ON EXERGY DESTRUCTION AND MAXIMUM POWER OUTPUT IN HEAT ENGINES.

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Abstract. *It was repeatedly reported that, in the linear transfer approximation, finite size mechanical, hydraulic and electrical engines achieve maximum power at an efficiency of 50%. However, according to the well-known Curzon-Ahlborn formula, heat engines apparently fail to follow this rule. This paper demonstrates that in the Newton heat transfer law approximation, maximum power from heat engines is achieved provided that out of the total exergy expended in the machine, half of it is destroyed due to irreversibility while the other half is transformed into exergy power. Otherwise said, at the point of maximum power, the efficiency of a heat engine is half the Carnot efficiency. The relationships between the total exergy power expended, the exergy produced and the exergy destruction rate is illustrated using a particular Carnot factor- heat diagram.*

Keywords: *Exergy, Maximum Power, Heat Engine, Modeling*

1. Introduction

Some authors have previously reported an interesting result regarding finite size steady state mechanical (Odum and Pinkerton, 1955), hydraulic and electromechanical engines (Bejan, 1996). They demonstrated that in the linear model approximation, the maximum power of the engines is achieved when half of the available input power is dissipated and the other half is transformed into output power. In other words, this means that maximum power corresponds to a thermodynamic efficiency of 50%. LeGoff and Tondeur (2002) confirmed these results and emphasized the fact that only heat engines do not abide by this rule. Indeed, Curzon and Ahlborn (1975) demonstrated that, in the linear heat transfer approximation, maximum power of a heat engine occurs at efficiency different from 50%. Here, from exergy balance considerations, it is demonstrated that heat engines follow the same 50% rule provided that the efficiency is defined in terms of the exergy power produced and the exergy rate expended within an engine.

2. Exergy Balance around a Heat Engine and Maximum Power Production.

The simple heat engine model used by Novikov (1958) can be represented as a combination of irreversible and reversible subsystems (see figure 1). Q_{in} flows irreversibly through a fixed thermal resistance with a corresponding temperature drop from T_1 to T_2 and through a reversible Carnot engine. Power output W is produced and outlet heat Q_{out} is discarded at T_3.

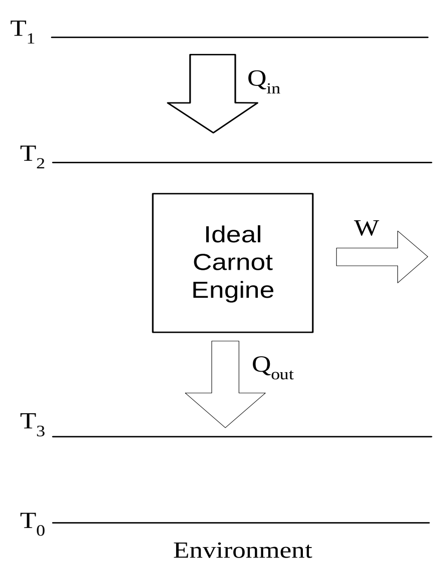

Figure 1. The endoreversible heat engine model proposed by Novikov.

The exergy balance around the system yields:

$$Q_{in}\theta_1 = Q_{out}\theta_3 + W + D \tag{1}$$

where D represents the exergy destruction rate. $\theta_i = 1 - T_0 / T_i$ is the Carnot function (Brodyansky et al., 1993) with i = 1 to 3 and T_0 is the environment temperature.

By combining the energy balance around the engine,

$$Q_{in} = Q_{out} + W \tag{2}$$

with equation (1), one obtains:

$$Q_{in}(\theta_1 - \theta_3) = W(1-\theta_3) + D \tag{3}$$

The term $W(1-\theta_3)$ is the produced exergy power, P_{ex} while the term $Q_{in}(\theta_1 - \theta_3)$ is the total exergy rate expended, ΔE_{exp}. Thus, equation (3) has the general form:

$$\Delta E_{\exp} = P_{ex} + D \tag{4}$$

The exergy destruction rate D s given by:

$$D = Q_{in}(\theta_1 - \theta_2) \tag{5}$$

In the Newton linear law approximation (Hoffmann et al., 1997), the input heat may be expressed as:

$$Q_{in} = \frac{k}{T_0}(\theta_1 - \theta_2) \tag{6}$$

where k is the phenomenological Onsager coefficient.
Solving equations (3) (5) and (6) for W, one gets:

$$W = \frac{k}{T_0(1-\theta_3)}(\theta_1 - \theta_2)(\theta_2 - \theta_3) \tag{7}$$

For given values of k, T_0, θ_1 and θ_3 the produced power depends upon only one variable θ_2. The maximum power occurs when

$$\frac{dP_{ex}}{d\theta_2} = 0 \tag{8}$$

from which it follows:

$$\theta_2 = (\theta_1 + \theta_3)/2 \tag{9}$$

By substitution in equation (5) and using the result in equations (4), one obtains:

$$P_{ex}(\max) = D = \Delta E_{\exp}/2 \tag{10}$$

Hence, at the point of maximum power, half of the expended exergy rate is destructed and the other half is transformed into useful output power.

Another way of interpreting this result is obtained by considering the expression of the produced to expended exergy power ratio:

$$P_{ex} / \Delta E_{\exp} = \frac{W}{Q_{in}} \bullet \frac{(1-\theta_3)}{(\theta_1 - \theta_3)} = \frac{W}{Q_{in}} \bigg/ \left(1 - T_3 \big/ T_1\right) == \eta \big/ \eta_c \tag{11}$$

where $\dfrac{W}{Q_{in}} = \eta$ is the energy efficiency and η_c is the Carnot efficiency of a thermal engine operating between temperatures T_1 and T_3

Thus, the efficiency of the engine at maximum power is given by:

$$\eta = \frac{\eta_C}{2} \tag{12}$$

or, in other words, the produced to expended exergy power ratio (see equation (11)) is 50%. It is worth noting that this result is valid for a Newton heat transfer law (linear dependence on θ). If the Fourier heat transfer law is used, (linear dependence on temperature), the value of the thermodynamic efficiency at maximum power reduces to the well known Curzon –Ahlborn formula (1975). The usefulness of such a simple heat transfer law approach to the modeling of a complex Otto Engine was recently demonstrated by Fisher and Hoffmann (2004).

3. Graphical representation

Following the work of Sorin et al. (2000, 2002) the bilinear character of the exergy balance (equations (3) and (5)) with respect to intensive parameter θ and extensive parameters Q_{in} and W allows a very practical rectangular representation of the exergy transfer within a heat engine (see figure 2). Work is characterized by the intensity θ = 1, heat – by intensity θ_i. The dimensions of the rectangles are: the height - (θ_1-θ_3) and (1-θ_3); the width - Q_{in} and W correspondingly. So the areas of the rectangles have the dimension of exergy rate, P_{ex} and ΔE_{exp} correspondingly and are called exergy dipoles. The one that represents the produced exergy power is tagged with an up going arrow and named uphill dipole while the other that represents the expended exergy power is tagged with a down going arrow and is named downhill dipole.

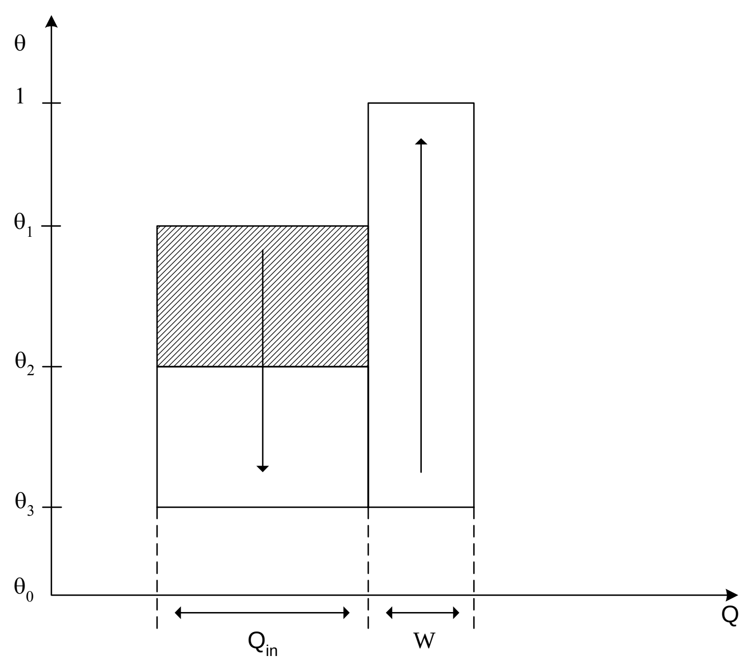

Figure 2. Graphical representation of exergy transfer within a heat engine on Carnot factor-heat diagram.

The exergy destruction rate due to heat transfer is represented by a dashed rectangular area in the downhill dipole. According to equation (3) the uphill dipole is always equal to the un-dashed area of the downhill dipole. According to equation (10), at the point of maximum power, all the dashed and un-dashed area are equal illustrating the fact that, at that point, the produced exergy power and the exergy destruction rates are both equal to half the expended exergy rate.

This simple visual representation may prove to be very handy in the analysis of real and more complex heat transformation systems.

4. Conclusion.

It is shown that, in the linear transfer law approximation, the maximum power from a heat engine is achieved provided that half of the exergy carried by the inlet heat flow and expended within an engine from the inlet to outlet temperature is destroyed due to irreversibility and the other half is transformed into the exergy power. This is a generalization that includes the 50% efficiency rule previously reported for pure work transformation devices different from heat engines and operated at maximum output power.

A visual representation that illustrates the exergy power balance around a heat engine is presented.

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