ROBOT PATH PLANNING: AVOIDING OBSTACLES

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Abstract. *Robot manipulators are programmable articulated mechanisms designed to execute a great variety of tasks in a repetitive way. His conception foresees versatility so that they can be adapted to several operational situations. In various cases the robot trajectory is constrained by the existence of obstacles in its workspace. Obviously, the presence of obstacles imposes additional difficulties because the end-effector as well as the other links of the robot is supposed to avoid the obstacles and, at the same time, to follow specifications of initial and final positioning, as previously define by the user. The present contribution is devoted to solve the problem of path planning and obstacle avoidance for 3D robot systems. Computational results demonstrate the applicability and efficiency of the methodology proposed.*

Keywords: *Robot manipulator, path planning, obstacle avoidance.*

1. Introduction

The simplest path planning consists in the determination of the movement of the end-effector of a robot from an initial to a final point. In this case the robot is driven from an initial to a final configuration without any concern about the intermediate points along the path.

In the context of real life applications, aiming at providing a more precise control for the movement, a preliminary planning (off-line calculation) should be done, including the analysis of kinematical and dynamical characteristics of the system along the movement.

Among the most important objectives of industry automation is to reduce production costs and to increase productivity. Consequently, to justify the use of robotic systems it is of great importance to consider both the planning and the optimization of the trajectories to be executed. Trajectory in this case is understood as the set of points that define the translation and rotation movement of robot manipulators. These points are written in joint coordinates by using kinematics transformation matrices. As repetitive processes are found in a variety of robotic applications in industry (inspection, welding, painting, assembling, pick-and-place operations, etc), off-line trajectory planning is justified.

The interpolation can be achieved through polynomial equations or, alternatively, by using more sophisticated techniques, such as *spline* functions. Trajectory interpolation allows the analysis of the geometrical characteristics of the movement and it is necessary to obtain information concerning kinematical and dynamical aspects of the movement.

Continuity conditions are written for the interpolating polynomial to guarantee a smooth movement for the manipulator, by using *spline* functions as described by cubic polynomial equations (Steffen and Saramago, 1997).

Moreover, it is important to point out that practical restrictions may avoid the effective implementation of a trajectory. For example, the movement that demand high torque in the joints should be not implemented because the actuators are in general limited to low torque. It is also desired that an acceptable amount of energy should be demanded due to technical and economical reasons. Also, under certain circumstances the velocity of the end-effector should be limited to preserve the integrity or safety of the manipulation process. Finally, the total time of the operation is a matter of concern because industrial tasks should be executed in the minimal possible time.

The present contribution proposes a new strategy to determine the trajectory for a required movement taking into account the requirements of torque, velocity, operation time and robot positioning along the movement.

2. Trajectory

Given the initial and final positions for a given task, it is necessary to determine a smooth trajectory in the joint space. The trajectory is defined according to the history of positions, velocities and accelerations of each joint, as functions of time.

First, n joints and their respective joint angles are defined q_i , $i=1, \ldots n$. Then, the interpolating *spline*-type polynomial $P_i(t)$ for each joint i, such that position specifications are fulfilled, is determined. This procedure requires the initial and final times t_0 and t_f , the initial and final position vectors $q_i(t_0)$ and $q_i(t_f)$, the initial and final velocities $\dot{q}_i(t_0)$ and $\dot{q}_i(t_f)$, respectively. The joint constraints are given by:

$$P_i(t_0) = q_i(t_0) \text{ and } P_i(t_f) = q_i(t_f) \text{ ,} \tag{1}$$

and velocity specifications are given by:

$$\frac{\partial P_i}{\partial q_i}(t_0) = \dot{q}_i(t_0) \text{ and } \frac{\partial P_i}{\partial q_i}(t_f) = \dot{q}_i(t_f) \text{ .} \tag{2}$$

The trajectory obtained fulfills smoothness requirements, however it does not necessarily represent an optimal configuration of the system, according to other considered criteria.

Then, m intermediate positions $q_{ij}(t)$ are defined for each joint $i=1,...n$, such that $q_i(t_0) < q_{ij}(t_t) < q_i(t_f)$ and $P_i(t_j)=q_{ij}(t_j)$, where $j=1,...m$ and $t_0 < t_j < t_f$. This means that intermediate positions of joint coordinates are defined such that they correspond to the value of the respective interpolating polynomial along m arbitrary time instants between the initial and final times.

The initial value of the intermediate positions can be easily obtained from $q_{ij}(t_j)=P_i(t_j)$.

The modification of the values corresponding to q_{ij} and t_j subjected to the interpolation scheme that satisfies Eq. (1) and Eq. (2), should now also satisfy Eq. (3):

$$P_i(t_j) = q_{ij}(t_j),\ i=1,...n,\ \ j=1,...m. \tag{3}$$

resulting a new trajectory for the robot, with new velocities, accelerations and torques associated.

The variation of the intermediate positions and their respective associated time instants for a given robot configuration as expressed in joint coordinates are used as design variables in the optimal design of the system.

3. Performance criterion

The optimization goal requires the definition of a performance criterion that incorporates in a single scalar objective function to be minimized, the energy, velocity and total time to move the robot from predefined initial and final positions, taking into account the presence of obstacles in the design space. In the following, all the necessary steps to construct the multi-criterion objective function, namely the global performance criterion, are explained.

3.1 Obstacles

Obstacle position is specified by a set of Cartesian coordinates $P_{obs,i}(x, y, z),\ i=1,...k_1$. An objective function is written by expressing the distance between the points that characterize the robot and those from each obstacle. This way, it is possible to analyze the motion executed by the manipulator as a whole (not only the end-effector, because obstacle avoidance is mandatory for all parts of the robot). $P_{man,j}(x,y,z),\ j=1,...k_2$ are points belonging to the manipulator contour, as obtained from its model discretization. The minimal Euclidian distance, $d_{min,obs}$, between a manipulator point $P_{man,j}$ and the point $P_{obs,i}$ belonging to each obstacle is defined as $d_{\min,obs} = \min_j | P_{obs,i} - P_{man,j} |$.

The following objective function is defined so that the simultaneous presence of several obstacles can be considered:

$$f_1 = \sum_j \left(\frac{1}{d_{\min,j}} \right)^2 \tag{4}$$

Observe that the denominator represents the smallest distance between a point of the manipulator contour and each obstacle (index j).

The above strategy is very efficient to detect possible collision, because in the case for which a manipulator point coincides with the obstacle, the denominator of Eq. (4) vanishes. Consequently the performance index tends to infinity. This criterion is similar to the one proposed by (Khatib and Le Maitre, 1978). However, they have considered the volume of the obstacle to be avoided in the optimization process. The proposed methodology can be used to consider the obstacle contour (volume) by taking into account a set of points that represent the desired contour. The technique can be repeated in the case that many obstacles are found in the workspace.

3.2 Total time

The total time of the operation is frequently considered to analyze the performance of robot manipulators (Constantinescu and Croft, 2000), (Bobrow, Dubowsky and Gibson, 1985), (Shiller and Lu, 1992). The total time can be minimized by simply minimizing the difference between the initial time t_i and the final time t_f, as represented by:

$$f_2 = (t_f - t_i)^2,\ \ t_f > t_i. \tag{5}$$

However, it should be pointed out that an arbitrarily small time interval results mechanical requirements that are impossible to be fulfilled by the manipulator. Besides, small time intervals may lead to collision, due to movement discontinuity. By including the total time together with other objective functions to form a single scalar performance index without any constraint function, makes the present methodology different from similar techniques.

3.3 Torque and mechanical power

The Lagrangian $L = K - P$ is defined by the difference between the kinetic energy *K* and the potential energy *P* of the system. The dynamics of the system can be described by the Lagrange equation as given by:

$$u_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} \tag{6}$$

where, q_i are the generalized coordinates (q_i for rotational joints and d_i for prismatic joints); $\dot{q}_i$ are the generalized velocities (angular velocity $\dot{q}_i$ for rotational joints and $\dot{d}_i$ for prismatic joints); and u_i are the generalized forces.

By using Eq. (6), the generalized forces u_i can be written as:

$$u_i = \sum_{j=1}^{n} D_{ij}\ddot{q}_j + I_{ai}\ddot{q}_i + \sum_{i=1}^{n}\sum_{k=1}^{j} C_{ijk}\dot{q}_j\dot{q}_k + G_i \tag{7}$$

where, D_{ij} is the inertia matrix of the system; C_{ijk} is the matrix that takes into account Coriolis and centrifugal effects; I_{ai} represent the inertia of the actuators; and G_i is the vector of gravitational forces.

The energy that is necessary to move the robot is an important design issue, because in real applications energy supply is limited and any energy reduction leads to smaller operation costs. Due to the relationship that exists between energy and force, the minimal energy can be estimated from the generalized force $u_i(t)$ that is associated to each joint *i* at time instant *t*. In the present contribution, the mechanical power is used for design purposes, as defined by:

$$f_3 = \int_{t_0}^{t_f} \sum_{i=1}^{n} \left[u_i^T(t)\,\dot{q}_i(t)\right]^2 dt \tag{8}$$

This expression is very representative about the phenomenon under study because it considers both the kinematical and dynamical aspects of the trajectory, simultaneously (Saramago and Steffen, 1998a) and (Saramago e Steffen, 1998b).

Following the methodology proposed, it was observed that by using a limited number of intermediate reference points for interpolation purposes, the resulting trajectory corresponds to robot configurations that are associated with the minimum mechanical energy with small variations of the torque along the trajectory. This means that it is not necessary to explicitly include a constraint for the maximum torque value in the formulation.

3.4 End-Effector Positioning

In the case the final Cartesian configuration specified can be obtained through various combinations of joint coordinates, i.e., the inverse kinematics problems have more than one solution, it should be chosen the configuration that best fulfills the optimality criteria presented before. In the present contribution, the final configuration $q_i(t_f)$ is not fixed; it can be chosen among the possible solutions for the inverse kinematics problem.

If the joint coordinate $q(t_f)$ satisfies the required positioning for the end-effector, then $T.p_0 = p_1$. This can be represented by the equation:

$$f_4 = \left| T_0^n(q(t_f)).p_0 - p_1 \right|^2 \tag{9}$$

that depends on the design variable $q(t_f)$. The specified positioning will be satisfied for all configurations that correspond to $f_4 = 0$.

4. Otimization Strategy

Each objective function defined above characterizes a separate optimization problem, resulting a distinct optimal value for the design variables. In general, each optimal solution does not coincide with the other solutions when the objective function is altered. This means that the minimization of a separate objective function does not guarantee that the other objective functions assume minimal values for the optimal robot configuration obtained for that separate objective function.

Consequently, a multi-criterion optimization problem is defined, so that it is possible to minimize several objective functions simultaneously. The final result is supposed to take into account conflict situations, by considering the Pareto optimal concept (Eschenauer, Koski, and Osyczka, 1990). In the present contribution the total traveling time, the positioning of the end-effector and kinematical and dynamical characteristics represent conflict aspects.

To formulate the performance criterion that takes into account all the objective functions in such a way that an overall multi-criterion objective function can be written, the Weighting Objective Method is used. The minimization process leads to a Pareto optimal solution or, alternatively, to a set of optimal solutions. The scalar objective function that represents the performance criteria altogether is written as:

$$f(x) = \sum_{i=1}^{k} a_i f_i(x) \tag{10}$$

where $a_i = 0$ are weighting coefficients that represent the relative importance of each separate criterion. From the numerical point of view the minimization process depends also on the numerical values that express the objective functions. Due to scaling problems, the numerical values that express the objective functions should be adjusted. Otherwise, a_i will not represent the relative importance of the objective functions (Deb, 2001). Consequently, Eq. (10) should be rewritten as follows:

$$f(x) = \sum_{i=1}^{k} c_i f_i(x) \tag{11}$$

where c_i are scaling factors. Usually, satisfactory results are obtained if $c_i = \frac{a_i}{f_i^0}$, where f_i^0 represents the minimum of the objective function f_i calculated separately (Oliveira e Saramago, 2004). Eq. (11) was used in the optimization processes shown in this paper.

5. Numerical Application

The following numerical experiments were developed in a microcomputer Pentium 4, 2.4 GHz, using Matlab ®.

The continuity of the movement between the initial ($t_0 = 0$) and the final time ($t_f = 10s$) is simulated through the discretization of the interval. For that purpose, 50 equally spaced intermediate points were considered. This way, both the possibility of collision between the robot manipulator and the obstacle, and the determination of kinematical and dynamical behavior of the system were determined at these discrete time instants. A larger number of points can generate more accurate results at the expense of higher computational costs.

In the present case, the optimization algorithm is based on the BFGS Quasi-Newton method (Vanderplaats, 1999).

5.1 Two link planar manipulator

5.1.1 Unrestricted workspace (obstacle free)

Table 1 shows the parameters of Denavit-Hartenberg for a two link planar manipulator. The mass of each link is 10 *Kg*, the rotations are limited to ± 1,700 *rad* for the first joint and ± 2,967 *rad* for the second joint. The moment of inertia of a cylindrical bar was used to represent the links, so that $I_x = 1/2\ mR^2$, $I_y = I_z = (1/12)m(3R^2 + L^2)$, where $R = 0.27\ m$ and $L = 1.0m$ (L corresponds to the parameter ***a*** in Tab. 1).

Table 1. Denavit-Hartenberg parameters.

Link	a (m)	α (rad)	d (m)	θ (rad)
1	1	0	0	θ_1
2	1	0	0	θ_2

The initial and final points were defined as $x=1.99m$, $y=0$, $z=0$, and $x=0$, $y=1.99m$, $z=0$, respectively. The movement was determined by using cubic interpolation between the values of the joint coordinates of the initial and final positions. This procedure leads to the determination of both the mechanical power and the total traveling time, which are used as f_i^0 parameters in Eq. (11). We consider that all objective functions are equally weighted ($\alpha_i = 1$).

5.1.2 First optimization test: obstacle avoidance

The most important path-planning criterion for a manipulator evolving in a restricted environment is to avoid obstacles in such a way that the resulting trajectory is feasible. It can be observed that if an obstacle is placed at the point given by $x = 1.75m$, $y = 1.0m$ and $z = 0.0$ the movement obtained in the previous analysis becomes unfeasible. Besides, another obstacle was included in $x=1.75m$, $y = -0.5m$ and $z = 0.0$, corresponding to a point that belongs to the trajectory of the manipulator for the case in which it avoids the first obstacle considered.

The multi-criterion optimization problem was defined according to Eq. (11). The obstacles are taken into account by using Eq. (4); the total traveling time is included by considering Eq. (5), and the final end-effector positioning is included by Eq. (9). The optimal trajectory is shown in Fig. 1.

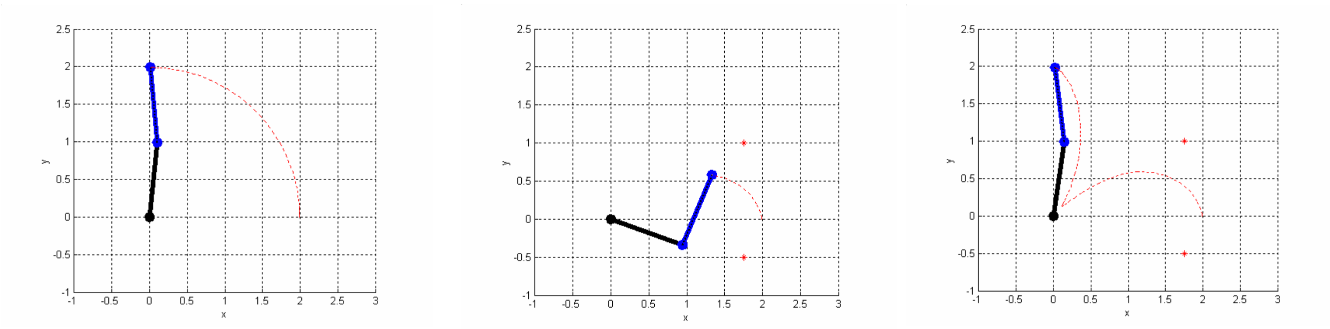


Figure 1. Trajectory in the unrestricted and restricted workspace

By comparing the characteristics of the obtained trajectory and the initial one, it is possible to see that the total traveling time increased from 10s to 11.9s. Similarly, the mechanical power increased from 304.71W to 352.94W.

It is also possible to conclude that the methodology presented leads to a collision-free trajectory. In this case, an increase both in the total traveling time and in the mechanical power required to accomplish the movement, as in most the cases analyzed. Then, the following question should be asked: is there the possibility to avoid collision and reduce time and mechanical power at the same time?

5.1.3 Second optimization test: mechanical power

Eq. (11) is now rewritten to include all the considered objective functions, i.e., obstacle avoidance - Eq. (4), total traveling time - Eq. (5), end-effector final positioning – Eq. (9) and mechanical power – Eq. (8). The optimal trajectory is presented in Fig. 2 for three different times.

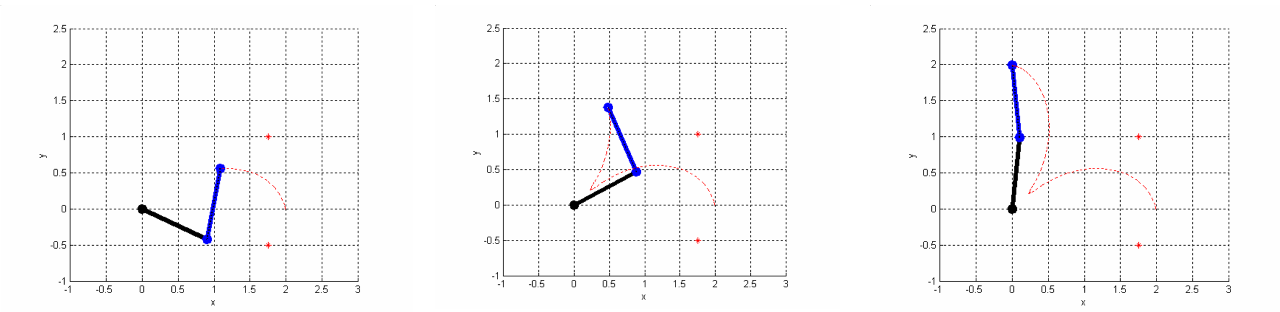


Figure 2. Optimal trajectory along the restricted workspace (minimal power included)

The optimal values for the optimization tests are shown in Table 2. It is possible to observe that the total traveling time obtained for the second optimization test was increased 65.1% with respect to the initial trajectory. However, the corresponding mechanical power reduced to 69.6% with respect to the initial trajectory. By manipulating the parameter c_i in Eq. (11) it is possible to obtain other optimal configurations that can be evaluated for specific design interests.

Table 2. Optimal values for each objective function corresponding to the optimal performance index

Parameter	*Initial trajectory*	*1st optimization test*	*2nd optimization test*
Total time (sec)	10.00	11.92	16.51
Mechanical power (watts)	304.71	352.94	254.38
Obstacle avoidance index	3.38	1.84	1.94
CPU time (sec)	1	81	83

In this analysis a single intermediate control point was used for the determination of the joint coordinates. It should be studied the case for which a larger number of intermediate points are taken into account.

5.1.4 Third optimization test: including another intermediate control point

In the following, two intermediate control points were used in the determination of the joint coordinates that describe the position of the robot manipulator with respect to time. The ordinates of the control points are obtained by computing a cubic spline from the initial to the final joint coordinates and getting their respective joint angles, whose abscissas are given by 1/3 ($t_f - t_i$) and 2/3 ($t_f - t_i$), respectively. These abscissas and ordinates define the design variables for the optimization process.

By keeping the same parameters as in the previous case, the resulting optimal trajectory is shown in Fig. 3 for three different times.

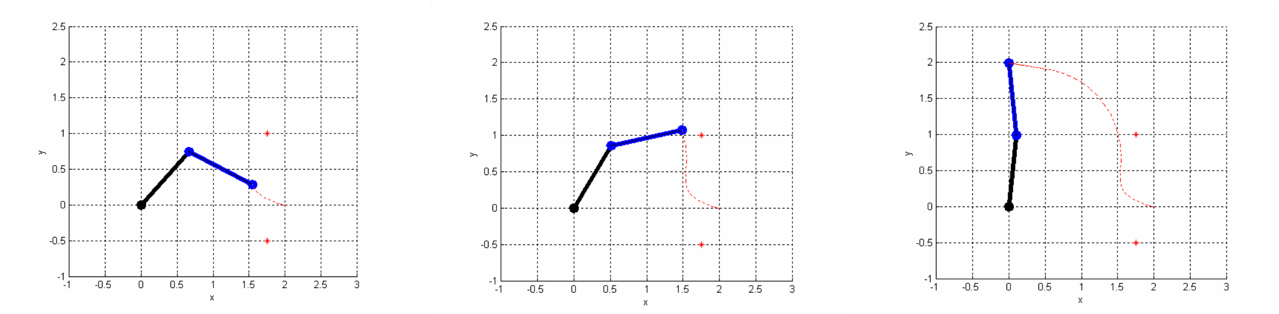


Figure 3. Trajectory in the restricted workspace (2 control points)

The optimal values obtained for the optimization tests are shown in Tab. 3.

Table 3. Optimal values for each objective function corresponding to the optimal performance index

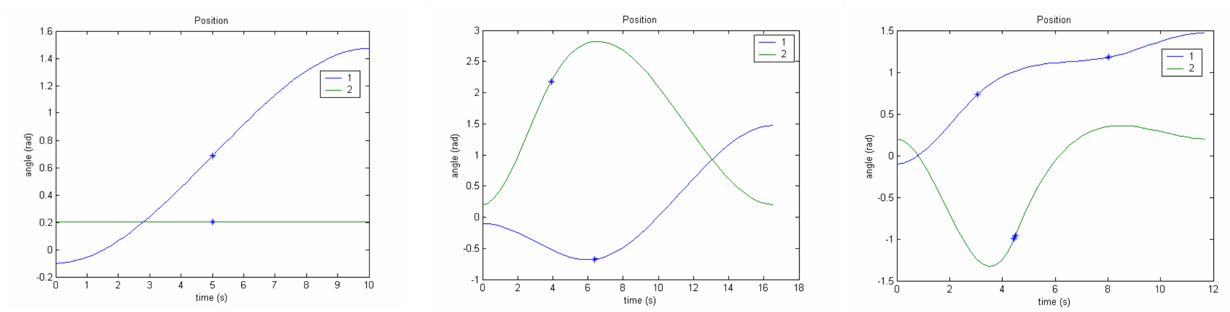
Parameter	*Initial trajectory*	*1st optimization test*	*2nd optimization test*	*3rd optimization test*
Total time (sec)	10.00	11.92	16.51	11.69
Mechanical power (watts)	304.71	352.94	254.38	219.77
Obstacle avoidance index	3.38	1.84	1.94	3.03
CPU time (sec)	1	81	83	129

In this case a 16.9% increase was observed for the total traveling time with respect to the value found for the initial trajectory. The mechanical power was reduced to 72% of the initial value. The optimal values for the objective functions that participate of the performance criterion are better than those obtained for the case in which a single intermediate control point was used. This behavior is due to the fact that a larger number of intermediate control points leads to less smooth trajectories. A large number of intermediate control points may lead to unacceptable trajectories from the smoothness point of view.

In Tab. 2 and Tab. 3 the value of the obstacle avoidance index represents a proximity parameter of the robot contour with respect to the obstacles. Larger this index, the larger is the distance between the robot contour and the obstacles.

By analyzing the results shown in Tab. 3, it is possible to conclude that smaller values for the objective functions are obtained when the obstacle avoidance index is also small (the robot manipulator passes close to the obstacles when describing its trajectory).

Figure 4 shows the initial and optimal trajectories in joint space coordinates for the two joints of the two-link planar manipulator. These trajectories were determined in such a way that initial and final velocities are zero, i.e., $\dot{q}_i(t_0) = \dot{q}_i(t_f) = 0$.



(a) initial configuration (b) optimal trajectory-2nd opt. test (c) optimal trajectory-3nd opt. test

Figure 4. Joint space configuration

5.2 Elbow manipulator

The following application is devoted to a three dimensional robot, namely the Elbow manipulator (three rotational joints). In this application the mass of each link is *1.0 kg* and each rotational angle is limited to ± *3.0 rad*. We have used the moment of inertia of the cylindrical bar to represent the dynamics of the links, i.e., $I_x = 1/2\ mR^2$, $I_y = I_z = (1/12)m(3R^2 + L^2)$, where $R = 0.27\ m$. The parameters of Denavit-Hartenberg for the Elbow manipulator are shown in Tab. 4.

Table 4. Denavit-Hartenberg parameters

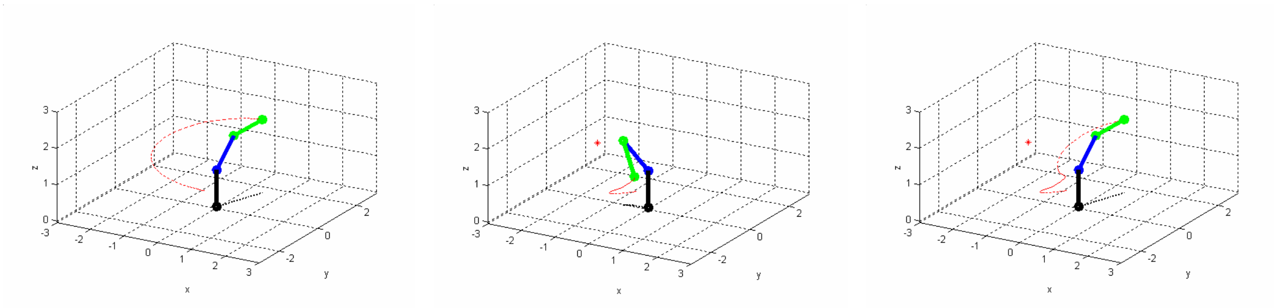
Link	a (m)	α (rad)	d (m)	θ (rad)
1	0	1.57	1	θ_1
2	1	0	0	θ_2
3	1	0	0	θ_3

The initial and final points are given by *x=0.5m, y=-1.5m, z=1.0m* and *x=0.5m, y=1.5m, z=2.0m*, respectively. The initial trajectory was obtained by performing a cubic spline interpolation between the initial and final points in joint coordinates. The corresponding values for mechanical power and total traveling time (7 sec) were used as f_i^0 parameters to formulate the performance index given by Eq. (11). The same priority is given for all the objective functions ($a_i = 1$). Tab. 5 gives the optimal values for the optimization tests obtained for the Elbow manipulator.

Table 5. Optimal values for each objective function corresponding to the optimal performance index

Parameter	*Initial trajectory*	*1st optimization test*	*2nd optimization test*	*3rd optimization test*
Total time (sec)	7.00	7.40	9.11	7.07
Mechanical power (watts)	101.22	144.39	75.35	75.97
Obstacle avoidance index	17.69	17.69	12.73	17.08
CPU time (sec)	25	251	259	403

In this application it can be observed a decrease in the mechanical power and an increase in the total traveling time in the case that a single intermediate control point is used (2nd optimization test), while a small increase in the time (1%) and a significant mechanical power reduction (to approximately 75%) were obtained for the case in which two intermediate control points were used (3rd optimization test). The resulting optimal trajectory is shown in Fig. 5 for two different times: cases (b) and (c). Case (a) corresponds to the obstacle-free workspace.



(a) (b) (c)

Figure 5. Initial trajectory and optimal trajectories for two different times

6. Conclusion

This contribution presented a methodology to obtain optimal trajectories for serial robot manipulators. The main characteristic of the proposed path planning formulation as an optimization problem is the definition of interpolation control points in the joint space as design variables. This approach allowed the simultaneous analysis of several objective functions, keeping track of movement smoothness. For that purpose, the objective functions were used to construct a scalar performance index to be minimized by considering conflict situations. The user according to specific applications can alter weighting factors and scaling factors.

The methodology performed satisfactorily in the cases of two and three-dimensional obstacle avoidance path planning applications.

Each time an obstacle has to be avoided the corresponding total traveling time and mechanical power values are increased. The physical insight in this case is that energy and time are consumed to avoid the obstacles. For this reason the performance criterion included total traveling time, end-effector positioning and mechanical power aiming at representing the dynamics of the system.

In the third optimization test, two intermediate control points were considered in the joint space. This procedure gives more freedom to the algorithm to obtain optimal trajectories and helps to minimize the associated mechanical power. However, it should be pointed out that the designer could not increase the number of intermediate control points arbitrarily, because this may lead to movement discontinuity or to non-smooth trajectories.

In the applications presented, the initial total traveling time was considered as an ideal time to fulfill a given robot task. This means that by increasing this time of a small value, it is possible to obtain better performance indexes. Another possibility would be to consider the total traveling time as a constraint. In that case, the algorithm would not consider total traveling times beyond the corresponding side constraints. In the present methodology the designer can consider the possibility of increasing or decreasing the reference value of the total traveling time for a given application and check if this procedure leads to better optimal results.

Also, it should be pointed out that the present methodology considers the possibility of collision for the entire manipulator (robot contour), not only for the end-effector. The final end-effector positional errors were lower than 0.001m for all cases studied.

The authors are encouraged to use the developed methodology for more complex path-planning applications of robot manipulators.

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