GRAY RADIATIVE CONDUCTIVE 2D MODELING USING DISCRETE ORDINATES METHOD WITH MULTIDIMENSIONAL SPATIAL SCHEME

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Abstract. Combined conduction and radiation heat transfer in multidimensional enclosure is a problem of considerable importance. Some examples are the heat transfer in glass fabrication, porous burners, volumetric solar receivers, fibrous and foam insulations, etc. To test the applicability of various radiative methods and also to study the effects of parameters such as the extinction coefficient, scattering albedo, the wall emissivity, etc., on the heat flux and temperature distributions, problems of combined conduction and radiation heat transfer in 2-D enclosures were analyzed in the last years. Included among others are the diffusion approximation, the methods of P_n, the method of discrete ordinates (DOM) and the method of finite volume. The angular and spatial discretization of the radiative heat transport equation (RTE) in the DOM are important to obtain accurate numerical results. In this work, two high order spatial discretization schemes are used and compared. One spatial discretization scheme is unidirectional and the other is multidimensional interpolating scheme. Different angular quadratures are selected and tested to obtain accurate results and reduce computational time. The RTE is solved using the conventional procedure of solution for DOM and the algorithm are validated by comparing the present results with exact solutions for different two-dimensional cases. The radiative source term in the energy equation is computed from intensities field. The radiative conductive model is validated by comparison with test cases solutions from the literature. To accelerate convergence an adequate relaxation factor in RTE and in energy equation is used. The model can be used to model cases with reflecting boundaries.

***Keyword**: radiation conduction, participating media, discrete ordinates method*

1. Introduction

In many engineering applications, the problem of radiation coupled with other modes of heat transfer is important. The heat transfer in glass fabrication and industrial furnaces are some examples of such applications. It is generally required to solve a multidimensional radiation field using computational techniques. In practice radiative heat-transfer calculations are complex and consequently many approximate solutions are proposed, including, among others, methods based upon diffusion approximation, the methods of P_n and simplified P_n , the method of discrete ordinates, the method of discrete transfer, and the method of finite volume.

The P_n approximation introduced initially by Krook (1955) and Cheng (1964) is simply taking the momentum of the radiative transport equation to obtain a system of equations free of the angular dependence. Each P_n approximation results in a system of n^2 equations and in the limit, that is, for large n, the P_n approximation results in the solution of the transport equation. The method of discrete ordinates DOM is an attractive simplified method to solve radiative transfer problems. This method was originally formulated by Chandrasekhar (1950), and developed by Lathrop and Carlson (1965) and Lathrop (1966). Fiveland (1984, 1988) formulated an accurate method of discrete ordinates of the first order based upon the method of control volumes for two-dimensional and three- dimensional enclosures and presented general outlines of the method. Ramankutty and Crosbie (1997) presented a more recent and extensive review and used this method to formulate the so-called modified discrete ordinates. Later, Sakami and Charette (2000) applied the modified discrete ordinates using triangular grids and finite elements methods. The techniques of total variation (TVD), presented by van Leer (1974), were applied by Jessee and Fiveland (1997) to solve radiation problems. Based upon the techniques TVD, Jessee and Fiveland (1997) formulated a second order interpolation CLAM scheme. Based upon genuinely two-dimensional advection schemes (Sidilkover and Roe, 2000), Balsara (2001) formulated a new second order scheme with multidimensional interpolation for RTE, and is called genuinely multidimensional (GM), and the resulting nonlinear system of equations is solved by using multigrid techniques in conjunction with the Newton-Krylov method.

The selection of angular quadrature is also one important aspect to consider to obtain accurate solutions as in Fiveland (1984, 1988), Balsara (2001), Thurgood et. al. (1995) and Koch et al. (2004).

The coupling numerical methods for radiative transfer with numerical methods for the energy equation proved to be a good alternative in the solution of combined heat transfer problems. Yuen *et. al.* (1988), had treated the radiation problem by a generalized exponential integral function and the coupled problem by one empirical additive approach. Kim *et. al.* (1991) had used the DOM with S_4 angular quadrature, diamond spatial scheme and uniform grid for the radiative part of the problem while the conductive term is discretized using the central difference schemes, and Sakami *et. al.* (1996) had used a modified discrete ordinates method based on the incorporation of directional ray propagation relations within the cells with triangular grids and in the conduction part of the coupled problem they used the finite element technique. Rousse (2000) and Rousse D. *et. al.* (2000) had used the control volume method in the radiative part and coupled with the finite element method in energy equation and adopted triangular grids. Lee and Viskanta (2001) compared solutions of combined radiation-conduction heat transfer in two-dimensional semi-transparent media using the finite volume method coupling with both DOM and diffusion approximation. They applied this approach in glass fabrication problem.

The objective of the present paper is to use an efficient method suitable for working with cartesian rectangular grids in 2D situations to solve the RTE and couple the developed code with problems including combined conduction and radiation. In the present study, to handle the problem of radiation-conduction within two-dimensional enclosure with diffusely emitting and reflecting walls the method of discrete ordinates based upon the method of control volume is used. Also the CLAM scheme and the GM scheme are used for the spatial discretization in conjunction with different angular quadratures. The energy equation is solved using control volume method considering the divergence of the radiant flux vector as a radiative source term. The predictions are validated with other results obtained by different approaches developed by other authors.

2. Formulation.

The energy equation at any location within a radiating medium is given by

$$\rho_f c_p \frac{DT}{Dt} + \nabla.(-k\nabla T) + \nabla.\boldsymbol{q}_r - q''' - \beta_f T\frac{DP}{Dt} - \phi_d = 0 \tag{1}$$

where, $\nabla.\boldsymbol{q}_r$, the divergence of the radiant flux vector $\boldsymbol{q}_r$ is considered as a radiative source term; $\frac{D}{Dt}$ is the substantial derivative; β_f is the coefficient of thermal volumetric expansion of the fluid; q''' is the local heat source per unit volume and time, and ϕ_d is the heat production by viscous dissipation.

The energy equation for coupled radiation conduction heat transfer of an absorbent, emitter and isotropic scatter media, under steady state and with constant thermal conductivity is as in Siegel and Howell (1992):

$$k\nabla^2 T - \nabla.\boldsymbol{q}_r = 0 \tag{2}$$

Writing the radiative source term as $\boldsymbol{S}_r = -\nabla.\boldsymbol{q}_r$, The energy equation is

$$k\nabla^2 T + \boldsymbol{S}_r = 0 \tag{3}$$

In two-dimensional cartesians coordinates, one can write

$$k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial y^2} + \boldsymbol{S}_r = 0 \tag{4}$$

To obtain the temperature distribution in the medium by solving equation (4) it is necessary to relate $\nabla.\boldsymbol{q}_r$ to the temperature radiation distribution within the medium. The local divergence of the radiative flux is related to the local intensities by

$$\nabla.\boldsymbol{q}_r = \kappa\,[4\pi \boldsymbol{I}_b(\boldsymbol{r}) - \int_{\Omega=4\pi} \boldsymbol{I}(\boldsymbol{r},\boldsymbol{\Omega})d\,\boldsymbol{\Omega}] \tag{5}$$

To obtain the radiation intensity field and $\nabla.\boldsymbol{q}_r$ it is necessary to solve the radiative transport equation (RTE). The RTE for an absorbing, emitting gray gas medium with isotropic scattering can be written as in Siegel and Howell (1992),

$$\left(\Omega.\nabla\right)I(r,\Omega)=-(\kappa+\sigma)I(r,\Omega)+\frac{\sigma}{4\pi}\int_{4\pi}I(r,\Omega')d\,\Omega'+\kappa I_b(r) \tag{6}$$

where $I(r,\Omega)$ is the radiation intensity in r, and in the direction Ω; $I_b(r)$, is the radiation intensity of the blackbody in the position r and at the temperature of the medium; κ and σ are the gray medium absorption and scattering coefficients; and the integration is in the incident direction Ω'.

For diffusely reflecting surfaces the radiative boundary condition for equation (6) is

$$I(r,\Omega)=\varepsilon I_b(r)+\frac{\rho}{\pi}\int_{n.\Omega'<0}|n.\Omega'|\,I(r,\Omega')d\,\Omega' \tag{7}$$

where r belongs to the boundary surface Γ, and equation (7) applies for $n.\Omega>0$. $I(r,\Omega)$ is the radiation intensity leaving the surface at the boundary condition, ε is the surface emissivity, ρ is the surface reflectivity and n is the unit vector normal to the boundary surface.

In the method of discrete ordinates the equation of radiation transport is substituted by a set of M discrete equations for a finite number of directions Ω_m, and each integral is substituted by a quadrature series of the form,

$$\left(\Omega_m.\nabla\right)I(r,\Omega_m)=-\beta I(r,\Omega_m)+\frac{\sigma}{4\pi}\sum_{k=1}^{M}w_k I(r,\Omega_k)+\kappa I_b(r) \tag{8}$$

where w_k are the ordinates weight. This angular approximation transforms the original equation into a set of coupled differential equations, with $\beta=(\kappa+\sigma)$ as the extinction coefficient. Equation (5) in discrete ordinates is,

$$\nabla.q_r=4\pi\kappa I_b(r)-\sum_{m=1}^{M}w_m\kappa I_m \tag{9}$$

Let,

$$S_m=\frac{\sigma}{4\pi}\sum_{k=1}^{M}w_k I(r,\Omega_k) \tag{10}$$

where S_m represents the entering scattering source term. The two-dimensional radiative transport equation in the m direction for an emitting, absorbing and scattering medium is

$$\mu_m\frac{dI_m}{dx}+\xi_m\frac{dI_m}{dy}=\beta I_m+\kappa I_b+S_m \tag{11}$$

where μ_m,ξ_m, are the directional cosines of Ω_m.

The reflection boundary condition in discrete ordinates can be written as

$$I_m=\varepsilon I_b+\frac{\rho}{\pi}\sum_{\substack{m'\\ \mu'_m<0}}w_{m'}\,|\mu'_m|\,I_{m'};\quad \mu_m>0\ \text{ in } x\in\Gamma \tag{12}$$

$$I_m=\varepsilon I_b+\frac{\rho}{\pi}\sum_{\substack{m'\\ \xi'_m<0}}w_{m'}\,|\xi'_m|\,I_{m'};\quad \xi_m>0\ \text{ for } y\in\Gamma \tag{13}$$

The discretization in finite volumes can be obtained by multiplying equation (11) by $dx.dy$ and integrating over the control volume *(i,j,)*

$$\mu_m A_x(I^m_{i+\frac{1}{2},j}-I^m_{i-\frac{1}{2},j})+\xi_m A_y(I^m_{i,j+\frac{1}{2}}-I^m_{i,j-\frac{1}{2}})=V_{i,j}(\kappa I_b-\beta I^m)_{i,j}+S_m \tag{14}$$

where $V_{i,j}$ is the control volume i,j in m^3 .

Assuming that the boundary conditions are given, the system of equations is closed and defines an interpolation system relating the intensities at the face to the nodal values.

3. Method of solution of RTE

In the present work the CLAM scheme of Jessee and Fiveland (1997) and the multidimensional scheme (GM scheme) of Balsara (2001) are used. In the single direction scheme CLAM, the interpolation in a given face involves three grid nodes and can be represented by three-point scheme. The multidimensional non-linear HR scheme of Balsara (2001) so-called genuinely multidimensional (GM), which was used in previous work (Ismail and Salinas, 2004) in radiative transport is used here in combined modes of heat transfer.

To discretize the radiative transport equation, one can rewrite equation (14) based upon the method outlined in Ismail and Salinas (2004) as,

$$(I_{i,j}^{m})^{n+1} = \frac{V_{i,j}(\kappa I_{bi,j})^{n} + |\mu_m| A_x (I_{i-\frac{1}{2},j}^{m})^{n+1} + |\xi_m| A_y (I_{i,j-\frac{1}{2}}^{m})^{n+1} + S_m^n + S_{df}^n}{|\mu_m| A_x + |\xi_m| A_y + \beta V_{i,j}} \tag{15}$$

where

$$S_{df}^{n} = |\mu_m| A_x (I_{i,j}^{m} - I_{i+\frac{1}{2},j}^{m})^{n} + |\xi_m| A_y (I_{i,j}^{m} - I_{i,j+\frac{1}{2}}^{m})^{n} \tag{16}$$

4. Validation of the radiative model

Several numerical experiments were realized to ensure that the algorithm does not have any directional march error or negative intensities. Tests with algorithm are realized and were validated by comparison with benchmark solutions, Crosbie *et al.* (1984). Also the numerical experiments were used to determine adequate value for the over-relaxation factor to accelerate the convergence. For comparison and validation of the model, two test cases having exact solutions were explored as in (Ismail and Salinas, 2004). One test case presented here is of pure absorption equivalent to the case of pure scattering. The medium is absorbing and emitting with three black cold walls while the top wall is black hot with diffuse emission $I_D = 1$. The results of figure (1) illustrate the convergence of the numerical method. To study the behavior of the converged solution for different quadratures S_n and T_n , the results of figure (2), show that for higher order quadrature the solution is smooth and the comparison with the exact solution of Crosbie *et al.* (1984), indicates good agreement.

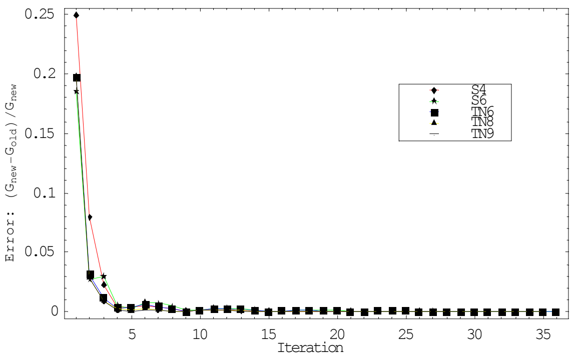

Fig. 1. Convergence for different quadratures.

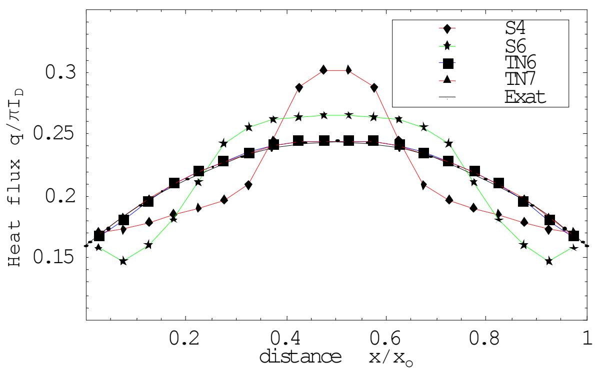

Fig. 2. Comparison of predicted heat flux at the bottom cold wall obtained by S_N and T_N quadratures for the 20x20 grid, isotropic scattering case, $\beta_{x/y}$=1.0 .

5. Method of solution of radiative conductive model

To solve the coupled radiation conduction problem, equations (2) and (5) are converted to non-dimensional form, as in Kim *et. al*. (1991),

$$\frac{1}{\tau_{L_x}^{2}} \frac{\partial^2 \Theta(r)}{\partial X^2} + \frac{1}{\tau_{L_y}^{2}} \frac{\partial^2 \Theta(r)}{\partial Y^2} = \frac{1-\omega}{N} \left(\Theta^4(r) - \frac{1}{4} \sum_{m=1}^{M} W_m G_m \right) \tag{17}$$

With the boundary conditions,

$$\Theta\,(0,\,Y) = 1;\;\; \Theta\,(1,\,Y) = 0{,}5;\; \Theta\,(X,\,0) = \Theta\,(X,\,1) = 0{,}5 \tag{18}$$

The parameters and non-dimensional variables are,

$X = \beta x / \tau_{Lx}$, $Y = \beta x / \tau_{Ly}$, $\tau_{Lx} = \beta L_x$, $\tau_{Ly} = \beta L_y$, $\omega = \sigma / \beta$, $N = k\,\beta / 4\sigma_o\,T_h^{\,3}$, $\Theta = T / T_h$, $G = I / \sigma_o\,T_h^{\,4}$

where $\boldsymbol{G}$ is the non-dimensional incident radiation intensity.

Let, the source term

$$\boldsymbol{S}_r = 1 - \omega\left(\Theta^{4}(\boldsymbol{r}) - \frac{1}{4}\sum_{m=1}^{M} W_m \boldsymbol{G}_m\right) \tag{19}$$

Then the energy equation is

$$\frac{1}{\tau_{L_x}^{2}}\frac{\partial^2\Theta(\boldsymbol{r})}{\partial \boldsymbol{X}^2} + \frac{1}{\tau_{L_y}^{2}}\frac{\partial^2\Theta(\boldsymbol{r})}{\partial \boldsymbol{Y}^2} = \frac{\boldsymbol{S}_r}{\boldsymbol{N}} \tag{20}$$

One can identify the energy equation of the problem as the Fourier conduction equation with one radiative source term. This equation is solved using the control volume method, in Patankar (1980).

The source term linearization is obtained by

$$\boldsymbol{S}_r = \boldsymbol{S}_C + \boldsymbol{S}_P\,T_P \tag{21}$$

$\boldsymbol{S}_P = 0$ and $\boldsymbol{S}_C = \boldsymbol{S}_r\left(T_P^{\,*}\right)$ were used and the symbol $T_{\boldsymbol{P}}^{\,*}$ denotes the previous iteration value of T_P. Due the non-linearities in the discretization equations which may cause large changes in the predicted solutions and produce oscillations and/or divergence in the iteration process, the relaxation technique (Patankar, 1980) in the dependent variable was used. The procedure of the numerical calculations starts by assuming that the radiative source term is zero and the conductive equation is solved to find the temperature field. The ADI line-by-line method is used to quickly bring the information from all boundaries to the interior. After, the radiative transport equation is solved using DOM until convergence of intensities field is obtained, then the radiative source term is calculated and the conductive equation is solved including the source term only once for the global iterative process to permit faster convergence. The iterative process continues until achieving convergence of the intensities radiative field and the temperature field. By this procedure, it was no necessary to set some chosen temperature at the corners of the enclosure in the hot wall side, for example $T=0.75T_h$ as it is reported in Rousse et. al. (2000).

The convergence of the solution was evaluated using a convergence criterion taken as the error in the intensity field in RTE solution and the error in the temperature field respectively as,

$$errorG = Max\left|\frac{\boldsymbol{G}_{i,j}^{n} - \boldsymbol{G}_{i,j}^{n-1}}{\boldsymbol{G}_{i,j}^{n}}\right| \le 10^{-6} \quad \text{and} \quad errorT = Max\left|\frac{\Theta_{i,j}^{\,n} - \Theta_{i,j}^{\,n-1}}{\Theta_{i,j}^{\,n}}\right| \le 10^{-5} \tag{22}$$

6. Validation of radiative conductive model

A test case of radiation conduction in a square cavity with participating media is solved. Figure (3) shows a simplified scheme of the cavity. Surface 2 is hot and is at temperature T_h; surfaces 1, 3 and 4 are cold at temperature T_c, that is equal to one half of T_h . The media in cavity is gray and the boundaries are black surfaces. The medium is further assumed to absorb and emit radiation, but not scatter radiant energy, $\omega = 0$.

This problem was solved using different methods by (Yuen et. al., 1988, Kim et. al., 1991, Sakami et. al. 1996, Rousse et. al. 2000). The method used in the present work is more simple than the ones cited before. It is used to handle combined heat transfer problems in cartesian space with rectangular grids and can be used to couple with the common volume control algorithms used in fluid dynamics. In this part of the work, the DOM is used with a non-linear high order interpolating scheme called genuinely multidimensional (GM) of Balsara (2001), which was used with rectangular uniform grids in radiative problem (Ismail and Salinas, 2004), and is extended here to use in the radiative-conductive problem. Also the accuracy of different angular quadratures such S_4, Tn_6, and the LC11 angular quadrature is investigated.

The results are shown for several values of Planck number $N = k\beta / 4\sigma_o T_h^3$, where k is the thermal conductivity of the medium and σ_o is the Stefan-Boltzmann constant. The results from (Yuen et. al., 1988, Kim et. al., 1991, Sakami et. al. 1996, Rousse et. al. 2000) show good agreement for $N > 0.01$, wherever for $N \leq 0.01$ the results of Yuen *et al.* (1988), shows no agreement with the other authors.

It was found that the numerical solution of the energy equation needs a relaxation factor (λ) to obtain convergence. This is important for lower *N* values. Figure (4) shows the variation of error for different values of *N* for the case of 20x20 uniform grid. One can observe that for the lowest *N* value the convergence of the algorithm is less quick, meanwhile in all tested situations the algorithm was efficient. The solution convergence for different angular quadratures was also investigated. The quadrature LC11 proposed by Lededev V., in (Koch R. *et. al.*, 2004) and recommended by Koch R. *et. al.* (2004), with 48 directions in 2D is included here with the S_n and T_N quadratures. Figure (5) shows the comparison of the numerical solution for $N = 0.001$, for *Tn6*, *LC11* and *S4* quadratures. One can observe that the agreement between the numerical results for *Tn6* e *LC11* quadratures is good and agree well with results of Yuen *et. al.* (1988) and Sakami *et. al.* (1996), while poor agreement is found for the *S4* numerical results.

Figure (6) shows the temperature distribution in the medium for Planck Number *N*, between 0.001 and 1.0 when using 20x20 uniform grid. The results show good agreement with those of Yuen *et. al.* (1988) Kim *et. al.* (1991) and Sakami *et. al.* (1996). As N decreases , the role of the radiation increases and the energy is transmited deeper into the medium, producing higher temperature gradients at both faces and increasing temperatures near the cold face. Figure (7) shows the isotherms in the medium for the case of $T_c = T_h / 2$ illustrating smooth isotherms curves in all domain.

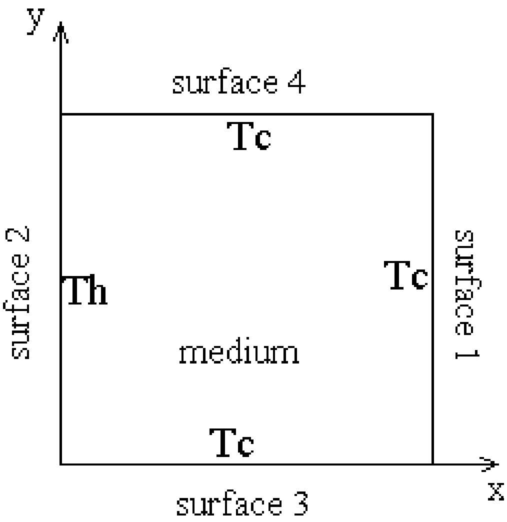

Figure 3 Two-dimensional cavity scheme, $T_c = T_h / 2$

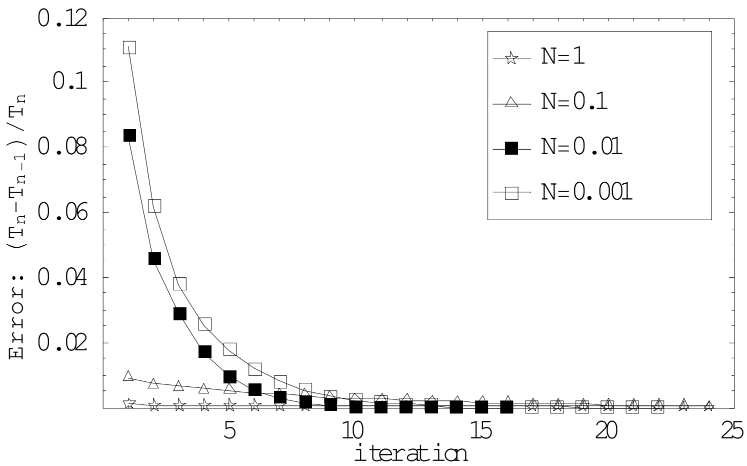

Figure 4 Comparison of the solution convergence for angular quadrature LC11, for N =1.0; 0.1; 0.01; 0.001.

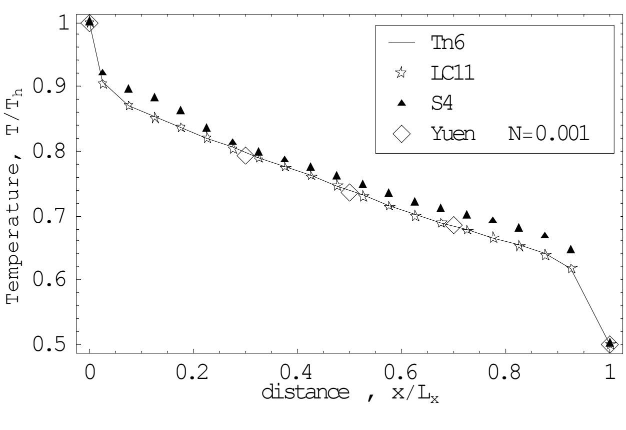

Figure 5 Comparison of solution convergence for different angular quadratures: Tn6, LC11 and S4, for N =0.001 and $\beta x_L = 1.0$.

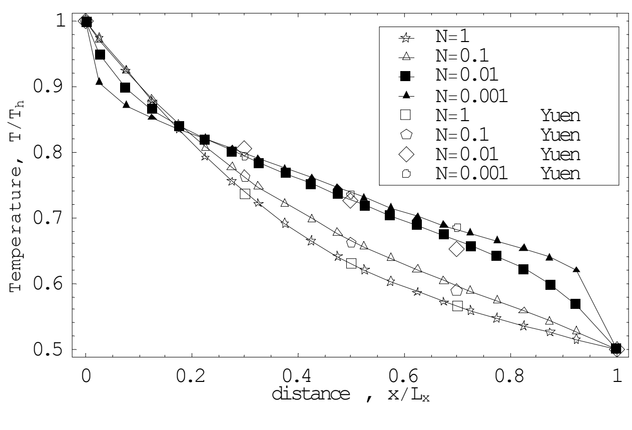

Figure 6 Comparison of dimensionless temperatures distribution at $y = y_L/2$, with the solution of Yuen et. al. (1988), for N =1.0; 0.1; 0.01; 0.001, and $\beta x_L = 1.0$.

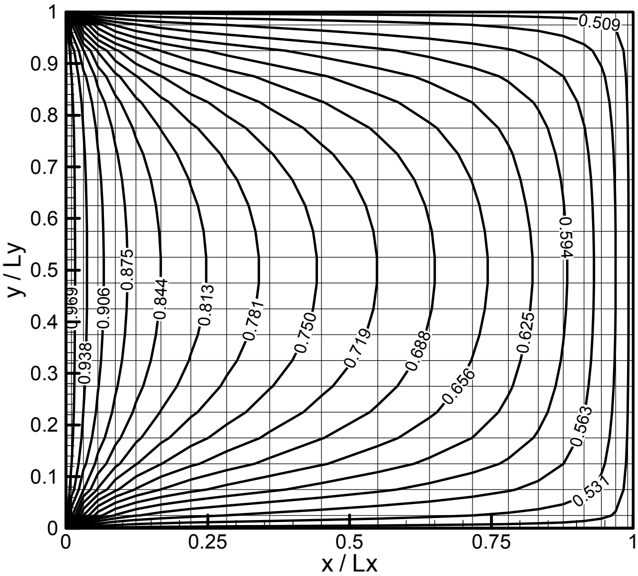

Figure 7 Non-dimensional temperatures distribution in the medium

7. Conclusions

In this study the multidimensional scheme (GM) in the classical discrete ordinates method is found to be suitable for accurate calculations of radiative transfer in 2D cases. The algorithm is used in radiation conduction case and the results are shown to be accurate. For lower values of *N*, it is necessary to use a lower relaxation value. The method used in this work is more simple than other methods cited before and is useful for to handle combined heat transfer problems in cartesian space with rectangular grids and can to used coupled with common volume control algorithms used in fluid dynamics. The use of rectangular grid in this case permits accurate calculation without additional computational time.

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