ON THE HYPOTHESIS AND LIMITATIONS OF TRADITIONAL METHODOLOGIES FOR MECHANICAL DESIGN OF A FILLET WELDED SIMPLE STRUCTURE

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Abstract. *Design methodologies for fillet welds, subjected to static loadings, using simple analytic models are widely available in textbooks of Mechanical Engineering Design. Some hypotheses are common for almost all analytic models: (1) the welds are homogeneous, isotropic and elastic; (2) the parts connected by welds are rigid; (3) only nominal stresses are considered (residual stresses and stress concentration are neglected). These methodologies are based in different concepts, with conservative approach and simplicity of utilization as common points. In this work a fillet welded structure submitted to a combined static loading is analyzed using two analytic models and two numerical models. The first analytic model is representative of the majority of methodologies present in textbooks of Mechanical Engineering Design. The second analytic model is based on traditional hypothesis of Strength of Materials. Finally, the numerical models are based on the Finite Element Method and consider elastic and elastoplastic behaviors. The stress predicted by the four models are compared and analyzed. The elastoplastic finite element model is used as a reference. It is considered that it furnishes a more precise analysis of the stresses present at the fillet weld of the welded structure, as it considers stress concentration effects and the stress redistribution effects promoted by the development of plastic strain.*

Keywords*: Finite Element Method, Mechanical Engineering Design*, *Modeling*, *Welded Components.*

1. Introduction

Design methodologies for fillet welds, subjected to static loadings, are widely available in textbooks of Mechanical Engineering Design (Ballio, 1983; Bresler, 1968; Shigley and Mischke, 2001; Faires, 1986). These design methodologies, called *regular* models in this work, have the main objective of furnish to the designer a simple and conservative tool to design simple structures with fillet welds. Some hypothesis established are common for almost all analytic models (Bresler, 1968): (1) the welds are homogeneous, isotropic and elastic; (2) the parts connected by welds are rigid; (3) only nominal stresses are considered (residual stresses and stress concentration are neglected).

It is well known that residual stress plays a preponderant part on the structural integrity of a mechanical component, especially in nucleation and propagation of cracks. Residual stresses result directly from the thermal cycle caused by the localized intensive heat input that promotes temperature gradients. High temperatures developed by the heat source promote phase transformation and plasticity. Mechanical properties present lower values at higher temperatures allowing the development considerable plastic strain. The temperature gradients developed through the piece results in a nonhomogeneous plastic strain distribution, which promotes residual stresses fields when the piece reaches room temperature. Due to the importance of estimate residual stresses in welding, several investigators had addressed this subject (Bang *et al.*, 2002; Fernandes *et al.*, 2003, 2004; Teng and Chang, 2004).

In this work some hypothesis normally used by traditional design methodologies are addressed. The presence of stress concentrations, plasticity and the form that the structure is attached is considered in the analysis developed. First, a brief description of a *regular* model presented by a classical reference book of Mechanical Engineering Design (Shigley and Mischke, 2001) is depicted. Also, an analytic model *based on Strength of Materials* is proposed, incorporating many of the aspects that are usually simplified in regular models, which increases its complexity. Finally, two numerical tri-dimensional finite element (FEM) models that consider elastic and elastoplastic behaviors are proposed. The elastoplastic FEM model is used as a reference, as it furnishes a more precise analysis of the stresses present in the mechanical component. A comparative analysis of the stress predicted by the four methodologies is developed considering a fillet welded tube as an example.

2. Fillet Welded Tube

The study is applied to a simple mechanical component composed by a tube SCH 40, of SAE 1020 structural steel, fillet welded to a base thought one end and loaded with a vertical load P of 887 N, pointed downwards, applied at an arm fixed at the other end, as shown in Fig.1. Table 1 presents the geometric and material parameters. The applied load produces a combined loading at the section fillet welded with a bending moment (M), a shear force (V) and a torque (T).

These parameters are chosen to satisfy a design of the mechanical component in accordance to the *regular model* presented by a classical reference book of Mechanical Engineering Design (Shigley and Mischke, 2001).

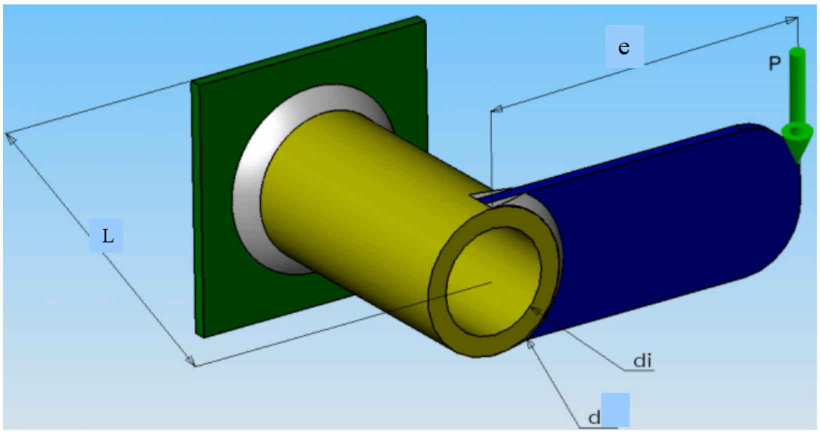


Figure 1. Geometry and loading of a welded tube submitted to a combined loading.

Table 1. Geometric and material parameters.

Geometric Parameter	
d (mm)	33.4
e (m)	0.385
h (mm)	3.83
L (m)	0.334
d_i (mm)	26.6

Material Parameter (SAE 1020)	
Young modulus - E (GPa)	200
Yield stress - S_Y (MPa)	250
Ultimate strength - S_{ut} (MPa)	450
Ultimate strain - ε_{ut} (%)	13
Poisson ratio - ν	0.29

3. Analytic Models

Two analytic models are considered in this section: a *regular* model (Shigley and Mischke, 2001) and a model *based on Strength of Materials*, for a fillet welded tube submitted to a combined loading. The *regular* model introduces an interesting idea of considering each fillet weld as a line. The advantage of this approach is that tables of bending and torsional geometric properties of fillet welds (specifically the unit second moment of area, I_u, and the unit second polar moment of area, J_u) can be used for any leg size, h. The model *based on Strength of Materials* applies classical concepts of Strength of Materials using less simplification hypothesis.

3.1. Regular model

For the design of fillet welds Shigley and Mischke (2001) consider a resultant shear stress acting on a throat area with an throat angle, θ, equal to 45º (Fig. 2), although for the situation analyzed the plane angle where maximum *von Mises* equivalent stress occurs is 62.5º (Kenedi *et al*., 2005).

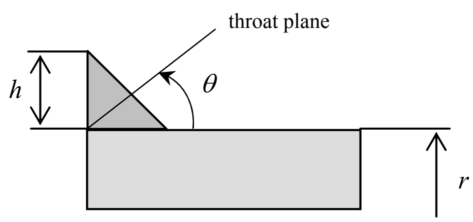


Figure 2. Throat area of fillet welds.

For the situation analyzed, three shear stress components, associated to the bending moment (M), shear force (V) and torque (T), are calculated (Shigley and Mischke, 2001):

$$\tau = \frac{M \cdot y}{I} \qquad ; \qquad I = 0.707\, h\, I_u \tag{1}$$

where h is the leg size, y is the vertical distance from the centroid to the weld (line) and I is the second moment of area.

$$\tau' = \frac{V}{A} \tag{2}$$

where A is the throat area.

$$\tau'' = \frac{T \cdot r}{J} \quad ; \quad J = 0.707\ h\ J_u \tag{3}$$

where J is the second polar moment of area.

For the geometry of fillet weld shown at Fig. 1, from (Shigley and Mischke, 2001):

$$A = 1.414\ \pi\, h\, r \quad ; \quad I_u = \pi\, r^3 \quad ; \quad J_u = 2\pi\, r^3 \tag{4}$$

Figure 3 shows a sketch of the fillet welded for an angle γ between 0 and 90º with the representation of the shear stresses present at the throat area according to the *regular* model.

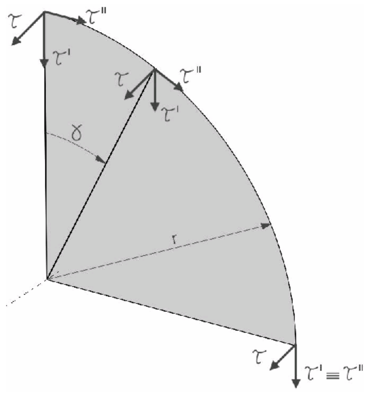


Figure 3. Representation of the shear stresses present at the throat area of *regular* model for the example of Fig.1.

The resultant shear stress τ_{result} for a point located at the circumference of radius r, for an angle γ between 0 and 90º is:

$$\tau_{\text{result}} = \sqrt{(\tau''\cdot\cos(\gamma))^2 + (-\tau' - \tau''\cdot\sin(\gamma))^2 + \tau^2} \tag{5}$$

3.2. Model based on Strength of Materials

This model uses a more conventional approach for elastic materials that considers well established principles of Strength of Materials, but it does not considers the effect of the stress concentration that promotes a stress increase in the regions where a geometric discontinuity is present. Figure 4 shows the geometric parameters used for this model. Note that in this model the fillet weld is not treated as a line.

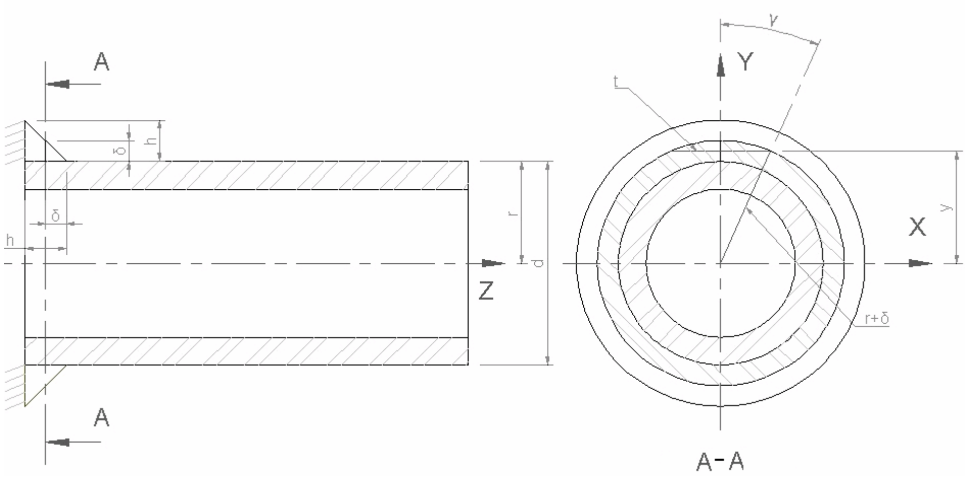


Figure 4. Geometric parameters used in the model *based on Strength of Materials*.

The expressions for the stress at the weld region associated to the bending moment (M), shear force (V) and torque (T) are represented by:

$$\sigma_{\mathrm{mx}}(\gamma) = \frac{M \cdot y(\gamma)}{I} \quad ; \quad y(\gamma) = (r + \delta)\cos(\gamma) \tag{6}$$

where y is the vertical distance from the center of the tube to the point of interest and δ is a distance inside the fillet weld.

$$\tau_V(\gamma) = \frac{Q(\gamma) \cdot V}{I \cdot t(\gamma)} \quad ; \quad t(\gamma) = 2\,(r + \delta)\sin(\gamma) \tag{7}$$

where, $Q(\gamma)$ is the first moment of area and $t(\gamma)$ is the horizontal width of the section at a distance $y(\gamma)$ from the neutral axis.

$$\tau_T = \frac{T \cdot (r + \delta)}{J} \tag{8}$$

For the fillet weld geometry shown in Figs. 1 and 4:

$$A = \frac{\pi}{4} \cdot \left[(d + 2 \cdot \delta)^2 - (d_i)^2\right] \quad ; \quad I = \frac{\pi}{64} \cdot \left[(d + 2 \cdot \delta)^4 - (d_i)^4\right] \quad ; \quad J = \frac{\pi}{32} \cdot \left[(d + 2 \cdot \delta)^4 - (d_i)^4\right] \tag{9}$$

$$Q(\gamma) = \begin{cases} \left(\dfrac{2}{3}\right) \cdot (r + \delta)^3 \cdot (\sin(\gamma))^3 & for \quad y(\gamma) \geq d_i/2 \\ \left(\dfrac{2}{3}\right) \cdot (r + \delta)^3 \cdot \left[(\sin(\gamma))^3 - \left[\alpha^2 - (\cos(\gamma))^2\right]^{3/2}\right] & for \quad y(\gamma) < d_i/2 \end{cases} \quad \text{where} \quad \alpha = \frac{\mathrm{r}}{r + \delta} \tag{10}$$

4. Finite Element Models

Finite element models can describe more precisely the geometry of the welded region. Therefore, two important characteristics of the weld region that influence the stress distribution at this region can be analyzed with FEM models: the stress concentration effects and the attachment condition associated to the presence or absence of a chamfer.

The discontinuity at the junction between the weld fillet and the mechanical component promotes a stress concentration that is not considered by the traditional analytic models. For ductile materials this effect is reduced by a local stress redistribution resultant from the plastic strain that develops in this region. This stress redistribution depends on the plastic behavior of the material.

There are two possibilities for the attachment of the piece to the base: without and with a chamfer. The presence of a chamfer increases the attachment area of the mechanical component. On the other hand, the absence of the chamfer reduces the attachment area and promotes a stress concentration as it introduces a geometric discontinuity. Therefore the chamfer can influence the stress distribution in the weld region. Figure 5 illustrates the two attachment possibilities.

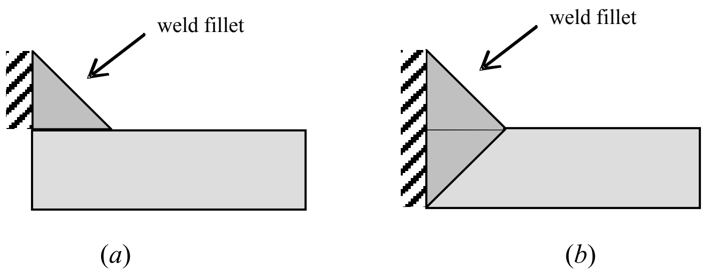
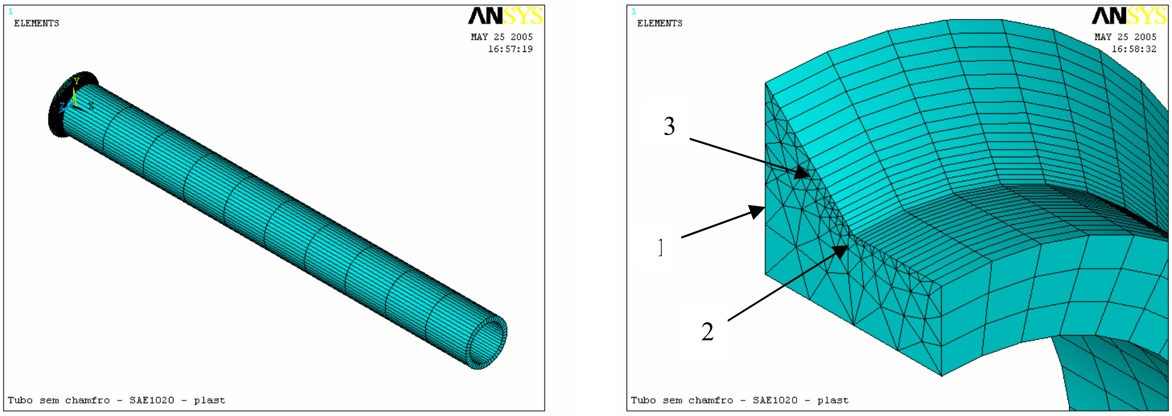


Figure 5. Types of attachments: (a) without and (b) with chamfer.

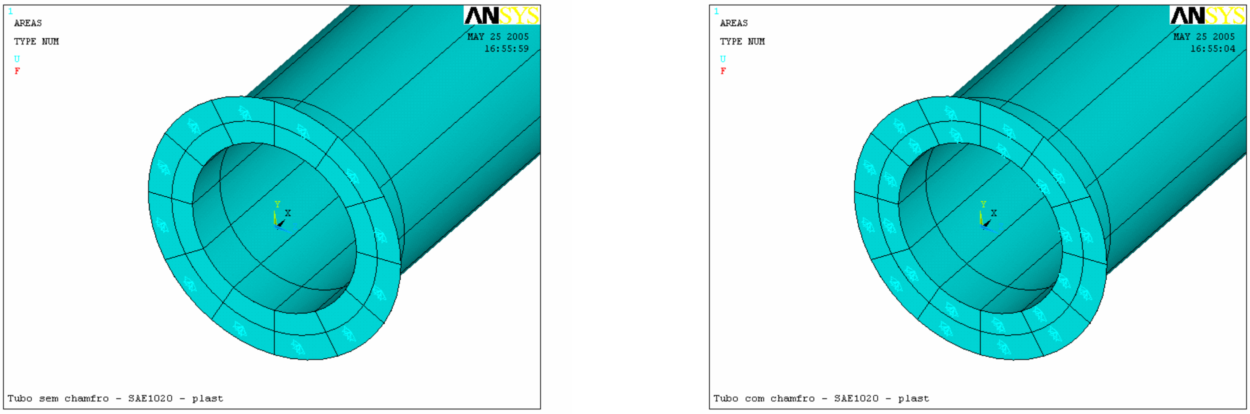
To study these effects, two tri-dimensional finite element models are developed: an elastic model and an elastoplastic model with kinematic hardening. Numerical simulations are performed with commercial finite element code ANSYS (ANSYS, 2004). Elements SOLID95 (20 nodes brick structural solid - 3 degree of freedom per node) and SOLID92 (10 nodes structural solid - 3 degree of freedom per node) are used (ANSYS, 2004). The final mesh is defined after a convergence analysis and is shown in Fig. 6. A bilinear kinematic hardening model BKIN (ANSYS,

2004) with a plastic tangent modulus of 1.55 GPa is employed to represent the material hardening. Two different boundary conditions are shown in Fig.7, where null displacements are prescribed at two different annular regions represented by the areas with blue triangles. The first one is used to represent a fillet weld for a tube without chamfer and the second one a fillet weld for a tube with chamfer. The loading of the mechanical component is modeled by an equivalent load applied in four points of the model free end: two vertical downward loads of 443.5 N and four loads of 5.11 kN in a circular clockwise pattern to represent the applied torque of 341.5 N.m promoted by the arm of 385 mm length. As the load is applied at a cross section far from the weld region (a distance correspondent to several diameters) the form that the load is applied does not affect the behavior of the material at the weld region. The red arrows in Fig. 8 represents the applied loads.



(*a*) (*b*)

Figure 6. Finite element model. (*a*) Mesh and (*b*) mesh detail at fillet region.



(*a*) (*b*)

Figure 7. Finite element model. Boundary conditions: (*a*) tube without chamfer and (*b*) tube with chamfer.

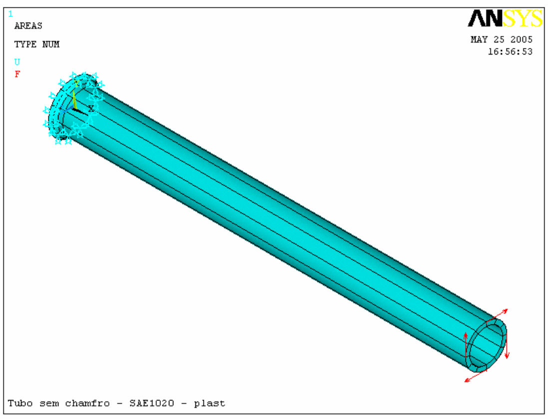
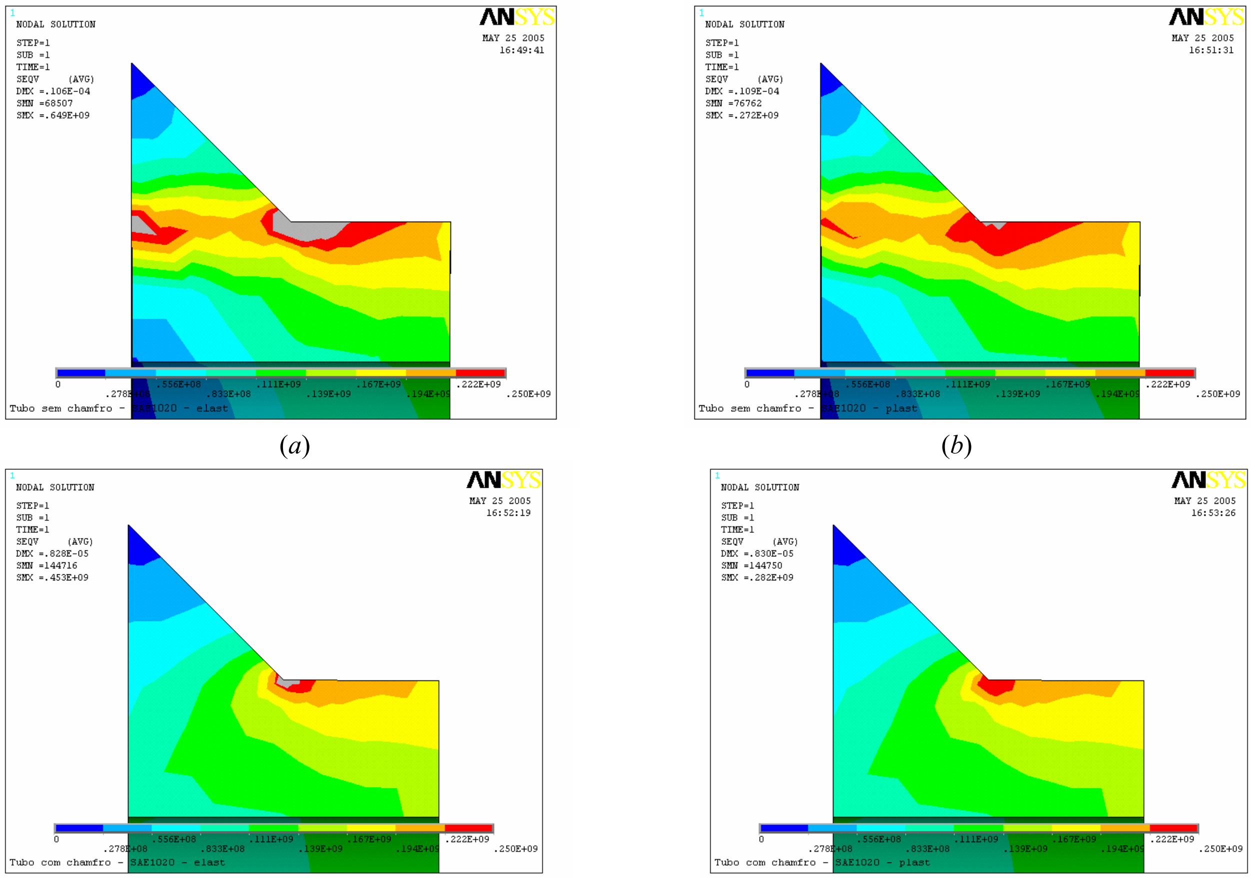


Figure 8. Finite element model. Loading conditions.

4.1. Numerical Simulations

The numerical simulations comprise four situations: elastic without chamfer, elastoplastic without chamfer, elastic with chamfer, elastoplastic with chamfer. Figure 9 shows the *von Mises* equivalent stress in the critical region of the mechanical component developed during the application of the load for the four different situations. The critical region occurs in the plane *yz* ($\gamma = 0$) of Fig.4. The gray area represents stresses levels higher than the yield stress (S_Y).



(*a*) (*b*)

(*c*) (*d*)

Figure 9. *von Mises* equivalent stress distribution. (*a*) Elastic without chamfer, (*b*) elastoplastic without chamfer, (*c*) elastic with chamfer and (*d*) elastoplastic with chamfer.

Some interesting results can be observed from the four cases of Fig. 9. Comparing cases without a chamfer ((*a*) and (*b*)) with the cases with chamfer ((*c*) and (*d*)) is apparent the effect of the type of attachment on the distribution of *von Mises* stresses. While in the first two cases, where there is no chamfer, the geometric discontinuity has a major effect on the *von Mises* stresses distribution, in the last two cases, where there is a chamfer, the disturbance is not present at all. Comparing the elastic models ((*a*) and (*c*)) with the elastoplastic models ((*b*) and (*d*)) is apparent that the redistribution of stresses promoted by a localized yielding is very effective to minimize the increase of stress promoted by stress concentration. In the first case is observed a stress reduction of about 60% and in the second case about 40%.

5.Comparision

A comparison between the analytic and numerical models can be done comparing the *von Mises* equivalent stresses. Note that for the case studied the ratio L/d is greater than 10 and the shear stress due to shear force can be considered negligible when compared to bending stress. Therefore, the shear stresses of Eqs. (2) and (7) are neglected in the analysis. For the two analytic models, *von Mises* equivalent stresses can be calculated by substituting the stresses indicated by Eq. (5) for the *regular* model (which gives τ_{result} = 144 MPa) and by Eqs. (6-8) for the model *based on Strength of Materials* in the following expressions:

$$\sigma_{eq} = \sqrt{3} \cdot \tau_{\text{result}} \tag{11}$$

$$\sigma_{eq}(\gamma) = \sqrt{\sigma_{mx}(\gamma)^2 + 3 \cdot (\tau_T)^2} \tag{12}$$

For the *regular* model a value of 250 MPa is obtained, as the parameters are chosen to satisfy the design of the mechanical component in accordance to the *regular model*. For the model *based on Strength of Materials* the calculations are developed for the three points shown in Fig. 6*b*. Points 1 and 2 are localized at surface of tube and point 3 is localized at the end of the 45º throat line. The following values are obtained: 192 MPa for point 1, 191 MPa for point 2 and 112 MPa for point 3.

Figure 10 shows *von Mises* stress distribution along the 45º throat line for γ = 0º (plane *yz* of Fig. 4) for the two numeric models with the two type of attachment. Points 1 and 3 are respectively at positions 0 and 2.71 mm of the 45º throat line and point 2 is located in Fig. 9 at stress concentration region (as show at Fig.6*b*).

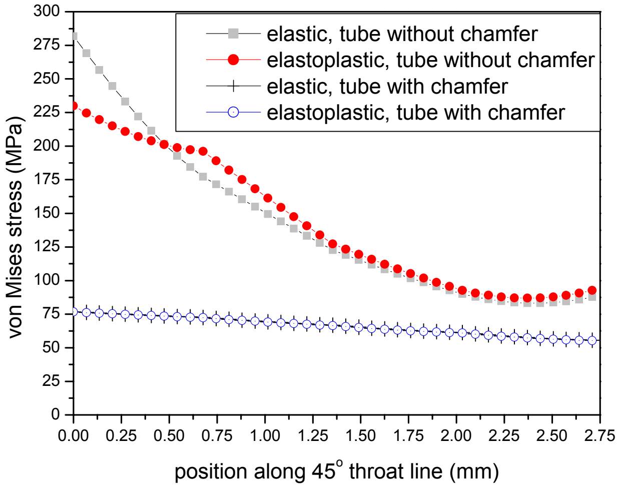


Figure 10. *von Mises* stress distribution along the 45º throat line at γ = 0º (plane *yz* of Fig. 4).

It is interesting to note in Fig. 10 that the two models without chamfer present higher values in the region associated with the geometric discontinuity promoted by the attachment. The elastic model presents a stress level slightly higher than the yield stress and for the elastoplastic model the stress redistribution promoted by the plastic deformation has lowered the stress levels to levels below the yield stress. For the two models with chamfer a similar stress distribution is observed and the stress levels are considerably below the yield stress.

Figure 11 presents a comparative study between all the models considered. The stresses are normalized in relation to the yield stresses (S_Y).

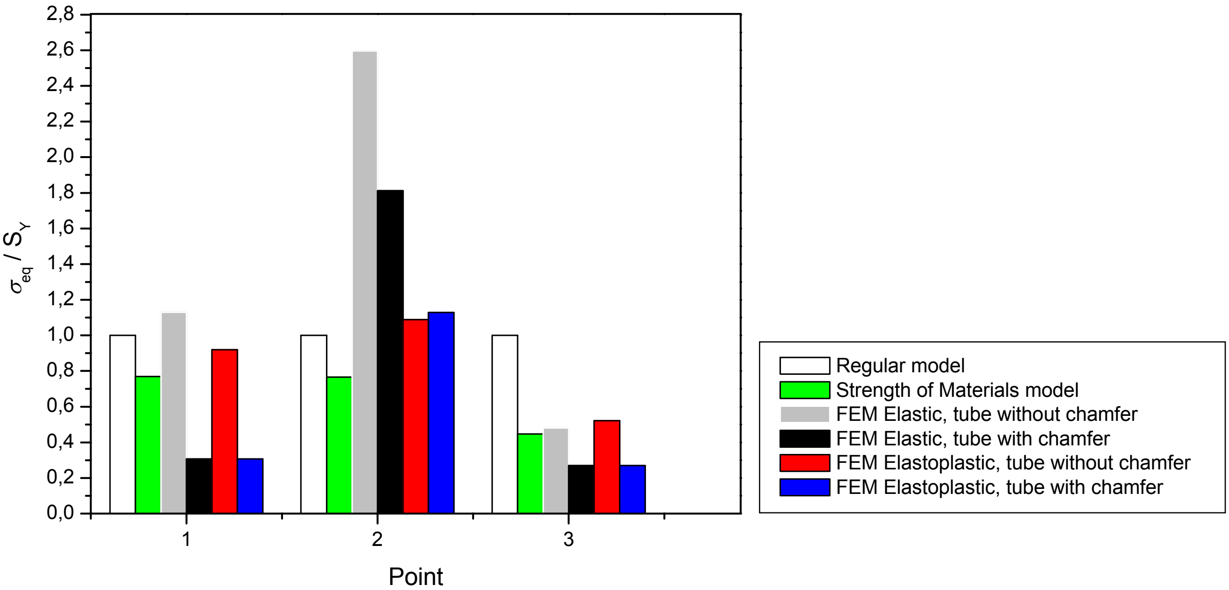


Figure 11. *von Mises* stress observed in the three points depicted in Fig. 6*b* for the models considered.

It can be observed that the two FEM elastic models predict very high and unrealistic stress levels at point 2. They consider the stress concentration associated to the geometric discontinuity but cannot represent the stress redistribution promoted by the plastic strain in this region, which turn a doubtful choice for modeling fillet welds. The *regular* model is represented with the same value of stress for all points because it restricts the analysis to a constant shear stress at the throat area. It has a conservative performance in comparison with other models (not considering the elastic ones). The analytic model *based on Strength of Materials* is less conservative than the *regular* model, and presents stresses values near the ones presented by the FEM elastoplastic models.

For the case studied, the *regular* model, in spite of its simplicity, furnishes a stress value prediction similar to the values obtained with the more sophisticated FEM elastoplastic models without chamfer, indicating that it is effective in the design of fillet weld structures of ductile materials for static loadings. This occurs as the material considered has sufficient ductility to develop plastic deformations to redistribute the stresses in the regions of stress concentrations. However it is important to note that for cyclic loadings a hysteresis stress-strain loop can develops at this region and low-cycle fatigue may occurs. As the material surrounding the regions of stress concentrations is in the elastic regime, this elastic region may subject the plastic region to a prescribed displacement loading. Therefore, for cyclic loadings a careful investigation of the material behavior in these critical regions must be considered.

6. Conclusions

This work presents a study of a fillet welded structure submitted to a combined static loading using two analytic models and two numerical models. The numerical models are based on the Finite Element Method and consider elastic and plastic behaviors. In this work some hypothesis normally used by traditional design methodologies are addressed. The presence of stress concentrations, plasticity and the use of chamfers is considered in the analysis developed. For the case studied, the *regular* model, in spite of its simplicity, furnishes a stress prediction similar to the ones obtained with the more sophisticated FEM elastoplastic models, indicating that it is effective in the design of fillet weld structures of ductile materials for static loadings. However it is important to note that for cyclic loadings a hysteresis stress-strain loop can develops at this region and low-cycle fatigue may occurs. Although the model *based on Strength of Materials* is promising it is not a substitute for the applicable standards. New developments are in course to check the applicability of the model in more situations.

7. Acknowledgements

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9. Responsibility notice