APPLICATION OF A MESOSCOPIC SCALE APPROACH ASSOCIATED WITH A NOTCH ANOLOGUE TECHNIQUE FOR FRETTING FATIGUE

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Abstract. *Fretting fatigue is a particularly severe form of fatigue that occurs when a component is subjected to small amplitude oscillatory movement between contacting surfaces. The goal of this work is to estimate the fatigue limit of components under fretting conditions by considering (i) a methodology conventionally used in notched fatigue analysis and (ii) a finite element computation of the fretting stresses. The proposed methodology is based on the application of the mesoscopic approach proposed by Dang Van in terms of the Theory of Critical Distance. The cyclic contact stress field was numerically determined by a home made finite element code. Available experimental data containing a size effect were considered to validate the analysis.*

Keywords: *fretting fatigue, mesoscopic approach, notch fatigue, finite element analysis, size effect*

1. Introduction

Many engineering materials face applications in which the component is subjected to freting fatigue conditions like, for example, screwed and riveted joints, couplings of shafts with gears or bearings, contacting surfaces of disks and blades in turbine engines or compressors, etc. The conventional fatigue limit is substantially reduced under the action of fretting, hence premature failures have been observed increasing the frequency of maintenance intervals and the cost associated with the change of spare parts. In this setting, it is important that studies are carried out in order to develop tools or models which can predict, more precisely, the fatigue strength of mechanical assemblies experiencing fretting. The development of these models has been conducted, generally, considering the use of simpler contact configurations, where the variables involved on the fatigue phenomenon (like stress and strain) could be easily obtained and the conduction of validation tests is less expensive.

As in notch fatigue, size effect phenomena have been often observed in fretting fatigue. In particular, the effect of the pad radius (or contact size) on fatigue life has been reported for different materials by many authors (Bramhall, 1973, Nowell, 1988 and Araújo, 2000). Recentely, Araújo and Mamiya (2003) used a mesoscopic scale approach to predict crack initiation conditions for these tests. They found the mesoscopic model proposed by Dang Van (1989) could correctly predict crack initiation for bigger contact configurations. Further, Araújo *et al.* (2004) verified that neither critical plane models or the mesoscopic criterion can correctly estimate the fatigue limit for smaller contacts, if the analyses consider only the point of maximum superficial stress state. On the hand, satisfatory results were obtained to fatigue parameters with high values sustained within a critical volume. More recently, Vallellano *et al.* (2003) testing sphere-plane contacts proposed the applicability of notch analogue to fretting fatigue.

The main goal of this work is to consider the Theory of Critical Distances (Taylor,1999), usually applied to notch fatigue problems, in association with Dang Van criterion (Dang Van, 1989), to define a fatigue limit condition for cylindrical contacts showing a size effect phenomenon.

2. Experimental data

The analysis presented in this paper will be validated considering the work conducted by Nowell (1988). A schematic representation of the configuration tested is shown in Fig. 1 where R is the radius of cylindrical fretting pad, P is the normal load, σ_B is the bulk stress and Q denotes the tangential load induced by the spring A. The contact loads P and

Q and the bulk stress σ_B were applied as shown in Fig. 2, i.e., P is a static load and Q and σ_B are in phase sinusoidal functions of time.

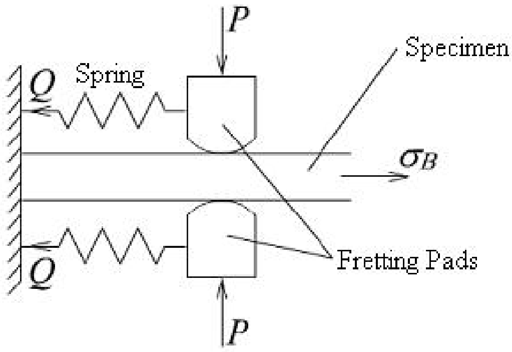

Figure 1. Experimental scheme used by Nowell.

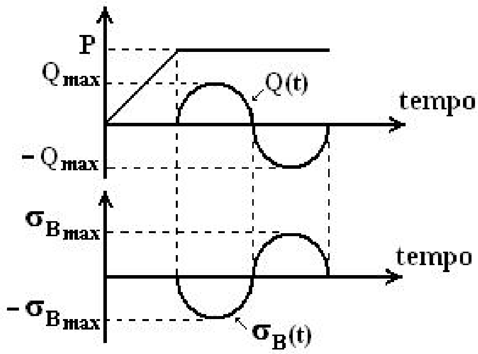

Figure 2. Experimental scheme used by Nowell.

Tests were carried out on Al4% Cu (HE15-TF). The basic mechanical properties for such alloy is provided in the next section. A series of seven tests were conducted. For all tests the parameters p_0 (the peak pressure), Q_{max}/P and σ_B^{max}, were kept constant, while the pad radius was varied from $12,5$ to $150mm$. Here the subscript max denotes the maximum value reached by Q and σ_B over time (Fig. 2). These parameters are presented in Tab. 1. The importance of varying R keeping p_0 constant is that it is possible to produce a data series where all specimens are submitted to the same superficial stress state although they experience different stress decays along the depth. The fretting fatigue experiments were all carried out in partial slip. The contact area is characterized by a central stick zone bordered by two slip regions (Hills and Nowell, 1994). As the test proceeds, surface modification will raise the coefficient of friction within the slip zones from the initial value, f_0, to a new value f_s. Although f_s can not be easily measured its value is required to conduct a stress analysis in the partial slip regime. To overcome such difficulty an expression has been derived (Hills and Nowell,1994) to estimate f_s from a measured mean value, f_m. Two tests were conducted to measure f_m and the average estimated value of f_s(hereafter denominated f) is reported in Tab. 1.

The tests carried out by Nowell revealed an effect of the pad radius (or contact size, a) on fretting life. It was observed that for small contact sizes fretting tests last infinitely while for large contacts specimens broke within a finite number of cycles. The range defined by the largest contact to show infinite life and the smallest contact to provide finite life was termed (Bramhall, 1973) the critical contact size range, a_{crit}. The critical contact size range for these tests is $a_{crit} = 0,54 - 0,72mm$.

Table 1. Experimental parameters.

p_0 (MPa)	σ_B^{max} (MPa)	Q_{max}/P	f
143	92,7	0,24	0,75

2.1 FE Modeling

The geometry adopted in the experimental programm was a simple one but the loads applied to conduct the tests were so that no analytical solution was available to work out the stress field. Therefore, the FE code $ef++$, developed by the Mechanics of Materials Research Group of the University of Brasília, was considered to simulate the tests. As a graphical interface this code uses the GiD plataform (Ribó, 2000), which carries out both, pre – geometry creation, meshing and boundary conditions assignment – and post processing – stress, strain and displacement field visualization. A contact element was recently implemented (Bernardo, 2003) into the FE code, which allows the calculation of the stress field under fretting situations.

In order to perform the FE analysis of the configuration depicted in Fig. 1 and considering its symmetry, the model showed in Fig. 3 was adopted. The fundamental parameters (Nowell, 1988) necessary to carry out the stress analysis and the fatigue strength data are: the Young's modulus $E = 74GPa$, yield stress $\sigma_{ys} = 465MPa$, ultimate tensile strenght $\sigma_{us} = 500MPa$, fatigue strenght coefficient $\sigma'_f = 1015MPa$, fatigue limit under alternate bending $f_{-1} = 124MPa$ e Poisson coefficient $\mu = 0,32$. The coordenated sistem *xy* adopted is defined in Fig. 3.

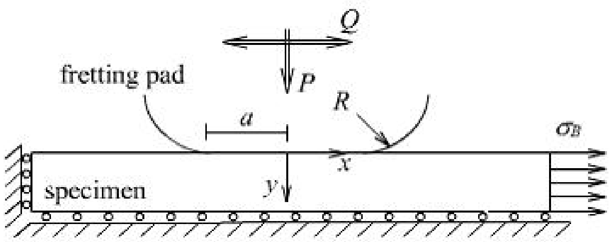

Figure 3. Scheme of the adopted model.

2.2 FE Mesh

An example of a characteristic mesh generated for both fretting pad ($150mm$ radius) and specimen is depicted in Fig. 4. Plane strain linear elastic triangular elements are used on the pad and specimen domains. To simulate the contact problem bi-dimensional two-nodes interface elements were considered, as shown in Fig. 4. In the same figure it can be noted that a structured mesh region was defined under the contact surface. Within such region the mesh refinement level and the finite element size are characterized by three parameters: the contact mesh width, l_c, and the element width and depth, l_e and h_e, respectively.

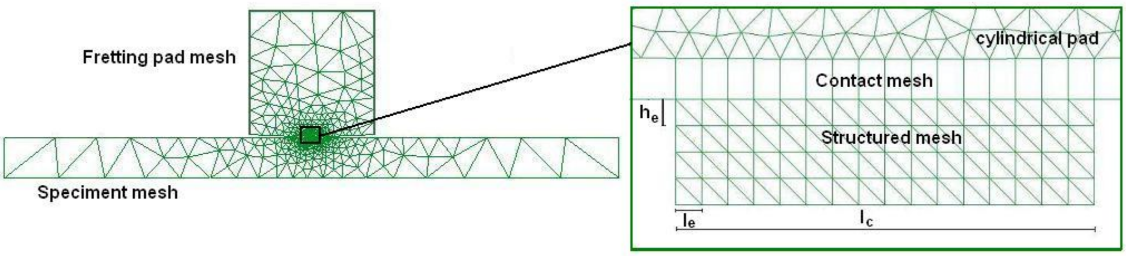

Figure 4. Mesh of both 150mm radius pad and specimen and mesh parameters.

In a previous work by Mendes *et al.* (2004) a mesh convergence analysis was carried out and it was shown that the adimensionalized parameters $l_c/a > 2.05$, $l_e/a < 0.0526$ and $h_e/a < 0.0117$ defined an appropriate mesh refinement level to compute the contact stresses. Futhermore, it should be reported that for l_e/h_e ratio far from the unit, the numerical simulation becomes unstable.

3. Dang Van multiaxial fatigue criterion

The Dang Van multiaxial fatigue criterion was chosen to carry out the analysis. The model (1989) assumes crack initiation is controlled by two parameters: shear stress and hidrostatic pressure.

$$\tau(t) = -\kappa p(t) + \lambda \quad (1)$$

Where the shear stress $\tau(t)$ is the Tresca equivalent stress τ_{eq} given by:

$$f(\tau) = \tau_{eq} = \frac{1}{2} \max_t \left(|s_1(t) - s_2(t)|, |s_1(t) - s_3(t)|, |s_2(t) - s_3(t)| \right) \quad (2)$$

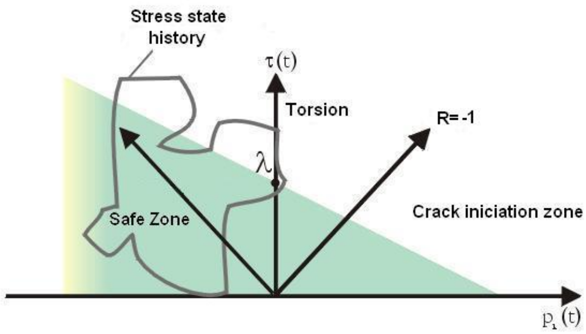

Figure 5. Dang Van criteria representation.

where: $s_i(t); i = 1, 2, 3$ are the eigenvalues of the deviatoric stress tensor $\mathbf{s} = \mathbf{S(t)} - \mathbf{S_m}$. being S_m the center of the minimum hypersphere circunscribing the deviatoric stress path (Dang Van, 1989).

And the hidrostatic pressure is given by:

$$p = \frac{tr(\sigma)}{3} \tag{3}$$

where σ is the Cauchy stress tensor.

If the load history is below the line that delimits the safe zone on the τ_{eq}, p plane the material fatigue lifetime tends to infinite.

The material parameters κ and λ can be evaluated taking the fatigue limits under fully reversed bending f_{-1} and for repeated bending f_0.

$$\kappa = \frac{3}{2}\left(\frac{f_{-1} - f_0}{f_{-1} - 2f_0}\right) \tag{4}$$

$$\lambda = \frac{f_{-1}}{2}\left(\frac{f_0}{2f_0 - f_{-1}}\right) \tag{5}$$

An error index that measures how far the limit load situation is from the real condition can then be defined as:

$$DV = \left(\frac{\tau_{eq} + \kappa \cdot p_{max} - \lambda}{\lambda}\right) \tag{6}$$

In this setting, when $DV < 0$, the model indicates that the solicitation is under the limit condition. On the other hand, when $DV > 0$ the model indicates that the component has failed (crack iniciation).

3.1 Multiaxial fatigue analysis in terms of the Critical Distance Method

Fatigue failure in mechanical components usually occurs due to geometry features that cause: (1) local stress concentration, (2) stress gradient from the notch tip to the specimen center and (3) multiaxial stress states below the notch root. It is well known that the behaviour of a notch can not be appropriately characterized by the maximum local stress (hot spot stress), but depends on other factors determined by the notch geometry and the local stress distribution. In this setting, many methods to calculate notch stress by introducting a factor based on the notch geometry or on the local stress gradient have been proposed (Neuber, 1958, Peterson,1959). These methods are based on the idea of a ”critical distance” or ”process zone” and propose that in order to occur fatigue failure it is necessary that the level of stresses are kept above a certain level within a material volume rather than in material point. The size of this volume is assumed not to be dependent on either the stress concentration feature weakening the component or the complexity of the stress field damaging the fatigue process zone. Tanaka (1983) proposed the value of this critical volume was related to the short crack parameter, a_0, previously defined by El Haddad *et al*. (1979).

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{\Delta \sigma_{fl}}\right) \tag{7}$$

where ΔK_0 e $\Delta \sigma_{fl}$ are the limit value of the stress intensity factor and the material fatigue limit, respectively.

Tanaka showed that the stress defined at a point in the center of this structural volume, whose radius was a_0, could successfully characterize the fatigue solicitation in the case of sharp cracks. This same discovery was made by Taylor (1999), who tested his theory firstly against experimental data from standard notches and later to predict the fatigue limit for an automotive suspension (Taylor *et al.*, 2000).

In a number of respects, fretting fatigue cracks present a notch analogue behaviour. Fretting cracks start at a point of extremely high local stress. The contact stress field is, however, extremely localised, and its influence will decay rapidly as the crack grows away from the interface. This suggests that a threshold condition for fretting crack initiation might be predicted by using methodologies similar to those employed to assess notched components (Susmel and Taylor, 2003). According to this idea, we seek to use the Dang Van criterion in terms of Critical Distance Theory. To be precise, our method takes as its starting point the idea that high-cycle fatigue damage in metals depends on both stress gradients and degree of multiaxiality of the stress field in the vicinity of crack initiation sites. Our understanding of the phenomenon is that this fact holds always true independently of the causes these two phenomena are originated from: Dang Van's model accounts for the multiaxiality of the stress field, whereas the TCD allows the stress gradient effect to be taken into account.

4. Results

Before presenting the estimates of fatigue strength it is useful to show some results for the numerically computed fretting stresses. In this setting, Fig. 6 shows the variation of the normalized stress components $p(x)/p_0$, $q(x)/fp_0$ and σ_{xx}/p_0 along the contact surface ($y/a = 0$) for $Q = Q_{max} = 0.6$ and $\sigma_B = \sigma_B^{max} = 0.648$. From these graphs it can be seen that (i)the pressure is Hertzian and reaches its maximum value at the center of the contact area(Fig. 6a), (ii) that reverse slip took place (Fig. 6b) and (iii) that the hot spot for the xx stress component is at the trailing edge of the contact at $x/a = -1$ (Fig. 6c), where fretting cracks have been reported to nucleate (Nowell, 1988). An interesting feature of this configuration is that, at this material point ($x/a = -1; y/a = 0$), σ_{xx} is the only non null stress component, being the zz component originated due to the plane strain assumption. This can be clearly observed in Fig. 7, which also shows the variation of all stress components against depth at $x/a = -1$ and at the instant of maximum tangential and bulk fatigue loads. Two important characteristics can be observed in this graph. First, the stresses decay very rapidly as the analysis get away from the interface and second that the state of stress becomes multiaxial just under the contact.

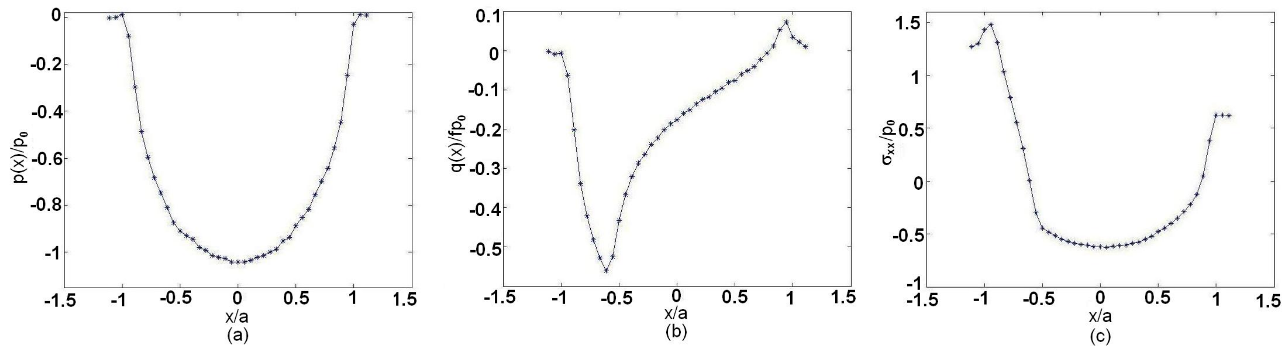

Figure 6. Distribution of different stress components along the contact surface for$Q = Q_{max}$ and $\sigma_B = \sigma_B^{max}$: (a) $p(x)/p_0$, (b) $q(x)/p_0$ (c) σ_{xx}/p_0.

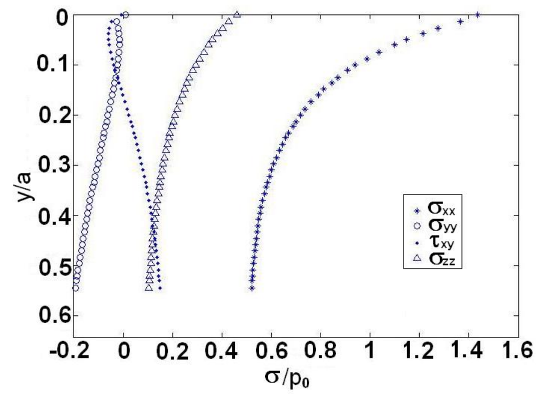

Figure 7. Stress components variation along the depth at the point $x/a = -1$ for the experimental configuration.

4.1 Fatigue strenght limit analysis

The Dang Van criterion will now be applied at the center of the structural volume in order to estimate the fretting fatigue limit for the experimental configuration considered in this work. Hence the first step in the analysis is to define the material parameter a_0. Susmel *et al.* (2004) reported $a_0 = 0.1mm$ for an Al 4% Cu alloy having the same fatigue limit ($\Delta\sigma_{-1} = 248MPa$) as the one tested by Nowell under fretting. The center of the the structural volume is then $a_0/2 = 0.05mm$. At this depth and at the trailing edge of the the contact zone (hot spot), the cyclic stress tensor was extracted at twelve different load steps by using numerical simulation. Here it is worthwhile mentioning that, in the setting of a finite element analysis, the trailing edge of the contact is somewhere between the position of the first superficial node where the pressure falls to zero and the position of the next node submitted to pressure. To be precise, we assumed that such pressure free node defined the position of the hot spot.

To continue the analysis the DV index (Eq. 6) needs to be computed from the stress history defined at this material point. Its calculation also requires two fatigue parameters at different loading conditions, such as the fatigue limits for fully reversed bending and torsion or the fatigue limits for stress ratios $R = -1$ and $R = 0$. In a recent work on the effects of mean normal stresses over the fatigue limit for a number of alloys, Dowling (2004) reported that, among a number of models assessed, the one proposed by Smith-Watson and Topper (1970) provided the best estimates of the fatigue limit for Al alloys tested under different R ratios. For the Aluminium alloy here considered this estimated value is then $87.7MPa$. The DV index calculated by applying the Dang Van criterion in terms of Critical Distances Theory to the available experimental data are reported in Tab. 2. It is also reported in this table information concerning the pad radius (R), the theorical and numerical contact semi-width (a_{theo} and a_{num} respectly) and the number of cycles in which the specimen failed for all tests. Tests that achieved 10^7 cycles were stopped and further investigation did not reveal the presence of cracks within the fretted zones or elsewhere within the contact region. At this stage the reader should be reminded that negative DV values mean that the solicitation is below the limit established by the multiaxial criterion and hence no failure is expected. From this table it can be noticed that the estimates provided for the tests of the experimental series considered do not agree with data in one case only ($R = 75mm$). Fortunately, the incorrect prediction is in the conservative side, i.e., it indicates that the specimen would failure while the test was a run out. Another form to visualize this result is depicted in Fig. 8. The DV index is plotted against the contact semi-with. The straight line which crosses the graph in $DV = 0$ defines the threshold condition for crack initiation. The points correspond to the DV estimates for each test considered. Arrows indicate run out tests. Therefore it can easily be seen that there is only one test where failure was uncorrectly predicted.

Table 2. Teorical and numerical contact sizes, experimental life and Dang Van error index for each pad radius.

Pad radius R (mm)	a_{theo} (mm)	a_{num} (mm)	Life (10^6 cicle)	DV
12,5	0,09	0,089	10	-0,4414
25	0,18	0,173	10	-0,2397
50	0,36	0,32	10	-0,0791
75	0,54	0,51	10	0,1823
100	0,72	0,68	5,06	0,2882
125	0,90	0,85	1,22	0,4054
150	1,08	1,02	1,28	0,4644

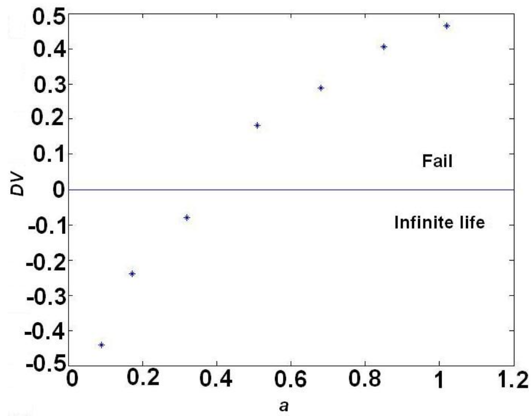

Figure 8. Dang Van error index DV and contact size relation.

5. Discussion and Conclusion

A fretting crack initiation threshold methodology was established. The work highlights the fact that the two phenomena of notch fatigue and fretting fatigue are linked, since both involve stress gradients; It is based on the application of the Dang Van criterion and on the Stress Point Concept. The methodology presented successful estimates in six of the seven fretting fatigue tests considered in this work. Further, for the case where the methodology failed it predicted crack initiation while the specimen experienced a run out, i.e. it was conservative. Compared with other notch analogue methodologies proposed for fretting fatigue, this approach has the advantage of defining the critical distance as a material parameter. Hence, if the basic fatigue parameters are appropriately defined for an specific alloy the crack initiation risk can be directly computed without the need to carry out further fretting fatigue calibration tests to define the size of the structural volume.

In the present work, the necessary material fatigue properties were taken from another source, however in this case the material composition and fatigue limit were identical (Araújo et al., 2004 and Susmell et al. 2004). It is important to highlight here that accuracy and reliability of the proposed method are strongly affected by the material constants used for its calibration. In fact, due to the locally high stress gradients which are always associated with contact problems, the predictions are more sensitive to the value of the crack propagation threshold $\Delta K_t h$ than to the plain-specimen fatigue limit. Provided $\Delta K_t h$ is accurately known, then any small errors in the measured value of the plain fatigue limit will be compensated by changes in the critical distance, a_0.

The methodology proved simple to implement and the fact that it requires only the linear-elastic stress state calculated at the centre of the structural volume to perform an accurate high-cycle fatigue assessment makes it extremelly appealing from a engineering point of view. On the other hand, it must be stated that further validation of the proposed approach considering different materials and contact configuration have to be carried out before using it to design real components.

6. References

Araújo, J. A., 2000, “On the Initiation and Arrest of Fretting Fatigue Cracks”, D.Phil.thesis, University of Oxford.

Araújo, J. A., and Mamiya, E. N., 2003, “The application of a Mesoscopic scale approach in fretting fatigue”, ABCM, Vol. 25, pp. 16-20.

Araújo, J. A., Nowell, D., and Vivacqua, R. C., 2004, “The use of multiaxial fatigue models to predict fretting fatigue life of components subjected to different contact stress fields”, Fatigue Fract Engng Mater Struct, 27, pp. 967-978.

Bernardo, A. T. S., 2003, “Fadiga por *fretting*: modelagem e simulação numérica”, Projeto final de graduação em Engenharia Mecânica, Universidade de Brasília.

Bramhall, R., 1973, “Studies in fretting fatigue”, D. Phil. thesis, University of Oxford, Oxford.

Dang Van, K., Griveau, B., and Message, O., 1989, “On a new multiaxial fatigue limit criterion: Theory and application”, Biaxial and Multiaxial Fatigue, EGF 3, Mech. Engng Publications, London, pp. 479-496.

Dowling, N. E., 2004, “Mean stress effests in stress-life and strain-life fatigue”, Society of Automotive Engineers.

El Haddad, M. H., Topper, T. H., and Smith, K. N., 1979, “Fatigue crack propagation of short cracks”, F. Engng Mater. Tech. (ASME Trans.), 101, pp. 42-45.

Hills, D. A., Nowell,D.,1994, “Mechanics of Fretting Fatigue”, Kluwer Academic Publishers

Mendes, M. F. R., Araujo J. A., and Mamiya, E. N., 2004, “Effect of the bulk stress on stresses produced by contact of cylinders under partial slip”, III Nacional Congress of Mechanic Engineering.

Nowell, D., 1988, “An analysis of fretting fatigue”, D. Phil. thesis, Oxford University.

Neuber, H., 1958, “Theory of Notch Stresses”, Berlin: Springer.

Peterson, R.E.,1959, “Notch sensitivity”, In: Sines G, Waisman JL, editors, Metal Fatigue, New York, McGraw-Hill, 293Ű306.

Ribó, R., Pasenau, M. A. R., and Escolano, E., 2000, “GiD Reference Manual”, International Center for Numerical Methods in Engineering (CIMNE), http:/gig.cimne.upc.es.

Smith, K. N., Watson, P., and Topper, T. H., 1970, “A stress-strain function for the Fatigue of Metals”, J. Mater., Vol. 5 No. 4, pp. 767-778.

Susmel, L., and Lazzarin, P., 2002, “A bi-parametric Wöhler curve for high cycle multiaxial fatigue assessment”, Fatigue and Fract Engng Mater. Struct., 25, pp. 66-78.

Susmel, L., and Taylor, D., 2003, “Two methods for predicting the multiaxial fatigue limits of sharp notches”, Fatigue and Fract Engng Mater. Struct., 26, pp. 821-833.

Susmel, L., Atzori, B., and Meneghetti, G., 2004, “Material fatigue properties for assessing mechanical components weakened by notches an defects”, Fatigue and Fracture Engng. Mater. Struct., 27, pp. 1-15.

Tanaka, K., 1983, “Engineering formulae for fatigue strength’reduction due to crack-life notches”, Int. J. Fract., 22, R39-R45.

Taylor, D., 1999, “Geometrical effects in fatigue: A unifying theoretical model”, Int. J. Fract., 21, 413-420.

Taylor,D., Bologna,P., and Bel Knani, K., 2000, “Prediction of fatigue failure location on a component using a critical

distance method", International Journal of Fatigue 22, 735-742
Vallellano, C., Dominguez, J., and Navarro, C., 2003, "On the estimation of fatigue failure under fretting conditions using notch methodologies", Fatigue and Fracture of Engineering Materials and Strutures, 26, pp. 469-478.

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