A NEW CORRECTION METHOD FOR UNSTEADY PRESSURE COMPUTATIONS AND TRANSONIC AEROELASTIC STABILITY ANALYSES

Roberto G. A. Silva
Instituto de Aeronáutica e Espaço – IAE - Centro Técnico Aeroespacial – CTA 12228-904 - São José dos Campos - SP – Brazil
rasilva@iae.cta.br

Olympio A. F. Mello
Instituto de Aeronáutica e Espaço – IAE - Centro Técnico Aeroespacial – CTA 12228-904 - São José dos Campos - SP – Brazil
oamello@iae.cta.br

João Luiz F. Azevedo
Instituto de Aeronáutica e Espaço – IAE - Centro Técnico Aeroespacial – CTA 12228-904 - São José dos Campos - SP – Brazil
azevedo@iae.cta.br

P.C. Chen
ZONA Technology, Inc., Scottsdale, AZ 85251
pc@zonatech.com

Danny D. Liu
ZONA Technology, Inc., Scottsdale, AZ 85251
danny@zonatech.com

Abstract. *The work presents a correction technique to compute unsteady transonic pressure distributions and aeroelastic stability in this flow regime. The methodology is based on corrections of pressure distributions by the weighting of the lifting surface self-induced downwash, resulting from aeroelastic structural displacements or prescribed motions. The correction of pressure distributions through the weighting of the lifting surface self-induced downwash is also known as downwash weighting method. This method has been enhanced leading to a new downwash correction technique. This extended downwash correction method led to a rational formulation named as "successive kernel expansion method" (SKEM). The unsteady pressures and aeroelastic stability boundaries computations using such method led to good agreement with experimental measurements. This procedure is a rapid form to compute the transonic flutter speed boundaries, compared to computational aeroelasticity and experimental techniques.*

Keywords*: Aeroelasticity, Unsteady Aerodynamics, Transonic Flow, Correction Methods*

1. Introduction

The application of discrete element kernel function methods are limited to purely subsonic or supersonic flows, since the governing equations over which the method were developed are based on a linearized unsteady potential flow hypothesis. However, the aeroelastic behavior of an aircraft is typically critical in the transonic flight regime where nonlinear phenomena related to embedded moving shock waves and viscosity, play an important role in aeroelastic stability. As discussed by Ashley (1980), the shock wave movement and strength profoundly affects the flexure-torsion flutter mechanism. One consequence of this behavior is the so called "transonic dip". This phenomenon is characterized by a decrease of the flutter speed as a function of the Mach number, when compared to flutter speeds obtained from a linear aeroelastic analysis. Therefore, it is necessary to pay special attention to the flutter phenomenon under these circumstances. This is especially significant as most modern aircraft fly under transonic flow conditions.

One of the feasible alternatives for analyzing the aeroelastic stability in nonlinear flow conditions is by the use of time accurate computational fluid dynamic (CFD) solutions of the nonlinear fluid equations coupled with structural dynamic representation of the vehicle. Another approach is obtained by wind tunnel testing of aeroelastic models, under transonic flow conditions. However, wind tunnel testing for aeroelastic investigation regarding flutter boundary computation is not usual because this class of experiments involves expensive models and high operational costs. The other way to evaluate the transonic aeroelastic behavior, is from flutter flight testing, which is the most hazardous and expensive option in terms of operational costs. This approach may be used either for experimental flutter boundaries identification or to verify the aeroelastic subcritical aerodynamic damping at specific flight envelope points to validate aeroelastic numerical models.

There have been several attempts to solve the transonic aeroelastic problem using combined procedures which relate linear models to measured data for the correction of unsteady linear aerodynamic models. Reviews on correction techniques were presented by Palacios *et al.*(2001) and Silva (2004), describing the most employed methods concerning transonic flutter prediction via combined procedures. Such approaches are named here as mixed procedures. The purpose of such procedures is to correct the linear aerodynamic models to take into account nonlinear effects, unpredictable by the linearized potential-based equations of the fluid flow.

The methodologies for the solution of the transonic aeroelastic problem based on mixed procedures are referred also as semi-empirical corrections. These corrections can be performed by the multiplication, addition or the whole replacement of the AIC matrix. This approach is an adequate tool for engineering applications, because the methodology employed is less expensive than direct use of CFD techniques.

Most of the known correction procedures employ steady state reference data, which may be loads or pressures. Some of them are based on the computation of corrected unsteady pressures from semi-empirical relations, or with the aid of computational fluid dynamics simulations. It was concluded in the work of Silva (2004) that downwash weighting procedures, are robust, inexpensive in terms of computational costs and is compatible with the physics of the problem. They also present the advantage in providing a simple way to modify the pressures obtained from the linear theory by the replacement of externally computed or measured pressures distributions. These procedures were developed using either steady or unsteady pressures as reference conditions. However, such approaches present drawbacks such as, wrong pressure phases computation in the case of the use of steady pressures as reference conditions and the dependency of expensive CFD simulations when unsteady reference conditions are used. Thus, the objective of this work will be the enhancement of such downwash correction procedures, to obtain correct transonic unsteady pressures using steady reference conditions, turning it inexpensive and best suited for industrial aeroelastic applications.

2. Aerodynamic model

Linear aerodynamic modeling techniques are based on discrete element solutions of the linear equations of the fluid flow. The fluid flow is represented by the linearized potential flow equation (Landahl, 1951), in the frequency domain as:

$$\beta^2\varphi_{xx}+\varphi_{yy}+\varphi_{zz}-2ikM_\infty^2\varphi_x+k^2M_\infty^2\varphi=0 \quad , \tag{1}$$

The aerodynamic modeling of unsteady linear potential flows may be performed by discrete kernel function methods, which are based on integral solutions of the small disturbances linearized potential flow equation. The integral solution is obtained by the application of Green's theorem to this equation (Chen *et al.*, 1993) in terms of unsteady source and doublet singularity distributions, over the body surface *S* and its associated wake surfaces *W*. The aerodynamic model for a given body is then approximated by the summation of elementary integrals solutions associated with each element (panels) which discretizes the body surface. These elementary integrals at each panel, as well as the aerodynamic interference of one panel onto others leads to a linear system of equations relating the pressure coefficient differences to downwash. The assembly of the elementary integral solutions results in a matrix which elements represent the aerodynamic influence of the panels into the control points. Each integral relationship between the downwash at a receiving point "*i*", and the pressures at a sending panel "*j*' may be written as a system of equations as:

$$\varphi_z^i = w_i = D_{ij}\Delta C_{p\,j} \tag{2}$$

where each of the matrix elements D_{ij} is the result of the integration of the kernel function over the given "j^{th}" lifting surface element geometry :

$$D_{ij}=-\frac{1}{8\pi}\int_{\bar{\xi}_{j-1}}^{\bar{\xi}_j}\int_{\bar{\eta}_{j-1}}^{\bar{\eta}_j}\left\{\lim_{z\to 0}\left[\frac{\partial}{\partial z}K_\psi\left(x_i-\bar{\xi}_n,y_i-\bar{\eta}_n,0,M_\infty,k\right)\right]\right\}d\bar{\xi}_n d\bar{\eta}_n \tag{3}$$

The inverse of the matrix operator *D* multiplied by the downwash vector yields the pressure distribution. In other words, the solution of the system of equations gives the doublet strength at each panel referred to a given downwash which is related to a displacement mode shape. The resulting inverse matrix is named as the aerodynamic influence coefficients matrix *AIC*, which is a function of the reduced frequency, and is related to the pressure distribution by:

$$\left\{\Delta C_p(ik)\right\}=\left[AIC(ik)\right]\left\{w(ik)\right\} \tag{4}$$

One should recall that a simple harmonic motion is assumed, hence the dependence on *ik*. The coefficients of this matrix may be interpreted as rates of pressure variation due to a given displacement amplitude input associated to the boundary conditions. Then, the determination of the pressure coefficient vector in Eq. (4) is performed from the known downwash, which is related to the amplitude of the pitch and plunge motion at each element. The substantial derivative of a given displacement mode is composed of a derivative of the normal direction displacement with respect to the main flow direction plus the associated velocity scaled by the undisturbed flow speed which, in a small disturbance sense, represents an angle of attack. Therefore, from the boundary conditions for those small perturbations, the relationship between the normalwash and a solid boundary displacement is rewritten as:

$$\left\{w\left(x,y,0,ik\right)\right\}=\frac{\partial h\left(x,y,0\right)}{\partial x}+ikh\left(x,y,0\right)=\left[F\left(ik\right)\right]\left\{h\left(x,y,0\right)\right\} \tag{5}$$

The substantial derivative applied to a given modal displacement vector $\{h\}$, which appears in Eq. (5) is denoted by the matrix operator $\left[F\left(ik\right)\right]$. The resulting aerodynamic loading vector, $\left\{L_a\left(ik\right)\right\}$ may be expressed as the multiplication of the pressures by an integration matrix, [S], which is constructed from the panel elements geometrical characteristics. The resulting final expression for the unsteady loading over the lifting surface is given by

$$\left\{L_a\left(ik\right)\right\}=q_\infty\left[S\right]\left[AIC\left(ik\right)\right]\left[F\left(ik\right)\right]\left\{h\right\}, \quad \text{with} \quad \left[F\left(ik\right)\right]\left(\cdot\right)=\left[\frac{\partial\left(\cdot\right)}{\partial x}+ik\left(\cdot\right)\right] \tag{6}$$

The subsonic discrete kernel function approach will be further employed as the unsteady aerodynamic theory for computation of unsteady pressures and loads for aeroelastic response and stability analysis.

3. Pressure Matching Correction Method

The basic idea of downwash correction methods is the modification of the downwash vector embedded into the pressure to downwash algebraic relationship, derived from the application of discrete element kernel function methods to model the linearized potential flow (Eq. 4). The transonic flow reference conditions can be based on either CFD solutions of the nonlinear fluid dynamic governing equations or experimental data. The downwash vector is related to the lifting surface displacements by boundary conditions defined in a small disturbances context, and it may be regarded as an effective dynamic angle of attack at each of the lifting surface elements. In the frequency domain, these boundary conditions are rewritten here as

$$\left\{w\left(x,y,0,ik\right)\right\}=\frac{\partial}{\partial x}\left\{h\left(x,y,0\right)\right\}+ik\left\{h\left(x,y,0\right)\right\}, \tag{7}$$

where $\left\{h\left(x,y,0\right)\right\}$ is an out-of-plane lifting surface displacement mode shape, and $\left\{w\left(x,y,0,ik\right)\right\}$ is the resulting downwash with respect to the modal motion. When steady state pressures are considered as reference conditions (Silva, 2004), the corresponding downwash is reduced to the derivative of the associated mode shape displacements with respect to the flow direction x, as

$$\left\{w\left(x,y,0\right)\right\}=\frac{\partial}{\partial x}\left\{h\left(x,y,0\right)\right\}. \tag{8}$$

This expression leads to a steady state pressure to downwash relationship given by

$$\left\{\Delta Cp\left(ik=0\right)\right\}=\left[AIC\left(ik=0\right)\right]\left\{h_x\right\}. \tag{9}$$

This approach presents good robustness and preservation of the mean steady nonlinear flow because the pressure are fully restored in steady state conditions. However, it fails to compute unsteady pressures due to absence of nonlinear unsteady pressure information in the reference conditions (Silva, 2004). As an alternative, the unsteady pressures matching, is a way to include the nonlinear unsteady information in the reference conditions. The drawback of such last correction approach is the dependence on time consuming computations to generate the unsteady reference pressures. The general form of the downwash correction method using either steady or unsteady reference pressures is given as

$$\left\{\Delta C_p^{nl}\left(ik_r\right)\right\}=\left[AIC\left(ik_r\right)\right]\left[WT\left(ik_r\right)\right]\left\{\overline{w}\left(ik_r\right)\right\}, \tag{10}$$

where k_r can be zero or not, depending on the nature of the reference pressure differences, the references $\left\{\Delta C_p^{nl}\left(ik_r\right)\right\}$, $\left\{\overline{w}\left(ik_r\right)\right\}$ is the downwash vector, and $\left[WT\left(ik_r\right)\right]$ is the weighting matrix which modify the downwash to match the nonlinear unsteady pressures after its multiplication by the downwash. The elements of the weighting matrix are the correction factors, and they are computed from the solution of the system of Eq. (10). For simplicity it is assumed that the weighting operator $\left[WT\left(ik_r\right)\right]$ is diagonal, and its multiplication by the downwash vector results in a modified downwash obtained from

$$\left\{w^{nl}\left(ik_r\right)\right\}=\left[AIC\left(ik_r\right)\right]^{-1}\left\{\Delta C_p^{nl}\left(ik_r\right)\right\}. \tag{11}$$

In the same way, the downwash associated to the lifting surface displacement can be written as the product between the AIC matrix and the linear pressures as:

$$\left\{\Delta C_p\left(ik_r\right)\right\}=\left[AIC\left(ik_r\right)\right]\left\{\overline{w}\left(ik_r\right)\right\}\Rightarrow\left\{\overline{w}\left(ik_r\right)\right\}=\left[AIC\left(ik_r\right)\right]^{-1}\left\{\Delta C_p\left(ik_r\right)\right\}. \tag{12}$$

The assumption of a diagonal matrix is sufficient since the nonlinear unsteady pressures vector can be reconstructed from the multiplication of this diagonal operator by the downwash vector. Therefore, the diagonal coefficients of this matrix can be computed from the ratio between the modified (weighted) downwash and the one corresponding to the lifting surface motion

$$\left(WT\left(ik_r\right)\right)_{ii}=\left(w^{nl}\left(ik_r\right)\right)_i\Big/\left(\overline{w}\left(ik_r\right)\right)_i\,. \tag{13}$$

The weighting matrix in then introduces in Eq. (6) leading to an approximate aerodynamic loading given by

$$\left\{L_a^{nl}\left(ik_r\right)\right\}=q_\infty\left[S\right]\left[AIC\left(ik_r\right)\right]\left[WT\left(ik_r\right)\right]\left\{w\left(ik_r\right)\right\}. \tag{14}$$

4. Successive Kernel Expansion Procedure

In order to circumvent the limitation imposed by the aforementioned procedures, the next step is to develop a new downwash correction method. The theoretical background is based on transonic flow behavior investigations in a small disturbances context. This investigation is presented in Silva (2004) and its purpose is to understand the physics of transonic flows, regarding its linearity (Dowell *et al.*, 1983 and Silva *et al*., 2002). The enhancement the downwash weighting procedure is motivated by the aforementioned investigation results. Linear unsteady aerodynamic theories, applied to the aeroelastic modeling and analysis, are developed considering small disturbances around mean angle of attack variations of the lifting surface. The linear/nonlinear behavior investigation indicated that unsteady transonic flow behavior is strongly dependent on the amplitude of the motion (Silva *et al.*, 2002). However, aeroelastic deformations are usually smaller than the linear limits identified in the linearity investigations.

Based on this observation, it is inferred that, for aeroelastic analysis in a small disturbance context, unsteadiness of transonic flows present a linear behavior with regard to aerodynamic derivatives and shock dynamics, when the amplitude of lifting surfaces undergoing unsteady motion are below the linear limits. In other words, unsteady transonic pressures behaves linearly around a steady nonlinear mean pressure distribution, for small amplitudes of the motion, such as aeroelastic deformations (Silva *et al.*, 2002). For this reason, it is possible to decouple as a linear contribution predicted by a small disturbance linear aerodynamic model, in superposition to a nonlinear steady mean flow. Thus the proposed procedure should be understood as an extension of the steady DWM, where the reference pressure distribution is composed by the superposition of an unsteady contribution predicted by the linearized potential flow equation in a steady nonlinear mean flow (Silva, 2004)). Therefore proposed procedure shall be divided in two steps, the first being a nonlinear steady mean flow correction as is performed for the correction of the steady downwash, when nonlinear pressure differences are considered as reference conditions. The second step is the correction of the unsteady downwash, where the unsteady counterpart of the reference pressures to be added to the steady nonlinear reference pressures will compose new reference pressure differences. These unsteady pressure contributions are predicted by a linear unsteady flow aerodynamic model.

In essence, this method can be understood mathematically as a successive kernel expansion algorithm that can inject the given steady pressure into the perturbed frequency-based integral equations to recover the out-of-phase pressure. To show the idea behind the successive kernel expansion method, consider the lifting surface formulation in which the integral equation according to the acceleration potential equation reads,

$$w(x,y,0,ik) = -\frac{1}{8\pi}\iint_A \Delta Cp(\xi,\eta)\,K(x-\xi,\,y-\eta,0,ik,M)\,d\xi d\eta \tag{15}$$

where $K(ik)$ is the kernel function of the acceleration potential, $w(ik) = h_x + ikh$, and h is the normal mode. It is assumed that it is possible to expand the subsonic kernel function as an asymptotic series around small reduced frequencies. This kernel is a function of the reduced frequency, and the domain of dependency is continuous and analytic at this circumstances. Consequently (Miles, 1959), the function may be expanded in an asymptotic series around reduced frequency values smaller than 1.0. Therefore it is possible to express all dependent variables in the equations in terms of the reduced frequency as the expansion parameter (Liu, 1976). Expanding $\Delta C_p(ik)$ and $K(ik)$ in terms of $(ik)^n$ gives :

$$h_x + ikh = (ik)^0 \iint_A \left(\Delta C_p{}^0 + ik\Delta C_p{}^1 + (ik)^2 \Delta C_p{}^2 + \ldots\right)\left(K^0 + ikK^1 + (ik)^2 K^2 + \ldots\right) dA \tag{16}$$

Collecting the $(ik)^n$ terms for n = 0, 1, 2, results in n equations:

$$\begin{aligned}
&O(ik)^0:\ \{h_x\} = \left[K^0\right]\{\Delta C_p\} \rightarrow \Delta C_p{}^0 = \left[A^0\right]\{h_x\}\\
&O(ik)^1:\ \{h\} = \left[K^1\right]\{\Delta C_p{}^0\} + \left[K^0\right]\{\Delta C_p{}^1\} \rightarrow \Delta C_p{}^1 = \left[A^0\right]\left\{\{h_x\} - \left[K^1\right]\{\Delta C_p{}^0\}\right\}\\
&O(ik)^2:\ \{0\} = \left[K^0\right]\{\Delta C_p{}^2\} + \left[K^1\right]\{\Delta C_p{}^1\} + \left[K^2\right]\{\Delta C_p{}^0\} \rightarrow \Delta C_p{}^2 = \left[A^0\right]\left\{\left[K^1\right]\{\Delta C_p{}^1\} + \left[K^2\right]\{\Delta C_p{}^0\}\right\}\\
&\vdots
\end{aligned} \tag{17}$$

where $[A^0] = [AIC(ik=0)]$ is the AIC matrix at $k = 0$

The successive kernel expansion method injects the given steady pressure into Eq. (17) by replacing $[A^0]$ by $[AIC^*]$, where $[AIC^*]$ is the corrected AIC matrix at k =0 obtained either by the downwash weighting matrix method. The unsteady pressure is then computed using Eq. (16) for $\Delta C_p{}^n$ in a successive manner. In this way, it was established a rational basis for DWM into which the successive kernel expansion method (SKEM) can be incorporated, thus the proper recovery of unsteady aerodynamics from the given steady aerodynamics.

5. Results

5.1 Validation of the Unsteady Pressure Distribution.

Two test cases are selected to validate the unsteady pressure coefficients computed by ZTAW with measured data. There are:

- F-5 wing pitching about 50% root chord at M = 0.948 and k = 0.264 (Tijdeman *et al.*, 1979)
- LANN wing in pitch mode about 62% root chord at M = 0.822 and k = 0.105 (Malone and Ruo, 1983)

The corrected AIC matrices at k=0 are first generated using the downwash weighting matrix method. The unsteady pressure coefficients are then computed by the successive kernel expansion method. For all test cases, the CFL3D N-S solver is used to compute the steady pressure coefficient C_p at two angles of attack; α_1 and α_2. Therefore the reference quasi-steady pressures, named here as $\left\{C_{p_{given}}\right\}$, are obtained as the ratio between pressure coefficient differences and the amplitude of the motion.

- ***F-5 Wing Pitching about 50% Root Chord at M = 0.948 and k = 0.264***

The CFL3D N-S computations are performed at M = 0.948 and α_1 =0.5° and α =0°. Figure 1 show that the steady C_p computed by CFL3D at M = 0.948 and α =0° which correlates very well with the test data. $\left\{C_{p_{given}}\right\}$ is then computed accordingly, (C_p (α =0.5°)- C_p (α =0°))/0.5° and it is presented in Fig. 2.

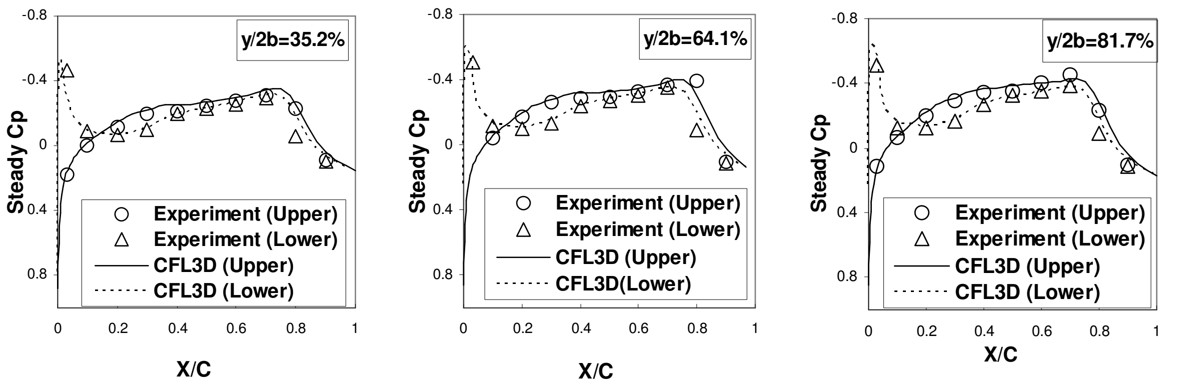

Figure 1. Comparison of Steady Cp between CFL3D and Wind-Tunnel Measured data on a F-5 Wing at M_∞=0.948 and α =0°

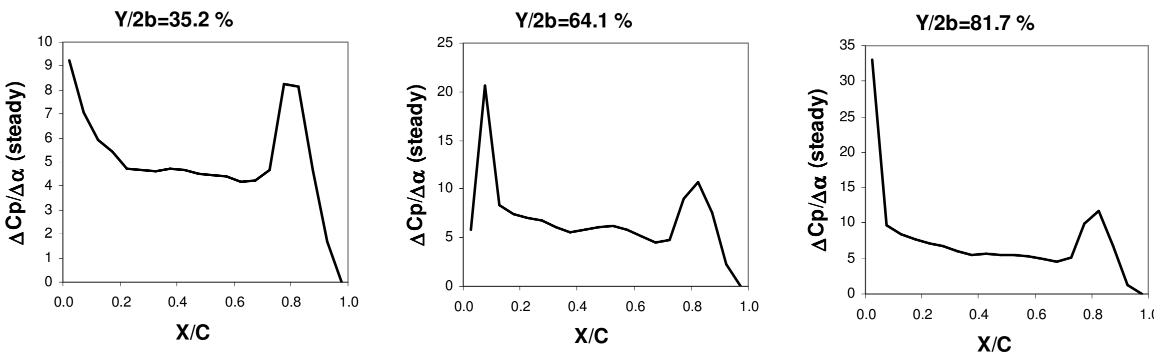

Figure 2. $C_{p_{given}}$ Computed by CFL3D of a F-5 Wing at M = 0.948

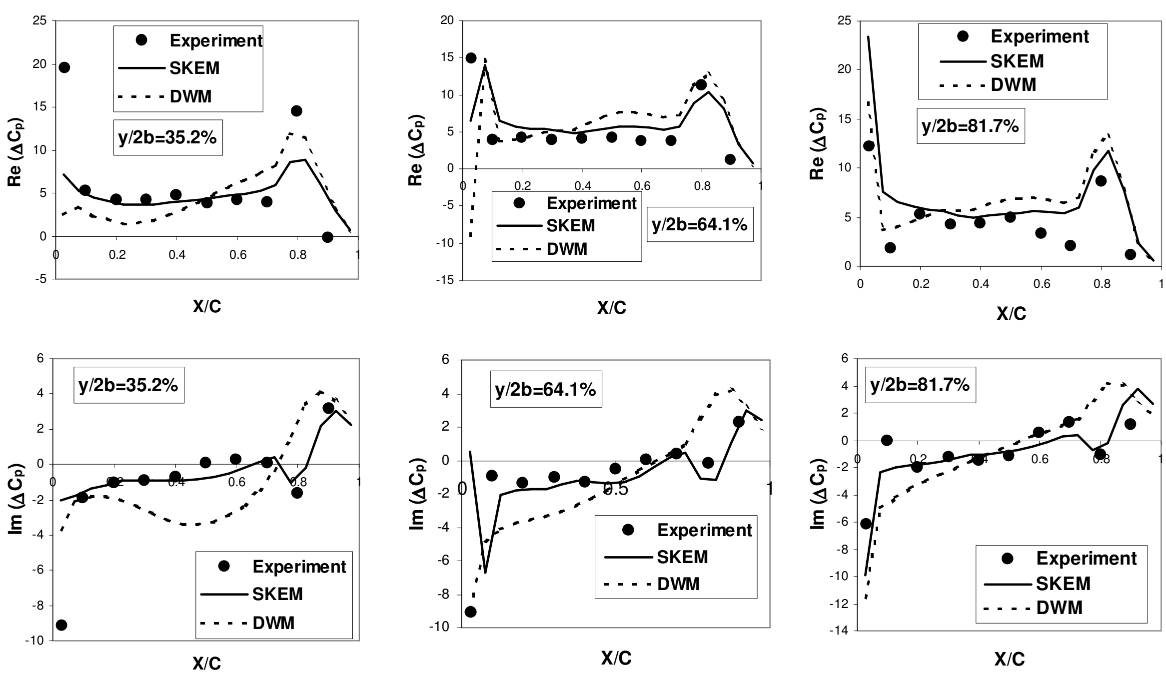

Figure 3. Unsteady ΔC_p on a F-5 Wing due to a Pitch Oscillation about 50% chord at M_∞=0.948 and k = 0.264

Shown in Fig. 3 is the comparison of the unsteady C_p computed by ZTAW and the downwash weighting matrix (DWM) method (due to the pitch oscillation about 50% root chord at $M = 0.948$ and $k = 0.264$) with the wind-tunnel measured data. It can be seen that the real parts of the unsteady pressures ΔC_p computed by ZTAW and DWM are very close to $C_{p_{given}}$ and they agree well with the wind-tunnel data. As discussed previously, this is expected because the in-phase ΔC_p of ZTAW and that of DWM can be essentially derived from $C_{p_{given}}$. However, the imaginary parts of the unsteady pressures ΔC_p computed by DWM do not seem to include the shock-jump behavior as indicated by the measured data. By contrast, the correct shock jump behavior is well predicted by ZTAW. This case clearly shows the shortcoming of the DWM method and the ability of the ZTAW method in extracting accurate out-of-phase pressures from the given steady C_p through the successive kernel expansion method (SKEM).

- ***LANN Wing in Pitch Mode about 62% Root Chord at M = 0.822 and k=0.105***

The CFL3D steady C_p presented in Fig. 6 on a LANN wing at M = 0.822 and α =0.6° shows a strong shock located at 40% chord. $C_{p_{given}}$ for this case is depicted in Fig. 5 which is computed by CFL3D at α =0.6° and 0.8° according $C_{p_{given}} = \Delta C_p / \Delta\alpha = \left[C_p\left(\alpha = 0.8^\circ\right) - C_p\left(\alpha = 0.6^\circ\right)\right]/0.2^\circ$.

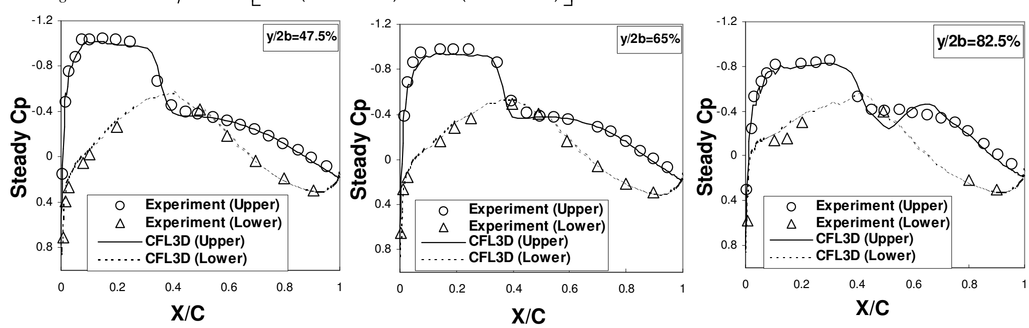

Figure 4. Steady C_p on a LANN Wing at M_∞ =0.822 and α =0.6°

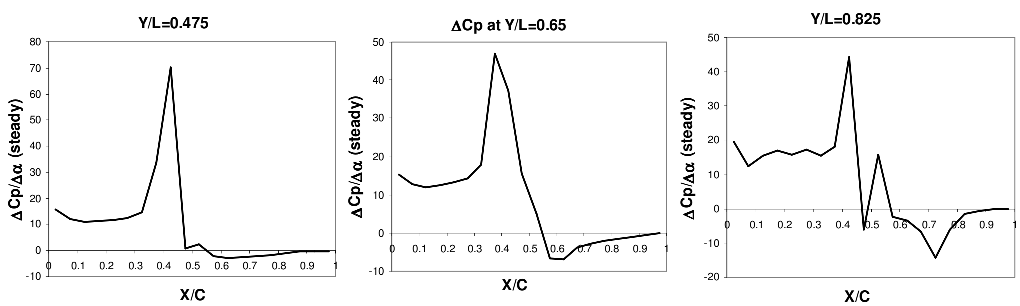

Figure 5. $C_{p_{given}}$ Computed by CFL3D of a LANN Wing at M = 0.822

The unsteady pressures ΔC_p on the LANN wing in pitch oscillation about 62% root chord at $M_\infty = 0.822$ and $k = 0.105$ computed by ZTAW and DWM are presented in Fig. 6. This time it is seen that the imaginary parts of the unsteady pressures ΔC_p as computed by DWM method result in erroneous shock jump behavior of opposite trend to that of the measured data. Out observation has been that of the opposite trend of the shock-jump would become obvious yielding from the results of most of the transonic AIC correction (TAC) methods, when a strong transonic shock jump in the given steady C_p is present.

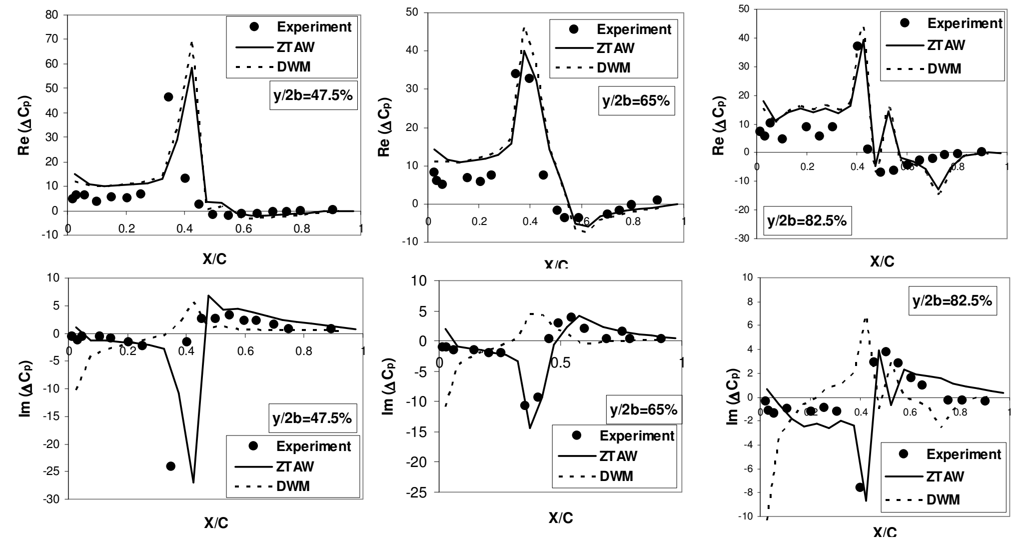

Figure 6. Unsteady ΔC_p on a LANN Wing due to a Pitch Oscillation about 62% Root Chord at M_∞= 0.822 and k = 0.105

Again, by contrast the computed results of unsteady ΔC_p by ZTAW predicted the correct trend in shock-jump behavior, and are in good agreement with measured data throughout all spanwise locations. It should be noted that the zig zag behavior of the ZTAW unsteady C_p at y/2b=82.5% is caused by the same zig zag behavior of $C_{p_{given}}$ at y/2b =82.5%. If more accurate $C_{p_{given}}$ is provided by CFL3D, it is believed that more accurate unsteady ΔC_p is expected to be obtained by ZTAW.

5.2 Validation of Flutter Boundary Predictions:

Two test cases are selected to validate the flutter boundary predictions of ZTAW with the wind-tunnel measurements. Again, $C_{p_{given}}$ is obtained by CFL3D computation at two angles of attack.

- *Flutter Boundaries of AGARD 445.6 Wing* (Yates, 1988)

Four flutter boundaries of the AGARD 445.6 weakened wing are presented in Fig. 7: those due to ZTAW, DWM, and ZONA6 as well as wind-tunnel measurements.

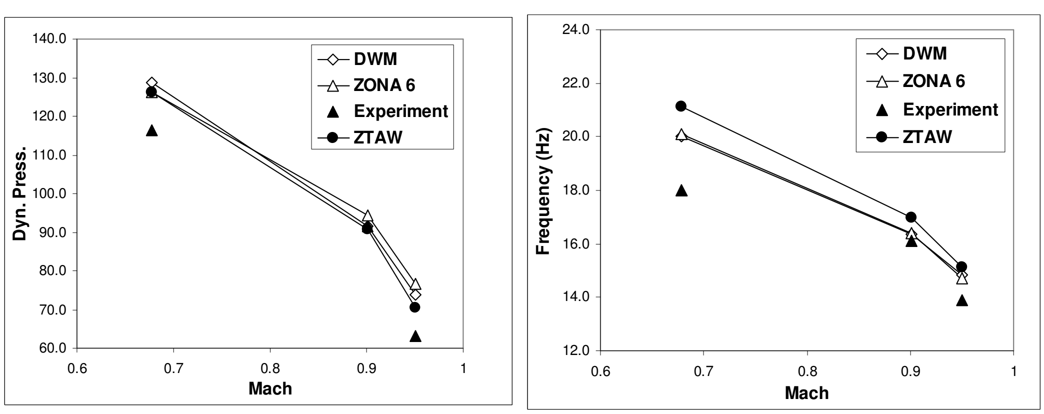

Figure 7. Flutter Boundary of the AGARD 445.6 Wing weakened model #3

In the steady downwash correction the pressure phases are not corrected, resulting in more conservative flutter dynamic pressures, when comparing with strictly linear results. In addition, one should note that the results of this method, for transonic Mach numbers, have the same behavior of the uncorrected linear ones, regarding the transonic dip curve slope. This is an indication that the transonic dip behavior is closely related to the contribution of the imaginary part of the pressures.

In the case of the successive kernel expansion procedure, the computed correction factors result from the matching of nonlinear unsteady pressures, composed by nonlinear steady mean flow pressures and a linear unsteady pressures contributions. Therefore, the correction factor will take into account in an approximate form the unsteady transonic flow behavior, since its computation is referred to those nonlinear unsteady pressures. In summary, the dip slope, predicted by the successive kernel expansion method is increased, because the correction procedure takes into account the influence of the real and imaginary part of the transonic unsteady pressures. Thus, this improvement in the reference conditions leads to better results in approaching the experimental measurements

- ***Flutter Boundaries of the PAPA Wing*** (Farmer and Rivera., 1988)

The flutter boundaries of the PAPA wing at $\alpha=1°$ and $\alpha=-2°$ computed by DWM and ZTAW (SKEM) are presented in Fig. 8, and compared to the wind-tunnel measurements. It can be seen that DWM largely underpredicts the flutter dynamic pressures, whereas they are well predicted by ZTAW. Similar trend of the flutter boundaries of the PAPA wing at $\alpha=2°$ is shown in the same figure. Again, ZTAW flutter results agrees better with the test data than that of DWM. One should observe that the downwash correction method largely underpredicts the flutter dynamic pressure while the flutter results of the successive kernel expansion method correlate well with test data, both for the subsonic and the transonic Mach numbers. This is so because the unsteady components of the nonlinear pressures, introduced by the correction procedure, play an important role in the flutter computation.

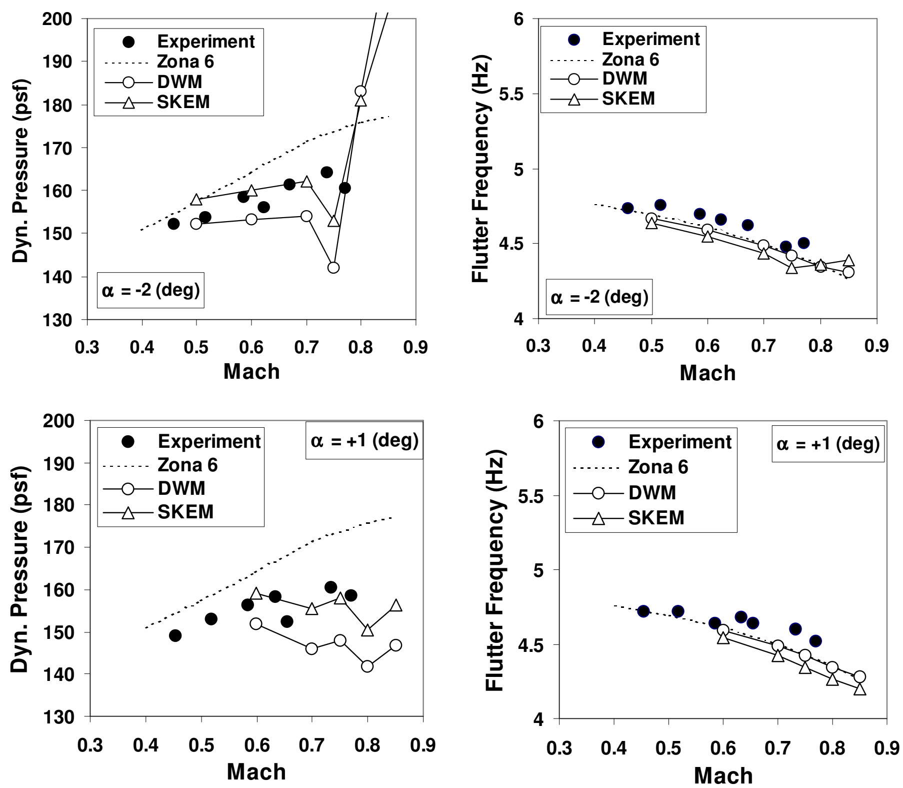

Figure 8. Flutter Boundaries of the PAPA Wing.

Another feature to be highlighted, with regard to the successive kernel expansion method, is the capability to predict the flutter of the wing at different initial angles of attack, instead of the linear theory, which can only predict the flutter margins for a zero angle of attack. This is so, because the steady mean flow can be computed at a given steady state angle of attack, and the nonlinear unsteady pressures, used to compute the correction factors, will be obtained from the contribution of the nonlinear steady mean flow pressures added to unsteady linear pressures. Therefore, the angle of attack contribution is included in the mean flow conditions, because unsteady linear pressures, predicted by the linear unsteady aerodynamic model is independent of the angle of attack

6. Conclusions

Almost all previous AIC correction methods based on steady reference conditions are found to yield erroneous out-of-phase pressures especially in terms of shock jump behavior. The procedure here proposed employs a successive kernel expansion method (SKEM) that forms a rational basis for an advanced AIC correction methods called downwash weighting matrix (DWM) method. With in-phase pressures per rigid mode given either by CFD or measurements, SKEM results are validated showing it can yield accurate out-of-phase pressures in general frequency range as well as well-correlated flutter solutions. In addition, SKEM can be extended to cover the full transonic range including $M \geq 1.0$ because the successive kernel expansion method is derived according to a unified acceleration potential formulation covering subsonic, sonic and supersonic Mach numbers.

7. Acknolegdments

The first author would like to thanks Zona Technology for the financial and technical support for this research. The second ens third authors acknowledge the partial support of Conselho Nacional de Desenvolvimento Científico e Tecnológico, CNPq, under the Integrated Project Research Grant No. 501200/2003-7. Support for this research was also provided by the Fundação de Amparo à Pesquisa do Estado de São Paulo, FAPESP under Research Grant No. 00/13768-4.

8. References

Ashley, H.,1980, "Role of Shocks in the 'Sub-Transonic' Flutter Phenomenon", *Journal of Aircraft*, Vol. 17, No. 3, March, pp. 187-195.

Chen, P.C., Lee, H.W., and Liu, D.D.,1993, "Unsteady Subsonic Aerodynamics for Bodies and Wings with External Stores Including Wake Effect", *Journal of Aircraft*, Vol. 30, No. 5, Sep.-Oct., pp. 618-628.

Dowell, E. H. , Bland, S. R., and Williams, M. H.,1983, "Linear/Nonlinear Behavior in Unsteady Transonic Aerodynamics", *AIAA Journal*, Vol. 21, No. 1, January, pp. 38-46.

Farmer, M. G., and Rivera, J. A.,1988, "Experimental Flutter results of a Cambered Supercritical Wing on a Pitch and Plunge Apparatus", presented at Aerospace Flutter and Dynamics Council Meeting, NASA Langley Research Center, May.

Landahl, M. T.,1951, *Unsteady Transonic Flow*, Pergamon Press, New York.

Liu, D. D.,1976, "A Mixed Kernel Function Method for Unsteady Transonic Flow Calculations", Northrop Tech-Brief PC-20-59, March.

Malone, J.B., and Ruo, S.Y.,1983, "LANN Wing Test Program: Acquisition and Application of Unsteady Transonic Data for Evaluation of Three-Dimensional Computational Methods," Air Force Flight Dynamics Laboratory, AFWAL-TR-83-3006, Wright Patterson Air Force Base, Ohio, February.

Miles, J.W.,1959, "Potential Theory of Unsteady Supersonic Flow," Cambridge Univ. Press, London.

Palacios, R.,2001, Climent, H., Karlsson, A., and Winzell, B., "Assessment of Strategies for Correcting Linear Unsteady Aerodynamics Using CFD or Test Results", Proc. of the CEAS/AIAA International Forum on Aeroelasticity and Structural Dynamics, Madrid, Spain, June, pp. 195-210.

Rumsey, C., Biedron, R., Thomas, J.,1997, "CFL3D: Its History and some Recent Applications", NASA-TM 112861, May, presented at the "Godunov's Method for Gas Dynamics" Symposium, Ann Arbor, MI, May, 1-2,1997.

Silva, R. G. A.,2004, "A Study on Correction Methods for Aeroelastic Analysis in Transonic Flow," Doctorate Dissertation, Aeronautics Dept., Technological Institute of Aeronautics (ITA), São José dos Campos, Brazil.

Silva, R. G. A., Mello, O. A. F., and Azevedo, J. L. F,2002, "A Navier-Stokes Based Study into Linearity in Transonic Flow fort Flutter Analysis, AIAA Paper 2002-2971, In: Proceedings of the 32nd AIAA Fluid Dynamics Conference, St. Louis, USA, June.

Tijdeman, H., van Nunen, J. W. G., Kraan, A. N., Persoon, A. J., Poestkoke, R., Roos, R., Schippers, P., and Siebert, C. M.,1979, "Transonic Wind Tunnel Tests on an Oscillating Wing with External Stores, Part II: Clean Wing," Air Force Flight Dynamics Laboratory, AFFDL-TR-78-194, Part II, Wright-Patterson AFB, Ohio. (Also NLR-TR-78106U, Part II).

Yates Jr., E. C.,1988, "*AGARD Standard Aeroelastic Configurations for Dynamic response I-Wing 445.6"*, AGARD Report No. 765.