MODELING OF A VEHICLE SUSPENSION WITH NON LINEAR ELEMENTS AND PERFORMANCE COMPARISON TO A SEMI-ACTIVE MODEL

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Abstract. *This work describes the analysis and comparison of a vehicle suspension semi-active controlled to a non linear passive system. The mathematic model of a suspension system, detailed in this work, has seven degrees of freedom in order to represent a full vehicle system. The two semi-active control laws used in this work are based on the skyhook theory. In the first one the damping coefficient is continuously variable (semi-active CVD) and, in the second one the damping coefficient can assume a maximum or a minimum value (semi-active ON-OFF). To analyze the systems, it was developed a program using the SIMULINK computational tool. This program can evaluate different situations of vehicle suspension systems. The results show that the non linear passive system, semi-active ON-OFF and semi-active CVD alternate the better performance. To have a better understanding of semi-active performance, an optimization of the parameters used in the control laws is needed. This work also explains the importance of considering the non linear behavior of passive systems elements.*

Keywords: *Vehicle suspension, Non linear system, Semi-active control, Skyhook theory*

1. Introduction

The development of control laws for intelligent suspensions has been object of research in the last decades. These systems are basically requested to provide comfort, keeping the ability in providing security to the passengers of the vehicle.

The main studied systems can be divided between the active systems and the semi-active ones. An actuator generally represents the active systems that, with an associated control law, it substitutes in efficient way, the shock absorber and the spring of the passive system. The semi-active systems control only the dissipative elements, in which the dissipative law can be actively modulated. These systems, theoretically, present performance, in comfort terms, similar to the completely active systems, with lower costs, lesser weight of the control system and minor energy demand.

In turn, the passive suspensions systems are still sufficiently competitive, due to its relative simplicity, reliability, reduce costs and not needing a power supply. However, the performance of this system, in term of comfort, is theoretically inferior to the one achieved with active and semi-active controllers.

Most of researches in this area indicate that the controlled, active or semi-active systems, add value to the system, mainly, in terms of comfort for the passengers. The problem raised in this work is that these researches generally compare the active or semi-active systems with passive systems that present linear characteristics of spring and shock absorbers. However, one knows that such passive components have a not linear behavior, what already represents a relatively high profit for the performance of the system.

This work verify, using a model of seven degrees of freedom (full vehicle), the real benefits of semi-active control laws (ON-OFF and Continuous Variable Damper), when compared with a passive system, considering the nonlinear dynamics of the components spring and shock absorber.

2. Mathematical Model

2.1. Vehicle Model

For the mathematical analysis of the passive and semi-active suspensions, the full vehicle model was adopted, based in the proposal of Ikenaga (1999), as figure 1. This model represents a system of seven nonlinear degrees of freedom and consists of a "sprung mass" (body of the vehicle) connected to four "unsprung masses" (axles of the vehicle).

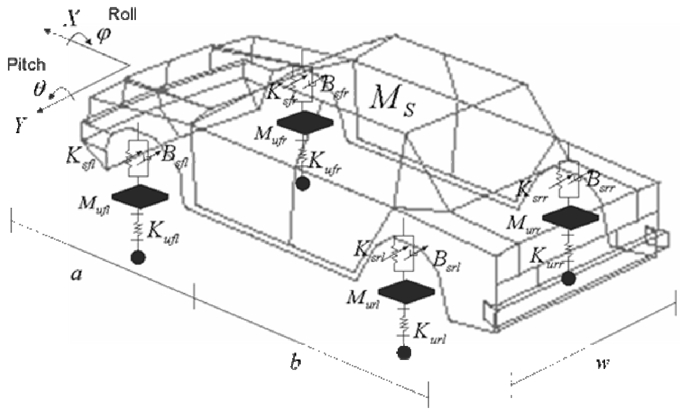

Figure 1: Full vehicle model

2.2. Mathematical equations

- Body Vertical Motion:

$$\begin{aligned} M_s\ddot{z} &= -M_s g - \left(K_{sfl}+K_{sfr}+K_{srl}+K_{srr}\right)z - \left(B_{sfl}+B_{sfr}+B_{srl}+B_{srr}\right)\dot{z} + \left(aK_{sfl}+aK_{sfr}-bK_{srl}-bK_{srr}\right)\theta + \ldots \\ &+ \left(aB_{sfl}+aB_{sfr}-bB_{srl}-bB_{srr}\right)\dot{\theta} + K_{sfl}z_{ufl} + B_{sfl}\dot{z}_{ufl} + K_{sfr}z_{ufr} + B_{sfr}\dot{z}_{ufr} + K_{srl}z_{url} + B_{srl}\dot{z}_{url} + K_{srr}z_{urr} + B_{srr}\dot{z}_{urr} \end{aligned} \tag{1}$$

- Pitch Motion:

$$\begin{aligned} I_{yy}\ddot{\theta} &= \left(aK_{sfl}+aK_{sfr}-bK_{srl}-bK_{srr}\right)z + \left(aB_{sfl}+aB_{sfr}-bB_{srl}-bB_{srr}\right)\dot{z} - \left(a^2K_{sfl}+a^2K_{sfr}+b^2K_{srl}+b^2K_{srr}\right)\theta - \ldots \\ &- \left(a^2B_{sfl}+a^2B_{sfr}+b^2B_{srl}+b^2B_{srr}\right)\dot{\theta} - aK_{sfl}z_{ufl} - aB_{sfl}\dot{z}_{ufl} - aK_{sfr}z_{ufr} - aB_{sfr}\dot{z}_{ufr} + bK_{srl}z_{url} + bB_{srl}\dot{z}_{url} + \ldots \\ &+ bK_{srr}z_{urr} + bB_{srr}\dot{z}_{urr} \end{aligned} \tag{2}$$

- Roll Motion:

$$\begin{aligned} I_{xx}\ddot{\varphi} &= -0.25w^2\left(K_{sfl}+K_{sfr}+K_{srl}+K_{srr}\right)\varphi - 0.25w^2\left(B_{sfl}+B_{sfr}+B_{srl}+B_{srr}\right)\dot{\varphi} + 0.5wK_{sfl}z_{ufl} + 0.5wB_{sfl}\dot{z}_{ufl} + \ldots \\ &- 0.5wK_{sfr}z_{ufr} - 0.5wB_{sfr}\dot{z}_{ufr} + 0.5wK_{srl}z_{url} + 0.5wB_{srl}\dot{z}_{url} - 0.5wK_{srr}z_{urr} - 0.5wB_{srr}\dot{z}_{urr} \end{aligned} \tag{3}$$

- Front Left Axle Vertical Motion:

$$M_{ufl}\ddot{z}_{ufl} = -M_{ufl}g + K_{sfl}z + B_{sfl}\dot{z} - aK_{sfl}\theta - aB_{sfl}\dot{\theta} + 0.5wK_{sfl}\varphi + 0.5wB_{sfl}\dot{\varphi} - \left(K_{sfl}+K_{ufl}\right)z_{ufl} - B_{sfl}\dot{z}_{ufl} + K_{ufl}z_{rfl} \tag{4}$$

- Front Right Axle Vertical Motion:

$$M_{ufr}\ddot{z}_{ufr} = -M_{ufr}g + K_{sfr}z + B_{sfr}\dot{z} - aK_{sfr}\theta - aB_{sfr}\dot{\theta} - 0.5wK_{sfr}\varphi - 0.5wB_{sfr}\dot{\varphi} - \left(K_{sfr}+K_{ufr}\right)z_{ufr} - B_{sfr}\dot{z}_{ufr} + K_{ufr}z_{rfr} \tag{5}$$

- Rear Left Axle Vertical Motion:

$$M_{url}\ddot{z}_{url} = -M_{url}g + K_{srl}z + B_{srl}\dot{z} + bK_{srl}\theta + bB_{srl}\dot{\theta} + 0.5wK_{srl}\varphi + 0.5wB_{srl}\dot{\varphi} - \left(K_{srl}+K_{url}\right)z_{url} - B_{srl}\dot{z}_{url} + K_{url}z_{rrl} \tag{6}$$

- Rear Right Axle Vertical Motion:

$$M_{urr}\ddot{z}_{urr} = -M_{urr}g + K_{srr}z + B_{srr}\dot{z} + bK_{srr}\theta + bB_{srr}\dot{\theta} - 0.5wK_{srr}\varphi - 0.5wB_{srr}\dot{\varphi} - \left(K_{srr}+K_{urr}\right)z_{urr} - B_{srr}\dot{z}_{urr} + K_{urr}z_{rrr} \tag{7}$$

The state equation is:

$$\begin{aligned} \dot{x} &= Ax(t) + Zz(t) \\ y &= Cx(t) \end{aligned} \tag{8}$$

being that matrix **A** represents the system dynamics, the matrix **Z** represents the system entrees, matrix **C** represents the system exit, the vector **x(t)** represents the state variables and the vector **z(t)** represents the entrees in each wheel.

In this work, the system is considered nonlinear. To model the non-linearity, it makes necessary a continuous variation of the coefficients of rigidity and damping for each one of the wheels, it means that the matrix **A** is not constant, varying in function of these coefficients. The matrix **A** was subdivided in nine matrices (aa, ab, ac, ad, ae, af, ag, ah and ai), being that the matrix aa is composed for the constant elements of the system and the others are related to the coefficients of rigidity and damping of the system, as equation 9:

$$\begin{aligned} \dot{x} &= aax(t) + K_{sfr}abx(t) + K_{sfl}acx(t) + K_{srr}adx(t) + K_{srl}aex(t) + \ldots \\ &+ B_{sfr}afx(t) + B_{sfl}agx(t) + B_{srr}ahx(t) + B_{srl}aix(t) + Zz(t) \\ y &= Cx(t) \end{aligned} \tag{9}$$

2.3. Spring non-linearity

One of the main contributions of the present work is the analysis of the suspension performance, considering the nonlinear behavior of its components. The determination of non-linearity, as much of the spring, as of the shock absorber, is based on the study developed by Rill (2002). For the non-linearity simulation of the spring, the following equations had been considered:

$$\text{If } X > 0 \Rightarrow K = K_{static} \cdot (1 + X) \tag{10}$$

$$\text{If } X < 0 \Rightarrow K = K_{static} \cdot (1 - X) \tag{11}$$

X is the rattle space (suspension deflection). Must be considered the pitch and roll influences.

K_{static} is the static stiffness constant.

As result of this spring non-linearity modeling, it can be noticed in the characteristic curve in figure 2, the dynamic behavior of this suspension component.

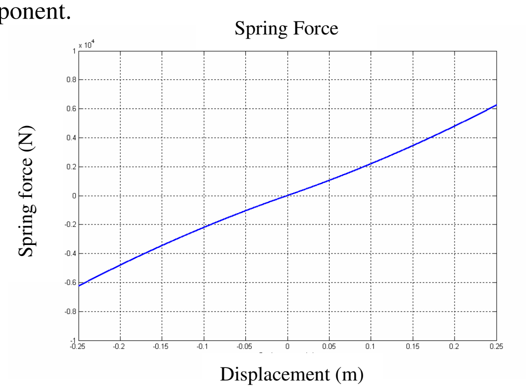

Figure 2: Nonlinear spring

2.4. Damper non-linearity

For the simulation of damper non-linearity, the following equation had been considered:

$$\text{If } X > 0 \Rightarrow \text{B} = \frac{C_{static}}{(1 + 0.3 \cdot X)} \tag{12}$$

$$\text{If } X < 0 \Rightarrow \text{B} = \frac{C_{static}}{(1 - 0.9 \cdot X)} \tag{13}$$

X is the rattle space (suspension deflection). Must be considered the pitch and roll influences.

C_{static} is the static damper constant.

As result of this non-linearity modeling of the shock absorber, it can be noticed in the characteristic curve in figure 3, the dynamic behavior of this suspension component

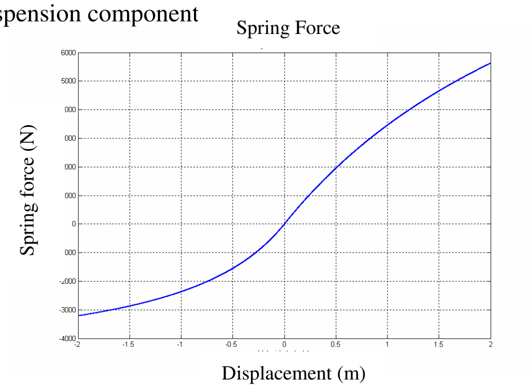

Figure 3: Nonlinear damper

2.5. Control laws

The main objective of a controller is the determination of the damping coefficient desired for the system. In this work, the control strategy is based on Hyvärinen (2004) and on the theory of skyhook. The coefficient of desired damping is function of three components:

- Vehicle Vertical Displacement;
- Pitch Angle;
- Roll Angle.

For the calculation of the vertical displacement component, the body is considered fixed in the "sky", as figure 4. In accordance with equation 14, the damping force created by the skyhook shock absorber is function of the absolute body vertical speed and the ideal skyhook coefficient:

$$f_{s,h} = c_{s,h} \cdot \dot{z} \tag{14}$$

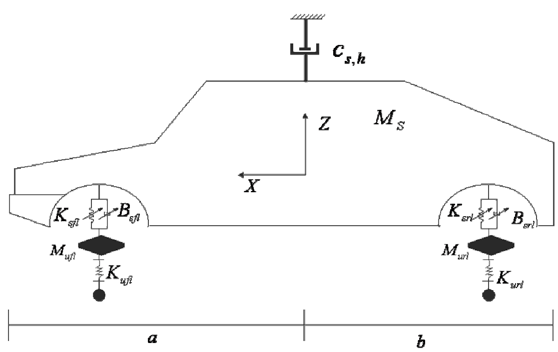

Figure 4: Skyhook vertical displacement

For optimization of the skyhook damping coefficient, it is usual to consider it function of the system critical damping coefficient, as equation:

$$c_{s,h} = \frac{\sqrt{2}}{2} \cdot c_{c,h} \tag{15}$$

where,

$$c_{c,h} = 2 \cdot \sqrt{k \cdot M_s} \tag{16}$$

For the calculation of the pitch angle component, the body is considered fixed in the "sky", as figure 5. In accordance with equation 17, the damping force created by the skyhook shock absorber is function of the absolute pitch angular speed and the ideal skyhook coefficient:

$$f_{s,p} = c_{s,p} \cdot \dot{\theta} \tag{17}$$

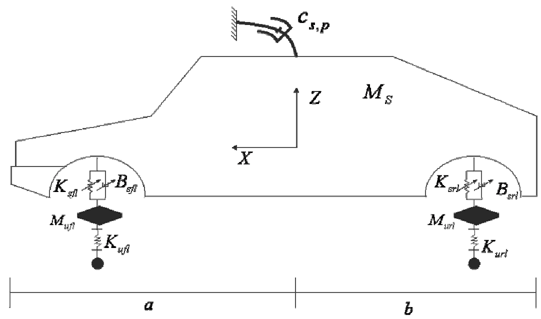

Figure 5: Skyhook pitch angle

The skyhook damping coefficient, considering the optimization is:

$$c_{s,p} = \frac{\sqrt{2}}{2} \cdot c_{c,p} \tag{18}$$

where, for the front wheels:

$$c_{c,p} = \frac{2 \cdot \sqrt{k \cdot I_{yy}}}{a} \tag{19}$$

and, for the rear wheels:

$$c_{c,p} = \frac{2 \cdot \sqrt{k \cdot I_{yy}}}{b} \tag{20}$$

For the calculation of the roll angle component, the body is considered fixed in the "sky", as figure 6. In accordance with equation 21, the damping force created by the skyhook shock absorber is function of the absolute roll angular speed and the ideal skyhook coefficient:

$$f_{s,r} = c_{s,r} \cdot \dot{\varphi} \tag{21}$$

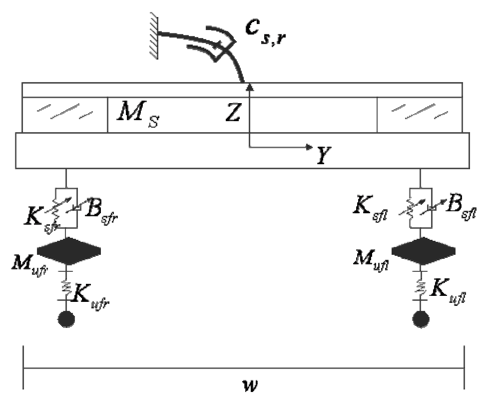

Figure 6: Skyhook roll angle

The skyhook damping coefficient, considering the optimization is:

$$c_{s,r} = \frac{\sqrt{2}}{2} \cdot c_{c,r} \tag{22}$$

where,

$$c_{c,r} = \frac{2 \cdot \sqrt{k \cdot I_{xx}}}{\left(w/2\right)} \tag{23}$$

Calculating the moment sum equal zero in one determined wheel, the value desired for the real coefficient damping is determined:

$$C_{s,fl}=\frac{\left[\left(1.4142\cdot\sqrt{k\cdot M_s}\right)\dot{z}-\left(\frac{1.4142\cdot\sqrt{k\cdot I_{yy}}}{a\text{ or }b}\right)\dot{\theta}+\left(\frac{1.4142\cdot\sqrt{k\cdot I_{xx}}}{\left(w/2\right)}\right)\dot{\varphi}\right]}{\left(\dot{z}_s-\dot{z}_u\right)}$$
$$C_{s,fr}=\frac{\left[\left(1.4142\cdot\sqrt{k\cdot M_s}\right)\dot{z}-\left(\frac{1.4142\cdot\sqrt{k\cdot I_{yy}}}{a\text{ or }b}\right)\dot{\theta}-\left(\frac{1.4142\cdot\sqrt{k\cdot I_{xx}}}{\left(w/2\right)}\right)\dot{\varphi}\right]}{\left(\dot{z}_s-\dot{z}_u\right)} \quad (24)$$
$$C_{s,rl}=\frac{\left[\left(1.4142\cdot\sqrt{k\cdot M_s}\right)\dot{z}+\left(\frac{1.4142\cdot\sqrt{k\cdot I_{yy}}}{a\text{ or }b}\right)\dot{\theta}+\left(\frac{1.4142\cdot\sqrt{k\cdot I_{xx}}}{\left(w/2\right)}\right)\dot{\varphi}\right]}{\left(\dot{z}_s-\dot{z}_u\right)}$$
$$C_{s,rr}=\frac{\left[\left(1.4142\cdot\sqrt{k\cdot M_s}\right)\dot{z}+\left(\frac{1.4142\cdot\sqrt{k\cdot I_{yy}}}{a\text{ or }b}\right)\dot{\theta}-\left(\frac{1.4142\cdot\sqrt{k\cdot I_{xx}}}{\left(w/2\right)}\right)\dot{\varphi}\right]}{\left(\dot{z}_s-\dot{z}_u\right)}$$

For the semi-active control of the suspension system, two control laws had been adopted, one considers that the damping coefficient varies continuously (CVD - Continuously Variable Damper) and to another one considers that the damping coefficient can assume only two values (ON-OFF).

2.5.1. CVD control law

This control strategy considers that the damping coefficient varies continuously. This variation is limited to a maximum value ($C_{\max}$) and a minimum value ($C_{\min}$).
In this work, the coefficients maximum and minimum had been defined as:

$$\begin{aligned} C_{\min} &= \min\left\{0{,}1\cdot c_{c,h},0{,}1\cdot c_{c,p},0{,}1\cdot c_{c,r}\right\} \\ C_{\max} &= \max\left\{0{,}25\cdot c_{c,h},0{,}25\cdot c_{c,p},0{,}25\cdot c_{c,r}\right\} \end{aligned} \quad (25)$$

In the following equations, it can be verified the adopted laws of control, for each one of the vehicle wheels:

$$\text{If } \dot{z}\cdot\left(\dot{z}-\dot{z}_u\right)\le 0 \quad \Rightarrow \quad B_s = C_{\min} \quad (26)$$

$$\text{If } \dot{z}\cdot\left(\dot{z}-\dot{z}_u\right)> 0$$
$$\Rightarrow B_s=\begin{cases} C_{\max}, & C_s > C_{\max} \\ C_s, & C_{\min} < C_s \le C_{\max} \\ C_{\min}, & C_s \le C_{\min} \end{cases} \quad (27)$$

2.5.2. ON-OFF control law

This strategy considers that the damping coefficient is a maximum value ($C_{\max}$) or a minimum value ($C_{\min}$).

In the following equations, it can be verified the adopted laws of control, for each one of the vehicle wheels:

$$\text{If } \dot{z}\cdot\left(\dot{z}-\dot{z}_u\right)\le 0 \quad \Rightarrow \quad B_s = C_{\min} \quad (28)$$
$$\text{If } \dot{z}\cdot\left(\dot{z}-\dot{z}_u\right)> 0 \quad \Rightarrow \quad B_s = C_{\max} \quad (29)$$

3. Results

The systems are evaluated in relation to the comfort and the security for the passengers. In this work the comfort is represented by the acceleration of the movements: vertical; pitch and roll and the security is represented by the vertical displacement of the axle, the space of work demanded of the suspension and the tire-way contact.

3.1 Body vertical acceleration

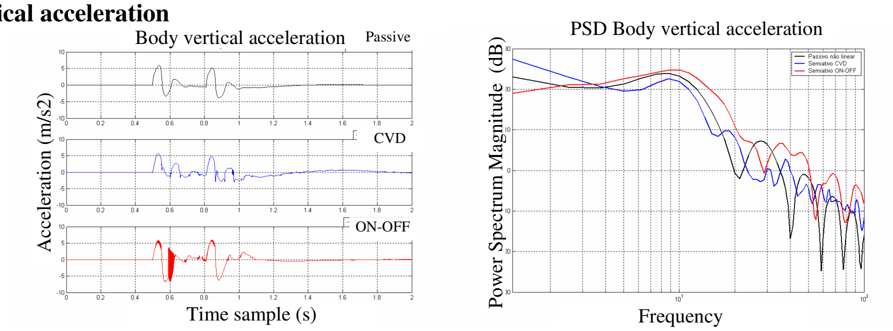

Figure 7: Body vertical acceleration - step excitation

3.2 Pitch acceleration

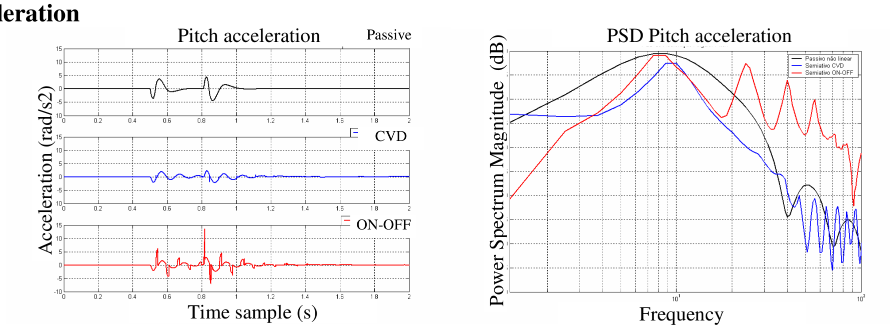

Figure 8: Pitch acceleration – rump excitation

3.3 Roll acceleration

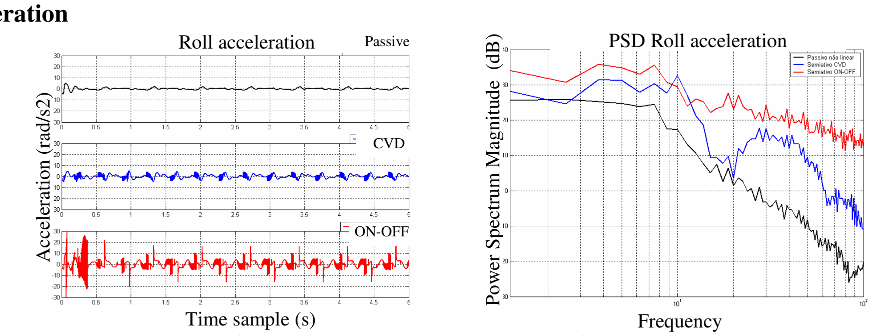

Figure 9: Roll acceleration – sinusoidal excitation

3.4 Axle vertical displacement

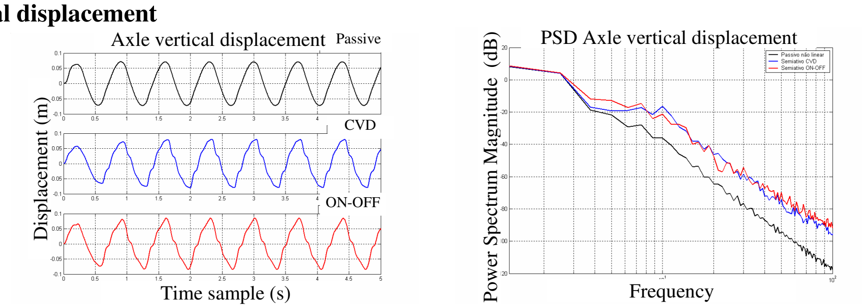

Figure 10: Axle vertical displacement – sinusoidal excitation

3.5 Rattle space

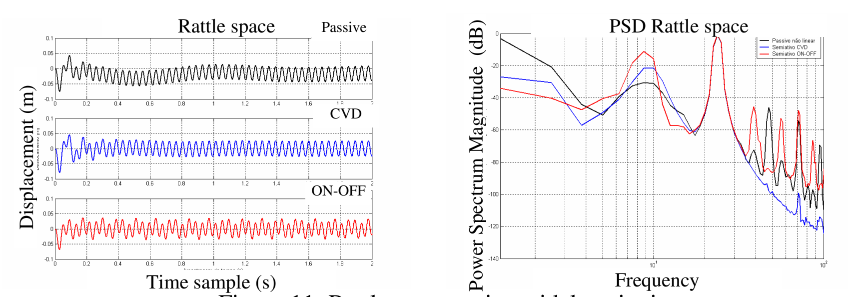

Figure 11: Rattle space – sinusoidal excitation

3.6 Tire-way contact

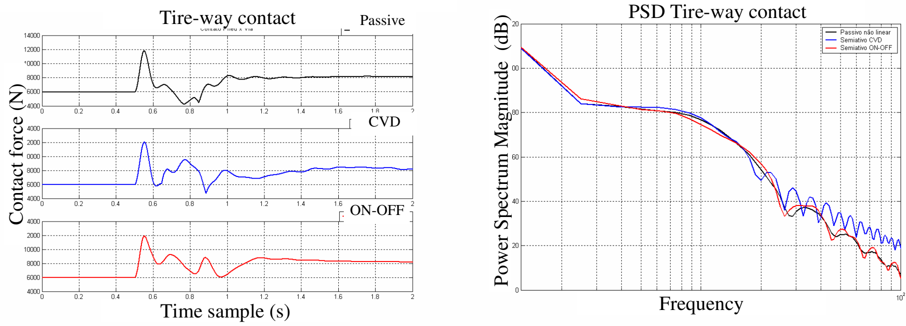

Figure 12: Tire-way contact – step excitation

4. Conclusion

The results demonstrate that an alternation of better performance exists between the systems passive non linear, semi-active ON-OFF and semi-active CVD.

For the passenger security and comfort, except to the roll movement, one semi-active control law (ON-OFF or CVD) presented a better performance that the non linear passive system, however, as evidenced in the results, the gotten profit is not so great, as told in the majority of the bibliographies that deal with the respect this subject, therefore, as already commented previously, in this work was considered the non linear dynamics of the system, what, by itself, already represents an increment in the performance of the system.

The consideration of the non linear dynamics of the shock absorber and spring, for the passive suspension is essential, therefore the comparison of a linear passive system with semi-active and active systems, does not represent the real effect of this system in a vehicle.

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