Topology optimization method applied to design hydro generator components under body forces

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Abstract. *. In the structural design, usually only concentrated loads are considered. However, there are some structures in which the body forces have a major importance in its design. Unfortunately, the optimal design of these structures is not intuitive, once both load and stiffness depend on the material distribution. Besides that, in the litarature, only few works deal with the optimization of structures considering density dependent body-forces. Thus, in this work a Topology Optimization formulation for designing three-dimensional structures under self-weight and inertial forces has been developed. As objective function, the traditional mean compliance design problem is considered. Material models parametrizing stiffness and density properties were implemented based on SIMP model. Optimality criteria method is applied as the optimization algorithm. To avoid the checkerboard problem higher-order 20 nodes elements are used, and the continuation method is applied to avoid local minimums. Concerning industry application the method was implemented in ANSYS™ software by using the APDL (ANSYS™ Parametric Design Language). To demonstrate the algorithm potentiality the design of a hydro generator rotor component subjected to self-weight and centrifugal force is shown and discussed.*

Keywords*: hydro generator rotor, self-weight, inertial forces, topology optimization, SIMP, ANSYS™*

1. Introduction

In heavy mechanical industries, the structural concept is a consequence of about a century of development. Such structural solutions have been exhaustively investigated and few chances of improvements applying conventional design methods remain. Nowadays, the great majority of new developments is based on the experience of a design group, or modifications upon an existing project. In this way, the structural optimization methods come as a powerful tool in the design of these components.

In structural design, usually only concentrated loads are considered. However, there are important mechanical structures in which the body forces have a major importance in the design. It is the case of rotational machines such as energy generator rotors, turbine runners and flywheels. The optimal design of these structures is not intuitive, once both load and stiffness depend on the material distribution. In this way, topology optimization method can be applied in conceptual design. The topology optimization method search for an ideal material distribution of a structure, such that the objective function is maximized. However, even though there are plenty of articles discussing topology optimization structural design, only a few of them consider the body force influence (Bruyneel and Duysinx 2005).

To consider density dependent body force in Topology Optimization Method it is necessary to modify the traditional formulation. The material model must be defined considering the penalization of spurious structures that can be present in the final solution, and the optimization algorithm, the optimality criteria in this case, must consider the possibility that the volume constraint is not active in the optimal solution.

The works of Liu, Parks and Clarkson (2005) cite Donath as the first to investigate the optimization of rotating structures with centrifugal forces. Donath approached the structure by a series of disks with constant thickness.

Stodola (1924) presented the rotor hub of a steam turbine. His works considered a disk, without central hole, subjected to centrifugal force. Stodola's work was based on the idea of constant stress level in the whole disk. Stodola suggested a hyperbolic curve to describe the best profile of the structure. By the advent of computational analysis methods based on mathematical programming, Bhavikatti and Ramakrishnan (1980) applied the method of sequential

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linear programming to solve the optimization problem of a disk subjected to the centrifugal force. While, Cheu (1990) applied with success the method of "feasible direction" to the solution of the same problem.

Kress (2000) applied the method "feasible direction" to optimize the thickness variation of a steering wheel with central hole.

In this work, the Topology Optimization Method is applied to optimize the hub of a hydro power generator. The formulation of maximum stiffness with volume constraint is considered. The problem is parametrized with the SIMP material model, and the optimization is solved by using the optimality criteria method.

This paper is organized as follows: In section 2, the basic Topology Optimization theory is presented, In section 3, the problem formulation employed and the optimization algorithm is described considering density dependent body forces. In section 4. the case study of optimization of a hydro generator shaft subjected to centrifugal forces, self-weight and concentrate loads using a three dimensional approach is presented. Section 5 presents the conclusion of this work

2. Topology Optimization Theory

Topology optimization is based on two main concepts (Bendsøe, and Kikuchi, 1998): the *extended design domain* and the *relaxation of the design domain*. The *extended design domain* is a large fixed domain that must contain the whole structure to be determined by the optimization procedure. The objective of topology optimization is to determine the holes and connectivity of the structure by adding and removing material in this domain. In fact, topology optimization problem is defined as a problem of finding the optimal distribution of material in the extended domain. As the extended domain is fixed, the Finite Element model domain is not changed during the optimization process.

The relaxation of the design problem is associated with the change of material from solid (one) to void (zero). The discrete problem, where the amount of material in each element can assume only values equal to either one or zero, is an ill-posed problem, that is, it does not present a solution. The problem must be relaxed by allowing the material to assume intermediate property values during the optimization procedure which can be achieved by defining a material model.

Essentially, the material model approximates the material distribution by defining a function of a continuous parameter (design variable) that determines a mixture of material and void. By allowing the appearance of intermediate (or composite) materials – rather than only void or full material - in the final solution. This provides enough relaxation for the design problem.

3. Topology optimization considering density dependent body forces

In this work, the traditional formulation for stiffness design problem is considered as objective function, where the objective is to find the material distribution that minimizes the mean compliance (Bendsøe and Sigmund 2003):

$$\mathrm{C}_{mean} = \int_{\Gamma} \mathbf{tu} \, d\Gamma + \int_{\Omega} \mathbf{pu} \, d\Omega \tag{1}$$

where **t**, **p** and **u** denote the traction, body forces and displacements, respectively, and Γ represents the boundary of the domain Ω Then, the general continuous form of the topology optimization problem for stiffness design can be defined as:

$$\begin{aligned}
&\underset{\eta}{\textit{Minimize.}} \quad \mathrm{C}_{mean} \\
&\textit{such that} \\
&\qquad \int_{\Omega} \eta(\mathrm{X}) \mathrm{d}\Omega \leq \overline{\mathrm{V}} \\
&\qquad 0 < \eta_{\mathrm{MIN}} \leq \eta \leq \eta_{\mathrm{MAX}} \\
&\qquad \textit{Equilibrium equation}
\end{aligned} \tag{2}$$

where η is the design variable which, for the SIMP material model, can be interpreted as the volumetric material fraction. To consider the body forces the material model must be extended to also parametrize the material density. Here the following relation was applied:

$$\rho = \eta^{p_f} \rho_0 \tag{3}$$

$$E = \eta^{p_k} E_0 \tag{4}$$

In this material model, two penalization factors p_f and p_k are introduced. In the literature (Bendsøe and Sigmund 2003, Rozvany and Zhou and Birker, 1992, Bendsøe 1989), the parameter p_k is usually set equal to 3 or 4. This penalization avoids the gray-scale regions (η around 0.5) in the optimized solutions. From a physical point of view, the density ρ has a linear relation with the volumetric fraction η, thus the parameter p_f should be set equal to 1.

However, as discussed by Bruyneel and Duysinx (2005) with the set $p_f = 1$ and $p_k = 3$ the results, in general, presents spurious structures. This happens because when the volume fraction tends to its inferior limit ($\eta_{MIN} = 10^{-3}$) the ratio between the density dependent force (**p**) and the material stiffness (E) increase, locally.

To prevent this problem, we propose to apply a penalization p_f different from 1. Analyzing the relation between penalizations factors, we can observe that $p_f \geq p_k$ should avoid the presence of spurious structures, however, the final solution tends to present gray scale because of intermediate values of η. As can be seen in Figure 1, the ratio between density dependent force (**p**) and the material stiffness (E) is favorable to decrease the mean compliance.

Regarding this situation, some numerical tests were performed, and it was concluded that the penalizations factors p_f and p_k must be equal $p_f = p_k$ to obtain results without gray scale or spurious structures.

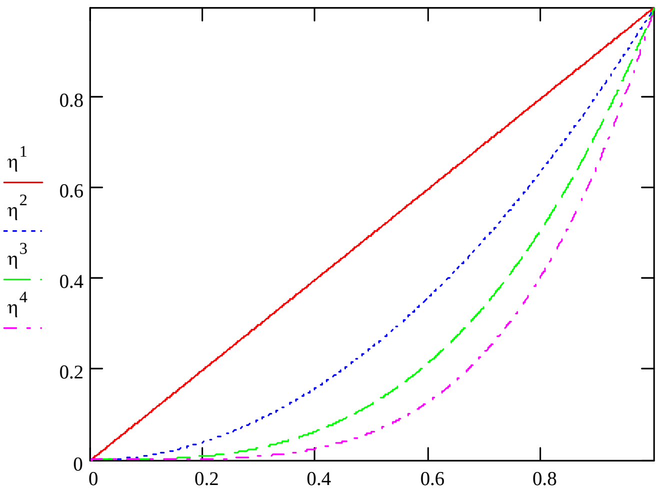


Figure 1: Interpolations functions

To avoid local minima a continuation method was employed, and the schedule of penalization was defined as $p_f = p_k = 1$, $p_f = p_k = 2$ and $p_f = p_k = 3$.

Regarding the checkerboard instability, the discretization of the density field was made in an element base, while the displacement field was approximated by a twenty-node element, avoiding the checkerboard problem (Jog and Haber 1996).

4.1. Optimization algorithm

To solve the optimization problem, the optimality criteria was applied (Sigmund 2001). Due to the possibility of positive values of the objective function sensitivity, small changes in the algorithm have been made, following the implementation of the optimality criteria for the design of flexible mechanism presented in (Bendsøe and Sigmund 2003). The following variable updating rule was applied:

$$\eta^{k+1} = \begin{cases} \max\{(\eta^k - \varsigma), \eta_{\min}\} & if \quad \eta^k B_e \leq \max\{(\eta^k - \varsigma), \eta_{\min}\} \\ \eta^k B_e & if \quad \max\{(\eta^k - \varsigma), \eta_{\min}\} \leq \eta^k B_e \leq \min\{(\eta^k + \varsigma), \eta_{\max}\} \\ \min\{(\eta^k + \varsigma), \eta_{\max}\} & if \quad \min\{(\eta^k + \varsigma), \eta_{\max}\} \leq \eta^k B \end{cases} \tag{5}$$

where B_e is given by:

$$B_e = \max\left\{0, \frac{-\frac{\partial C_{mean}}{\partial \eta}}{\lambda \frac{\partial V}{\partial \eta}}\right\}^{0.3} \tag{6}$$

This updating rule allows the method to obtain optimal solutions even when the volume constraint is inactive at the end of the optimization procedure.

4.2 Numerical Implementation

The optimization algorithm and the topology optimization procedure were implemented in ANSYS™ by using the APDL™ (ANSYS™ Parametric Design Language). This allows us to take all advantages of Finite Element code capabilities presented in the commercial code increasing the design tool flexibility.

The procedure implemented is characterized by some advantages:

- Multidisciplinary Analysis and Optimization - An advantage of using ANSYS™ as a FEA solver in optimization is that it is multidisciplinary. Considering the architecture mentioned above, the optimization program needs only a slight change when additional kind of analysis is added.
- Customization and Maintenance - An in-house optimization code integrated with ANSYS™ assures independent maintenance for each program. Another aspect is the possibility of customizing problem parameters (such as type of element, and outputs)

5. Numerical Results

In this work, we have studied the topology optimization of a hydro generator shaft subjected to centrifugal force. Three dimensional examples are presented here to examine the configuration of the optimal solution for the problem.

Properties of the isotropic material are:

- Young modulus = 210 MPa
- Poisson ratio = 0,3
- Base material density = 7.850 kg/m3

The applied loads are:

- Angular speed = 136 rad/s (1300 rpm)
- Gravity acceleration = 10 m/s^2

5.1 Description of the design problem of a generator rotor

The generator rotor is essentially composed by the shaft, poles, fan, rotating parts of the bearings, brake disc and slip ring (as in Figure 2). In the present design, the generator shaft has a segmented form. Two shaft parts are available (upper and lower), being directly flanged respectively at the top and at the bottom of the intermediate part. The shafts are made of forged carbon steel. The lower shaft has a flange integrally forged for direct coupling to turbine shaft with pre-tensioned bolts and tapered pins.

The shaft transmits the torque from the turbine to the hydro generator. To couple with turbine, a coupling flange is forged integrally with the shaft at the drive-end. The poles (pink and orange parts) are directly fitted onto the generator shaft (light green). The pole fitting slots are obtained by a longitudinal machining on the shaft.

The main dimensions of the studied shaft are:

- Shaft outer diameter (central part) = 1360 mm
- Shaft outer diameter (upper and lower part) = 360 mm
- Shaft inner diameter = 150 mm
- Coupling flange outer diameter = 600 mm
- Central part length = 1360 mm
- Total shaft length = 4700 mm

The shaft function is to transmit the torque from the turbine to the generator. However, in this work, the generator was considered operating in runaway speed. At runaway, the generator is electrically disconnected from the power grid, and there is no torque acting on the shaft. Then, in this operating condition, the generator rotor components are subjected only to inertial forces. The runaway speed is about twice the nominal speed.

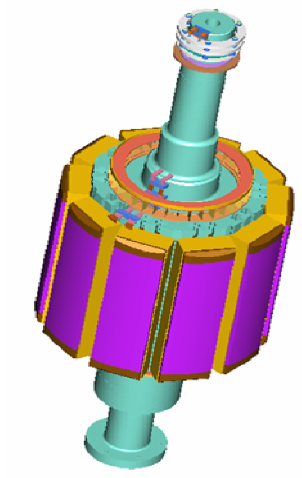
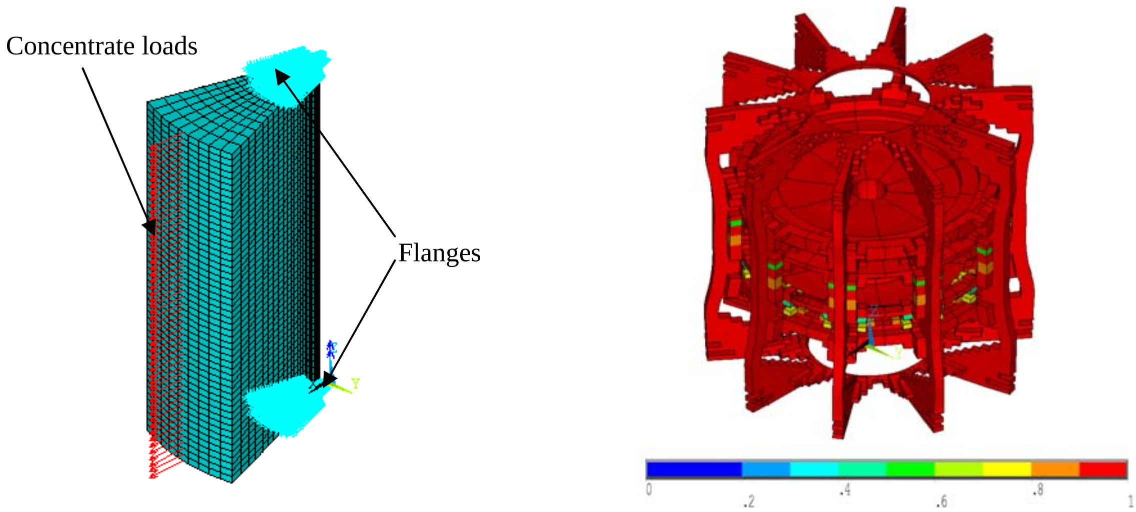


Figure 2. Generator Rotor

5.3 Three-dimensional model

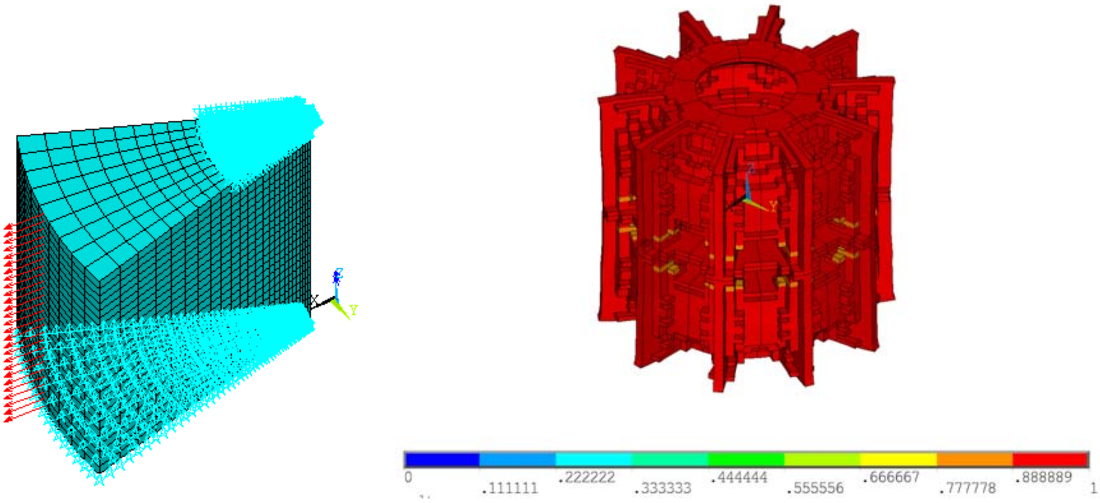
For the three dimensional models, the design domain is discretized into 20-node solid elements (SOLID95) subjected to inertia force due to angular velocity. Firstly, it was considered only the axial symmetry of the model. As can be seen in Figure 3 (a), the hub is rigidly fixed at the top and bottom of the domain, simulating the flanges and concentrate loads simulating the centrifugal force due to the pole mass, are applied,.



(a) (b)

Figure 3. Axisymmetric approach: (a) Extended Design domain and boundary conditions. (b) Synthesized structure considering self-weight, centrifugal force and concentrate loads, for the volume constraint of 20%.

Figure 3 (b), shows the optimum solution for the volume constraint 20%, observing the structure we note that the influence of the self-weight is small comparing with the centrifugal force. Thus, it was decided to neglect the gravity and a symmetry boundary condition was set to middle of the intermediate shaft. This new model is presented in Figure 4 (a) The upper and lower boundary of the design domain (only in the coupling region) was rigid support. In this case, it was also applied cyclic symmetry condition.



(a) (b)

Figure 4: Symmetry and Axisymmetry approach: (a) Design domain and boundary condition considering the load axisymmetric (b) Synthesized structure considering external forces due to the pole mass.

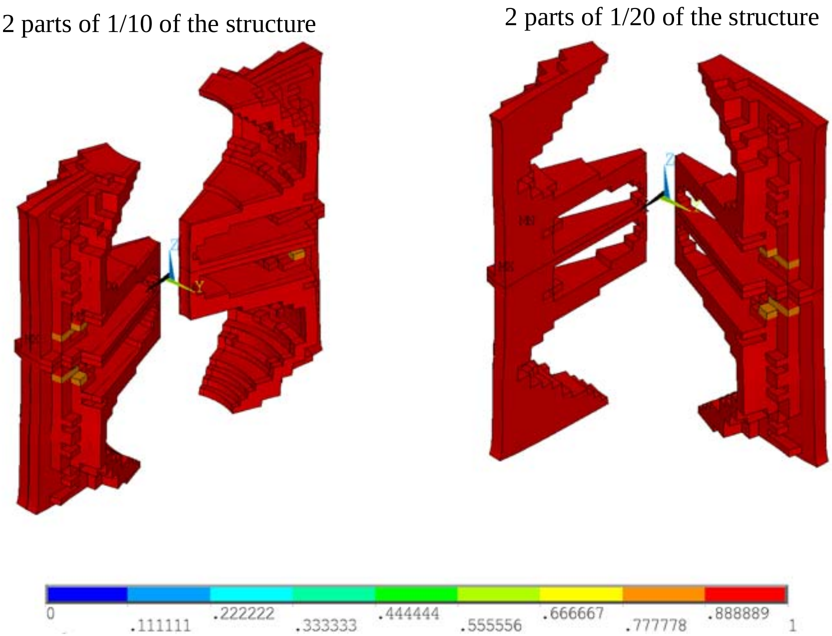


Figure 5: Longitudinal slices of the synthesized structure in the three dimensional approach.

The optimal structure for a constraint of 30% is presented in Figure (4) b. The Figure 5 presents the slices of the same structure, allowing too observe its inner part. Here its possible to observe how the structure must be defined in the center of the shaft.

Comparing both results presented it is possible to conclude that the increase of the volume fraction lead to a structures with the same topology, however with different thickness.

Considering the manufacture of the optimal structure, Forging is the traditional process for shaft manufacturing. However, observing the results obtained in Figure 5, we can suggest alternative shaft design configurations.

One feasible solution would be the replacement of the forged shaft by a group of cast disks. The disks would be fastened by bolts and have the same external diameter. The internal diameter would then change in a way to reproduce a similar configuration obtained in the optimized solution.

6. Conclusions

In this work, a Topology Optimization formulation for designing three-dimensional structures under self-weight and inertial forces was developed. Some particularities of topology optimization including inertial forces and a comparison between two and three dimensional results were presented.

The topology optimization procedure was implemented in ANSYS™ Language, which allows the above mentioned benefits.

The mean compliance design problem was the objective function and density properties were implemented based on SIMP model. A modified optimality criteria method was applied as the optimization algorithm, as the traditional optimality criteria method is not appropriate for density-dependent body forces.

The numerical applications have shown that the density-dependent body forces have a strong influence in the topology optimization result depending on their magnitude in relation to the applied concentrated loads. Hence, depending on the operating condition of the mechanical part, body forces cannot be neglected to obtain the optimized design.

7. Acknowledgements

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