

$$\mathbf{J} = \frac{1}{2}\int_0^\infty (\mathbf{z}^T\overline{\mathbf{Q}}\mathbf{z} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt \quad \text{, where } \overline{\mathbf{Q}} \text{ is given by } \mathbf{H}^T\mathbf{Q}\mathbf{H} \qquad (13)$$

In this way, the outputs from ADAMS (inputs to Simulink) as defined in Fig. 3 are the listed above. The inputs to ADAMS (outputs form Simulink) are the 4 actuator forces of the suspension, one for each suspension, that acts in parallel to the passive suspension.

With these definitions, a simulation was performed giving priority to the vertical acceleration response of the vehicle. The objective of the controller was to minimize vertical acceleration in order to optimize the passenger's comfort.

The Fig. 4 shows the road roughness as an input to the vehicle, with different entrances for the right and left wheel, in blue and red respectively.

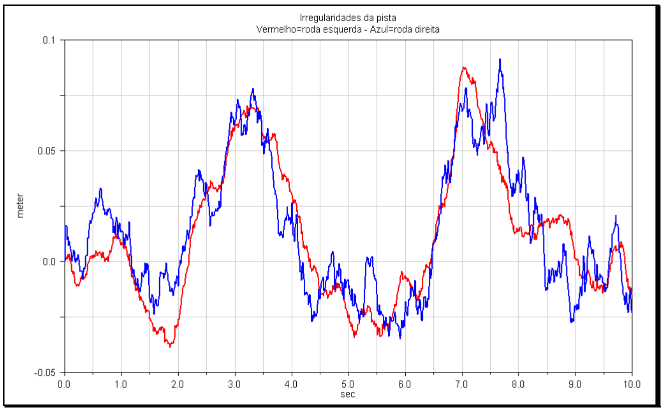
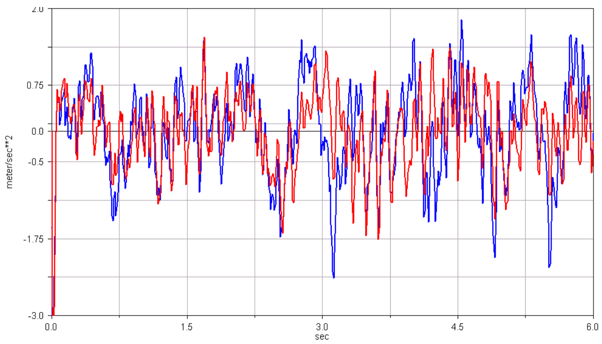
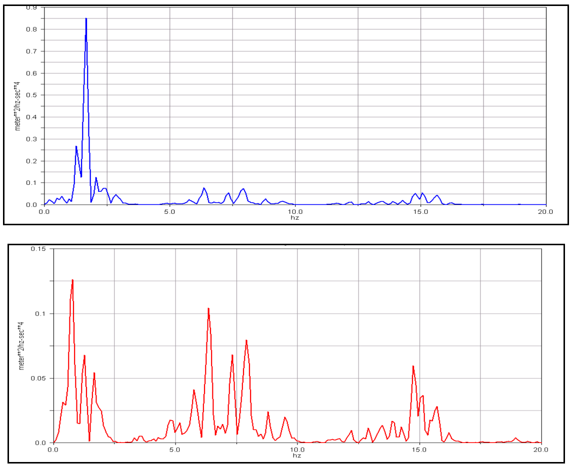


Figure 4. Road roughness

There are several possible measured outputs, but here the showed ones are the vertical acceleration at the sprung mass CG as the vehicle is traveling at 20 m/s, with and without suspension active control, in red and blue respectively. On Fig. 5 there are also showed the PSDs of the vertical acceleration for the two cases. The PI in this case priories the minimizing of the vertical acceleration, for better ride. It can be seen the better response of the vertical acceleration in terms of its PSD, reducing the low frequency components of interest for better ride. Many other responses can be obtained i.e. roll an pitch angular vehicle behavior, suspension displacement and the forces and energy consumption of the suspension actuators.



5(a)



5(b)

Figure 5. Vertical acceleration 5(a) and its PSD 5(b) of the sprung mass – Without control in blue and with suspension control in red.

5. Conclusions

The modeling approach adopted in this work, combining non-linear and linear vehicle models, is a powerful method for characterizing vehicle behavior in many different situations and makes possible the project of controllers considering vertical, lateral and longitudinal dynamics. A first attempt on control co-simulation studies is made considering a suspension active actuator based on optimal control theory and the first results proved to be very encouraging. The possibility of linearizing the vehicle model in many different operation conditions, give us the possibility to design controllers for a large band of vehicle maneuvers. If for determined bands of operation we could obtain robust controllers for that band, it is possible to think about some kind of gain scheduling control to optimize the behavior of the vehicle in many kinds of maneuvers, considering various control systems, i.e. ABS, TCS, 4WS and active suspension acting together. This is what we call integrated motion control.

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