STROUHAL NUMBER FLUCTUATIONS IN A CYLINDER WAKE

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Abstract. *The Strouhal number for the shedding frequency from a bluff body submited to a fluid flow is usually obtained from autospectral density functions computed by means of Fourier Transform. The vortex shedding frequency is calculated as time average value and than, a unique Strouhal number is calculated for each bluff body shape and flow regime. In this paper, the vortex shedding frequencies from cylinders with three different cross sections in cross flow were obtained from hot wire measurements and presented as time functions, using the wavelet transform technique. The statistics for the Strouhal number fluctuation range were calculated and presented showing that the shedding frequency is not a constant value, but it flutuates inside a frequency range as a function of the body geometry and Reynolds Number.*

Keywords*: Vortex street, Strouhal number, wavelets, turbulence.*

1. Introduction

The Strouhal number (S) is the dimensionless proportionality constant between the predominant frequency of vortex shedding and the free stream velocity (U) divided by cylinder width - or diameter (D) (Blevins, 1990);

$$S = \frac{f_s D}{U} \tag{1}$$

The Strouhal number of a cylinder in a subsonic flow is a function of Reynolds number and, to a lesser extent, surface roughness and free stream turbulence.

Roshko (1952) made an extensive study about the shedding frequency of two dimensional bluff bodies and showed graphics with Strouhal number versus Reynolds number for several geometries. After Roshko various studies were made for other geometries and arrays and for different flow regimes. In Blevins (1990) the Strouhal numbers for several noncircular cross sections are presented as a function of the Reynols number.

Usually, the Strouhal number is considered as a constant value for each regime and the shedding frequency is calculated using the spectrum obtained from Fourier Transform. The frequency obtained is therefore an average value corresponding to the whole observation time.

Nevertheless, recent studies (Indrusiak, 2004) showed that the frequency of the vortex shedding oscillates around a mean value along time, even if the flow is homogeneous and stationary. In this study the Strouhal number was presented as a time function, being the wavelet transform used to obtain the frequency at each time.

The purpose of this experimental work is to investigate the behavior of the mean shedding frequency oscillation along time as a function of bluff body geometry.

2. Experimental Technique

The test section, shown schematically in Fig.1 is a rectangular channel, with 146 mm height (H) and a width (T) of 195 mm. Air was the working fluid, driven by a centrifugal blower through a set of honeycombs and screens, before reaching the test section with about 1% turbulence intensity.

Before the test section, a Pitot tube was placed at a fixed position, to measure the reference velocity for the experiments. The velocity in the test section can set at any velocity value, from rest to the maximum of about 16 m/s, by means of a frequency inverter.

The velocities were measured by means of a DANTEC *StreamLine* constant temperature hot-wire anemometer. Two probes were positioned, as showed in the Fig.1, in the half way up the tunnel. The first one, positioned upstream the bluff body, measures the instantaneous incident velocity. The second one measures the wake velocity fluctuation and is positioned downstream the body, with its axis tangent to the body side. Data acquisitions were performed simultaneously by a Keithley DAS-58 A/D-converter board controlled by a personal computer.

Six geometries were studied: three circular cylinders with diameters (D) of 32 mm, 16 mm and 9 mm, a rectangular cross section prism of 9.5 mm x 16.5 mm, where 9.5 is the width (W) normal to the flow, and two flat plates positioned normal to the flow direction, with widths (W) of 20 and 32 mm. All the bodies were 146 mm high (L). Each geometry was studied for three flow velocities. The main characteristics of the bodies are shown in Tab.1.

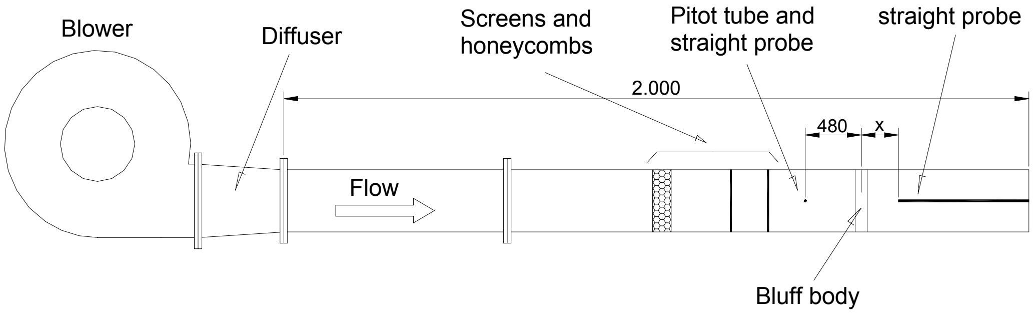


Figure 1. Schematic view of the wind channel. The x gap was set for each geometry.

Table 1. Geometric parameters of the bluff bodies studied.

geometry	dimensions (D or W) (mm)	length (L) (mm)	blockage ratio (D/T or W/T) (%)	aspect ratio (L/D or L/W)
circular cylinder	D 32,1	146	16.46	5.55
circular cylinder	D 16	146	8.21	9,13
circular cylinder	D 9.5	146	4.87	15,37
rectangular prism	9.5	146	4.87	15.37
plate	20	146	10.26	7.3
plate	32	146	16.46	4.55

3. Mathematical tools

3.1. Fourier analysis

The Fourier transform and the Fourier spectrum of a discrete time series enable the study of the bulk spectral behavior of the random phenomenon represented by the series. In practice, in order to minimize the random error, the power spectral density function (PSD) was used, this being the Fourier spectrum of the series smoothed over frequency intervals and over an ensemble of estimates (Bendat and Piersol, 1971).

3.2. Wavelet analysis

While the Fourier transform uses trigonometric functions as basis, the wavelet transforms use functions named wavelets. A wavelet is a finite energy function, with a zero average, also called mother wavelet, because it generates an entire set of wavelet basis:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\,\psi\left(\frac{t-b}{a}\right),\ a, b \in R,\ a > 0 \tag{2}$$

where a and b are respectively scale and position (or time) parameters.

The continuous wavelet transform (CWT) of a function x(t) is given by:

$$Wx(a,b) = \int x(t)\psi_{a,b}(t)dt \tag{3}$$

The respective wavelet spectrum is defined as:

$$P_{xx}(a,b) = |Wx(a,b)|^2 \tag{4}$$

While the Fourier spectrum gives the energy for each frequency over the entire time domain, in the wavelet spectrum, Eq. (4), the energy is related to each time and scale (or frequency). This characteristic of the wavelet transform allows the representation of the distribution of the energy of the transient signal over time and frequency domains.

According to Percival and Walden (2000), the discrete wavelet transform (DWT) is a judicious sub sampling of CWT, dealing with dyadic parameters, j and k, given by:

$$D(j,k) = \sum_t x(t)\psi_{j,k}(t) \tag{5}$$

$$A(J,k) = \sum_t x(t)\phi_{J,k}(t) \tag{6}$$

where $\phi(t)$ is the scaling function associated to the wavelet function , D(j,k) are the details of the signal, related to the frequency bands and A(J,k) is the approximation related to mean of the signal and also the lower frequencies.

The inverse transform of a discrete time series with sampling frequency Fs is done by:

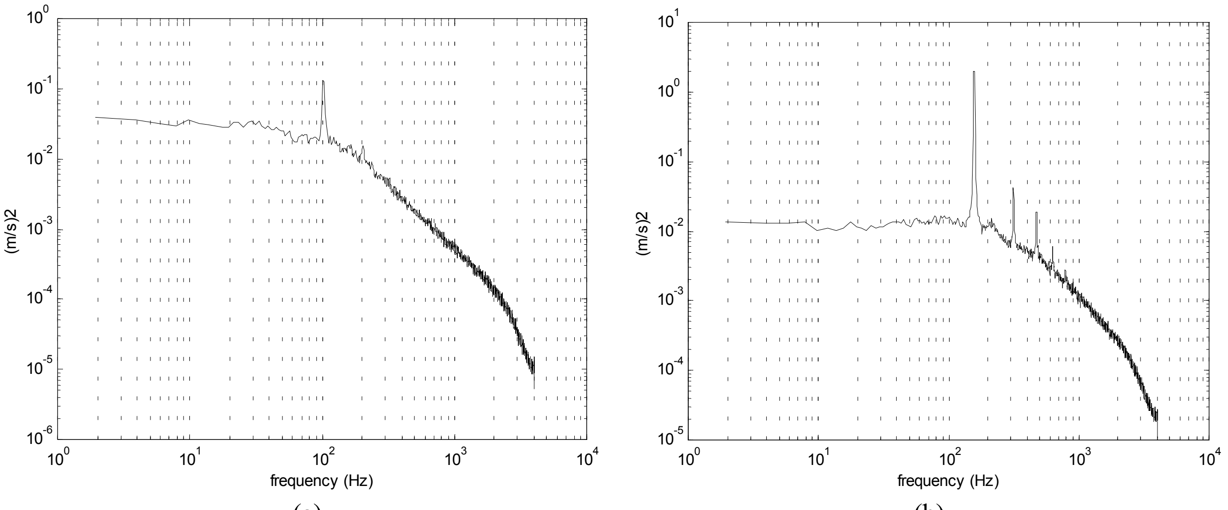
$$x(t) = \sum_k A(J,k)\phi_{J,k}(t) + \sum_{j\leq J}\sum_k D(j,k)\psi_{j,k}(t) \tag{7}$$

where the first term is the approximation of the signal at the scale J, which corresponds to the frequency interval $[0 , Fs/2^{J+1}]$ and the inner summation of the second term are details of the signal at the scales j ($1\leq j\leq J$), which corresponds to frequency intervals $[Fs/2^{j+1} , Fs/2^j]$.

The studies were made using the software Matlab version 5.3 and the toolboxes Signal Processing and Wavelets.

4. Results

Figures 2(a) and 2(b) show the Fourier spectra of the wake velocity fluctuations, obtained using the Welch algorithm (Welch, 1967), for the circular cylinder with 32 mm in diameter and Reynolds number equal to $3,4 \times 10^4$, and for the rectangular prism and Reynolds number equal to $9,8 \times 10^3$, respectively. The velocities were measured in the wake downstream the bluff bodies, as illustrated in Fig.1. The Fourier spectra were smoothed using a frequency band of 1.9 Hz. In both figures the higher peaks denote the shedding frequency. For the rectangular prism there are other two peaks corresponding to approximately the shedding frequency harmonics of 300 Hz and 480 Hz. The results for all bodies and Reynolds numbers studied are shown in Tab. 2. These results, calculated as the previous cases, agree with the results found in the fluid mechanics literature.



(a) (b)

Figure 2. Fourier spectrum for: (a) the circular cylinder with 32 mm in diameter and Re = $3,4 \times 10^4$, and (b) the rectangular prism with 9.5 mm in width and Re = $9,8 \times 10^3$.

Table 2. Experimental results for all bodies and Reynolds numbers studied.

body	dimensions (mm)	reference velocity (m/s)	Reynolds number	vortex frequency(Hz)	Strouhal number
circular cylinder	32	16,08	33903	101,56	0,202
		11,32	23862	70,31	0,199
		5,71	12040	37,11	0,208
	16	16,55	17441	195,31	0,189
		12,01	12660	136,72	0,182
		6,08	6406	66,41	0,175
	9.5	16,67	9883	332,03	0,189
		11,91	7063	228,52	0,182
		6,07	3597	111,33	0,174
rectangular prism	9.5	16,54	9807	156,25	0,090
		11,72	6949	107,42	0,087
		5,98	3547	54,69	0,086
plate	32	14,99	33989	103,52	0,206
		10,54	23895	56,64	0,160
		5,68	11968	29,30	0,165
	20	15,95	21013	121,09	0,152
		11,22	14785	82,03	0,146
		5,67	7474	41,02	0,145

Figure 3 shows the Strouhal numbers as a function of the Reynolds number. The results are in agreement with that of Roshko (1952), with a decrease of the Strouhal numbers at low Reynolds numbers, except for the higher blockage ratio. In this last case, the Strouhal number increases when the Reynolds number decreases, as found by West and Apelt (1982) and Indrusiak (2004).

The wavelet spectrogram calculated using the continuous wavelet transform allows the analysis of the behavior of the shedding frequency along time. Figures 4(a) and 4(b) show the spectrograms plotted on 3-D graphic for the same time series of the Fourier spectra of Fig 2. In this representation, it is possible to see that the higher peak positions, related with the shedding frequencies, vary along time, but it is difficult to identify the temporal localization. For the circular cylinder the higher peaks oscillate in the interval of 90 to 110 Hz, approximately. For the rectangular prism the higher peaks fluctuation occur in a narrow frequency range centered at about 155 Hz, as showed in Fig. 4(b).

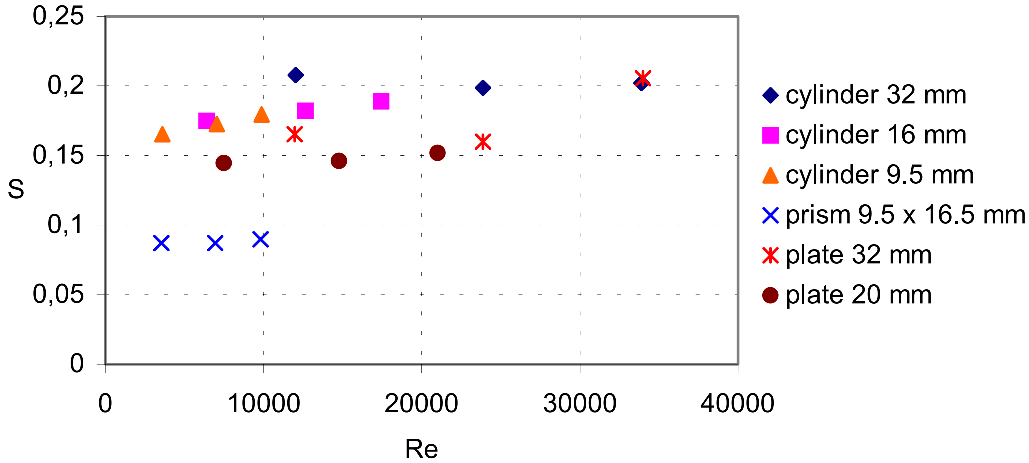
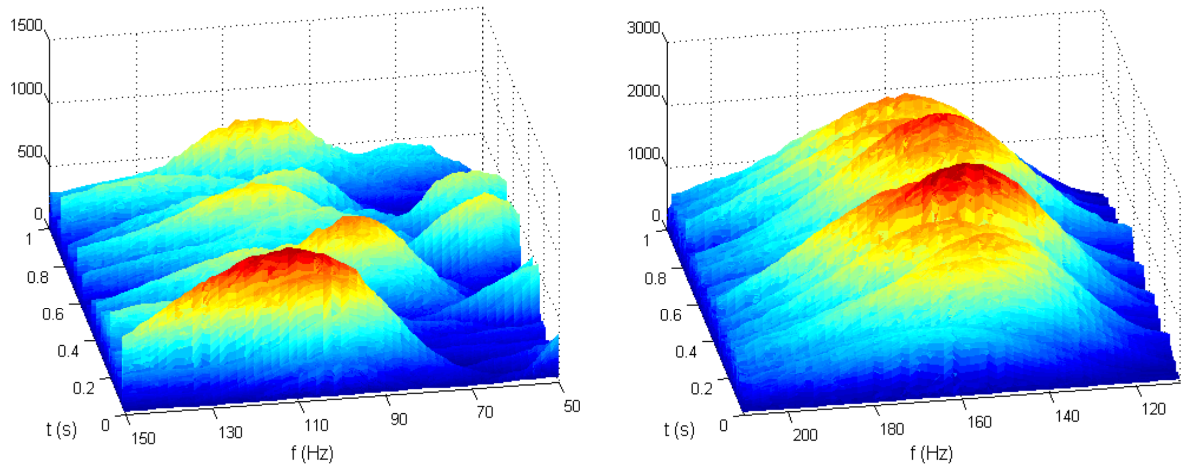


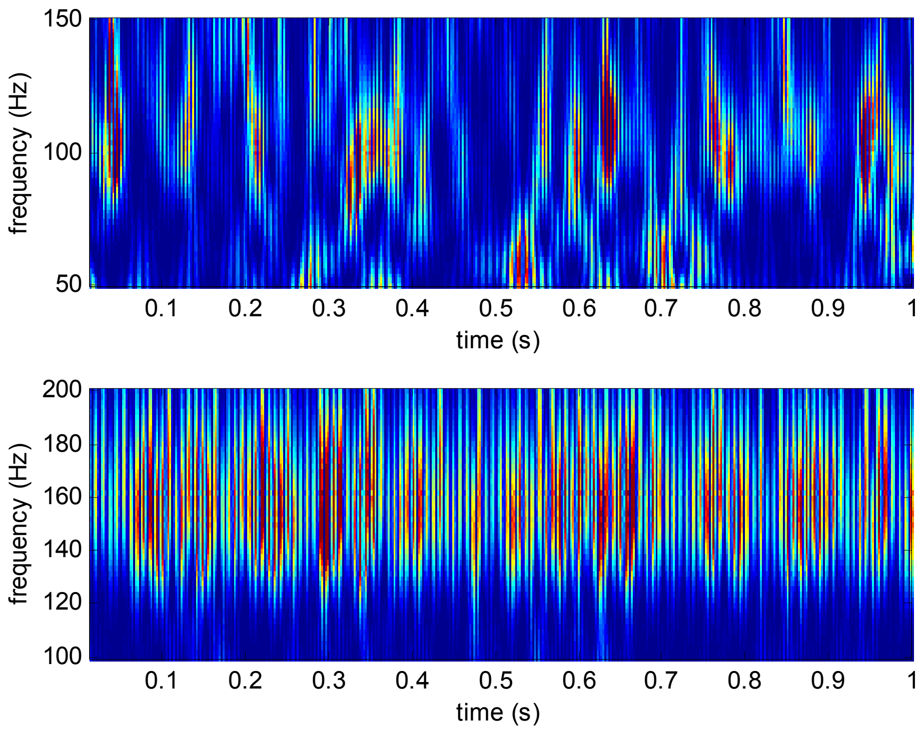
Figure 3. Strouhal number versus Reynolds number for the studied bluff bodies.



(a) (b)

Figure 4. Wavelet spectrum for wake velocity behind the tube with diameter of 32 mm (a), and the rectangular prism with 9.5 mm in width.

A better visualization can be made using a plane graphic, where the temporal localization is obtained more easily. Figures 5 illustrates the wavelet spectrum for the previous cases for a one second time interval, where the color scale is the same of the former figure, with the red color representing the higher coefficients and the blue color the lower ones. Thus, tracking the maximum energy points it is possible to visualize the behavior of the vortex shedding frequency fluctuation along time.



(a)

(b)

Figure 5. Wavelet spectrum for wake velocity behind of: (a) circular cylinder with diameter of 32 mm, (b) rectangular prism with width of 9.5 mm.

To calculate the variation of Strouhal number along time, the frequency associated with the higher wavelet coefficient for each instant is used in Eq.(1). Figures 6 and 7 show the Strouhal numbers for the circular cylinder with 32 mm in diameter and the rectangular prism with 9.5 mm in width, respectively. Notice that the Strouhal number does not correspond always to that expected. This occurs because in several points the energy in the lower frequencies are higher than those related with the vortex shedding, as show in Fig. 6, where the concentration of Strouhal number values in the lower frequencies are not related to the vortex shedding. The values related with the vortex shedding are the ones near S = 0.2 with a strong dispersion around this value. For the rectangular prism the concentration of Strouhal number values is more uniform around the number that corresponds to the vortex shedding - about S=0.09. Also in this figure there are many points concentrated around the lower frequencies that are not related to the vortex shedding.

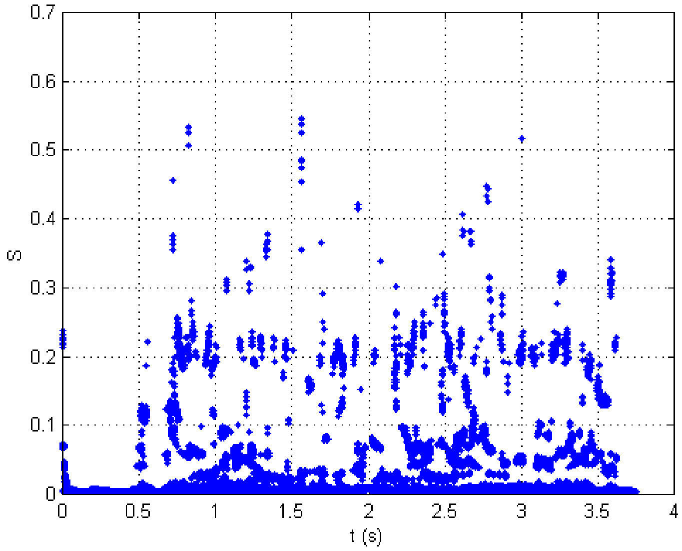


Figure 6. Strouhal number variation along time for the circular cylinder with 32 mm in diameter.

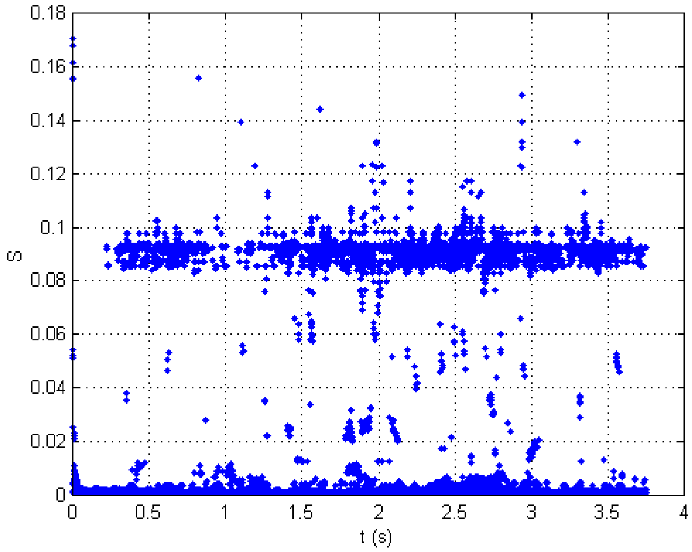
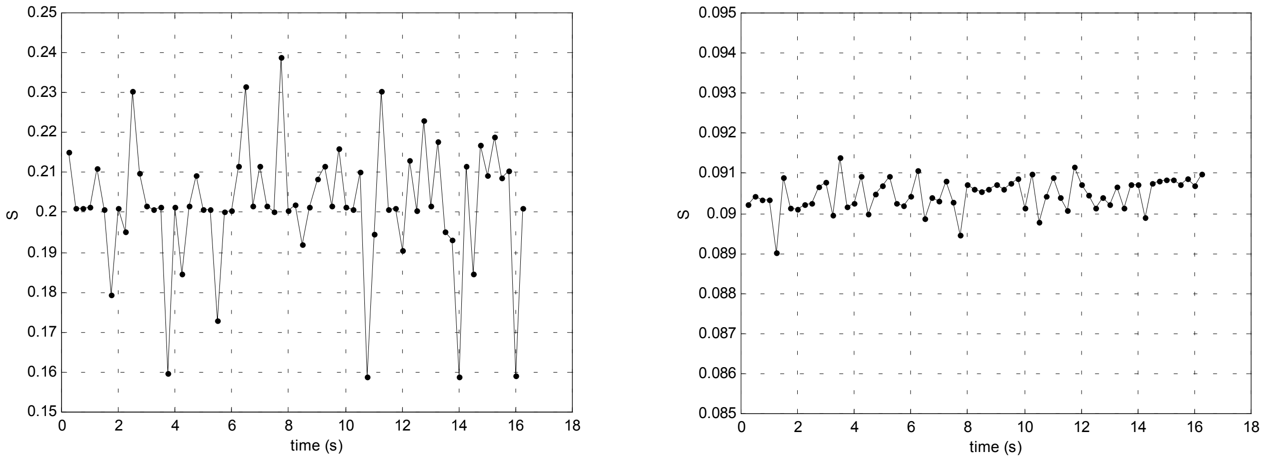


Figure 7. Strouhal number variation along time for the rectangle with 32 mm in width.

In order to make a comprehensive study about the behavior of Strouhal number along time, and considering the uncertainty principle, the wavelet coefficients were integrated in time periods. The resulting spectra show the oscillations and a mean value associated with the shedding frequency for each time period. This procedure was made for all cases studied, using wavelet decomposition in a frequency range of two decades upper and two decades lower them the shedding frequency found through the Fourier transform. Figure 8 shows the behavior of the Strouhal number obtained for the circular cylinder and for the rectangular prism, at Reynolds numbers of 3.4×10^4 and 9.8×10^3, respectively. The results for the other geometries are presented in Tab.3.



(a) (b)

Figure 8. Strouhal number fluctuation along time for: (a) 32 mm circular cylinder, and (b) the rectangular prism.

Table 3. Mean Strouhal number calculated from the wavelet analysis.

body	dimensions (mm)	reference velocity(m/s)	Reynolds number	Strouhal number	standard deviation
circular cylinder	32	16,08	33903	0.2019	0.0159
		11,32	23862	0.1975	0.0185
		5,71	12040	0.1924	0.0388
	16	16,55	17441	0.1870	0.0053
		12,01	12660	0.1790	0.0081
		6,08	6406	0.1758	0.0095
	9.5	16,67	9883	0.1865	0.0058
		11,91	7063	0.1811	0.0051
		6,07	3597	0.1736	0.0034
Rectangular prism	9.5	16,54	9807	0.0905	0.0004
		11,72	6949	0.0871	0.0006
		5,98	3547	0.0874	0.0009
Plate	32	14,99	33989	0.2009	0.0077
		10,54	23895	0.1527	0.0134
		5,68	11968	0.1489	0.0273
	20	15,95	21013	0.1462	0.0118
		11,22	14785	0.1403	0.0163
		5,67	7474	0.1368	0.0217

According to the results presented in Tab. 3, the mean Strouhal numbers obtained from the wavelet analysis are very close to those calculated by means of Fourier analysis. For all the cases the standard deviation indicates how much the values fluctuate with the time. Thus it is possible to associate the width of the peak band in Fourier spectrum with the mean Strouhal number and the respective standard deviation. If the basis of the vortex shedding frequency peak is wide the respective standard deviation is large and the shedding frequency has a large amplitude fluctuation. From Tab.3, also can be verified that for all but one case, when the Reynolds number increases the fluctuation intensity decreases, the largest fluctuation occurs for the circular cylinder, and the minor fluctuation occurs for the rectangular prism.

In order to understand the nature of shedding frequency fluctuation, the Strouhal number fluctuation along time was plotted together with the low frequency fluctuation of the incident velocity measured upstream the bluff bodies. The fluctuation of the incident velocity was obtained by means of a wavelet discrete approximation (Eq.6) for a frequency band of 0 to 1 Hz. The results, plotted in Fig. 9 for a seven seconds interval, for the circular cylinder with 32 mm in diameter and the rectangular prism with 9.5 mm in width, respectively, do not show a relation between the Strouhal number and the mean velocity fluctuations.

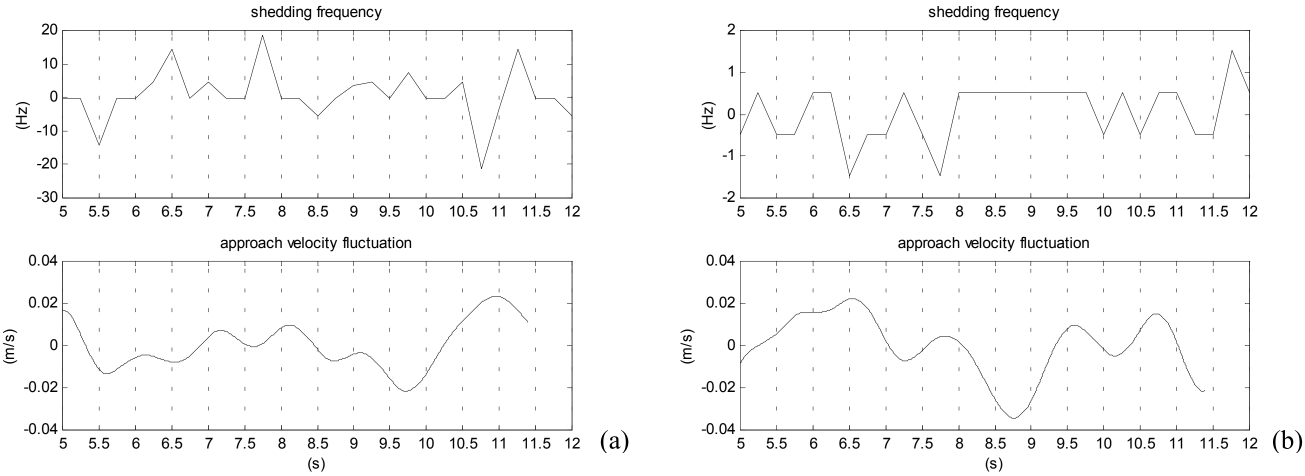


Figure 9. Comparison between Strouhal number oscillation and the incident velocity (a) circular cylinder 32 mm, (b) rectangular prism.

5. Conclusions

In this paper the oscillation of the vortex shedding frequency along time was experimentally investigated for various bluff body shapes. The main conclusions obtained were:

- The vortex shedding frequency for a bluff body in cross flow is not a constant value, but rather it fluctuates within a frequency range which is function of the body geometry and Reynolds number.
- The high blockage ratio is responsible for the increase of the Strouhal number values at lower Reynolds numbers. For low blockage ratio the Strouhal number decreases when the Reynolds number decreases.
- The oscillation of the vortex shedding frequency is not related to the oscillation of the mean incident velocity
- An increase in the Reynolds number causes a decrease in vortex shedding frequency oscillation.
- The circular cylinder wake presented the most fluctuant shedding frequency. In this case, the vortex shedding occurs along the cylinder surface caused by the negative pressure gradient and the point where the shedding occurs can vary as a function to the local energy balance, thus varying the free shear layer distance, and also the vortex shedding frequency. On the plates and the rectangular prism, the shedding point is determinated by the edges being thus expected a more stable shedding frequency. For the plate case, the shedding points are the plate edges, where also occur the flow division, which can cause instabilities and recirculations. In the case of prism the upstream edges are involved in the process of division of the flow, however along of the sidewalls the boundary layer is formatted and regularize the flow again. Thus the vortex shedding occurs at the downstream edges in a more steady form. Accordingly, the oscillations of the Strouhal number are more pronounced for the plate than for the prism.

The results of this study show that the shedding frequency is not a fixed value, and thus, for such geometries, it is very important a judicious choice of the design safety margin that consider this phenomenon.

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