H_∞ Control with Pole Placement Constraints for Flexible Structures Vibration Reduction

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Abstract.
The vibration control of a flexible structure presents some difficulties for the controller design. The flexible structure is continuous and presents infinite vibration modes. The mathematical model, obtained by finite elements or experimentally identified, is truncated generating model uncertainties. These uncertainties are reflected at the positions of the closed loop poles provoking the spillover phenomenon and instability. Besides, the controllers are in general non-collocated which characterize a more difficult control problem. These aspects require the use of a design technique that considers this kind of difficulty. The H_∞ technique aims the reduction in the highest peak of the respective transfer function. This ensures some robustness to the project. But the H_∞ controller based on a reduced model can not be enough to control the real structure, which motivates the use of additional constraints related to the closed loop pole placement. The active vibration control problem can be formulated using the LMI (linear matrix inequalities) approach which consists of minimizing a linear objective subjected to LMI constraints. The H_∞ control using output feedback with regional pole placement constraints in terms of LMIs is investigated in this work for a flexible beam. Some numerical results are presented and discussed in the paper.
Keywords: *H_∞ Control, Flexible Structures Control, Vibration Control, Linear Matrix Inequalities, LMI region.*

1. Introduction

The vibration control of flexible structures can be considered a difficult problem for the controller design because of the following aspects (see Balas, 1989):

- The flexible structures are continuous and present infinite vibration modes with little natural damping and modes densely packed through the frequency domain.
- The mathematical model, obtained by finite elements or experimentally identified, is truncated generating model uncertainties. These uncertainties are reflected at the positions of the closed loop poles provoking the spillover phenomenon and instability. Therefore, robust control design methods must be developed to account for model uncertainties.
- The controllers are in general non-collocated which characterize a more difficult control problem. The non-collocation of the actuators and sensors lead to performance limitations on the control system and introduces potential stability problems.

Some usual approaches to active vibration suppression include: direct output feedback, linear quadratic gaussian (LQG) optimal control, LQG with loop transfer recovery (LQG/LTR) and pole placement (see Balas, 1989). But none of these methods guarantee the necessary robustness to avoid the above mentioned difficulties, which requires the use of a design technique that considers all these issues. The H_∞ technique aims the reduction of the highest peak of the respective transfer function (see, for example, Skogestad and Postlewaite, 1996). This ensures some robustness to the project, but the H_∞ controller based on a reduced model may not be general enough to control real structures. This motivates the use of additional constraints related to the closed loop pole placement. The control problem can then be formulated using the LMI approach, which consists of minimizing a linear objective subjected to LMI constraints (Boyd et al., 1994). Using LMI-based design methods has the convenient advantage of easy mixing of different design objectives (Gahinet et al, 1995).

The H_∞ control using output feedback with regional pole placement constraints in terms of LMIs (Chilali and Gahinet, 1996) is investigated in this work for a flexible beam. Numerical results are presented and discussed in the paper.

The next sections of this paper present a brief description of the H_∞ concepts, the mathematical model of a flexible beam, the regional pole placement LMI-based design, simulated results and finally some discussions and conclusions.

2. H_∞ control scheme

The purpose of active vibration control is to design a controller to attenuate the vibration of the natural modes to acceptable levels ensuring the closed-loop stability. Based on the reduced model of the plant, an important objective is to avoid the influence of the upper residual vibration modes in the system performance. This means not to excite out-of-bandwidth natural frequencies, but also to guarantee that in the bandwidth frequencies the poles remain stable.

In H_∞ control the objective is to minimize the ∞-norm of the transfer function between the disturbance and the desired performance index, and an increase in the robust stability of the system is achieved in general. The solution of the H_∞ problem was originally based on the solution of Riccati equations. More recently, this problem was formulated using linear matrix inequalities. This method searches the solution as a convex minimization problem, and allows the inclusion of additional constraint equations to the problem. For the present case, the closed-loop poles placed in specific regions of the complex plane, written also as LMIs, characterizes the concept of LMI region (Chilali and Gahinet, 1996).

The control problem can be summarized in the block diagram of the Figure 1.

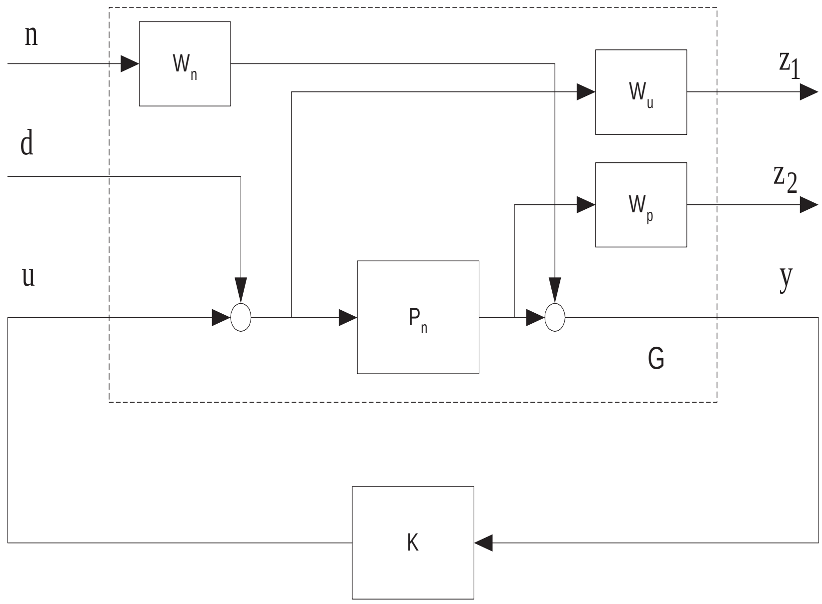

Figure 1. Block diagram of the H_∞ design scheme

Referring to Figure 1, the control signal $u(t)$ is weighted through the filter $W_u(s)$ in order to avoid the spillover, and the filter $W_p(s)$ is applied to the output of the plant $y(t)$ to achieve the vibration level reduction. Two performance signals $z_1(t)$ and $z_2(t)$ respectively the outputs of the filters $W_p(s)$ and $W_u(s)$ are considered. The input signal $d(t)$ represents an input disturbance and $n(t)$ represents a measurement noise. s is the Laplace transform variable.

The nominal plant is here denoted by $P_n(s)$, the real plant is denoted by $P_r(s)$ and the controller is $K(s)$.

The H_∞ design finds a central controller $K(s)$ that minimizes the transfer function norm from the disturbance to the performance output, $||T_{zd}||_\infty$. Given a $\gamma > 0$, it is usual to solve a sub-optimal problem to find $||T_{zd}||_\infty \leq \gamma$, and to minimize γ through iteration.

The robust stability can be analyzed using the small-gain theorem (see Shahian and Hassul, 1993) which states that:

$$||\Delta(s)T_{zd}(s)||_\infty < 1 \tag{1}$$

where $\Delta(s)$ is the uncertainty.

The filter $W_u(s)$ may be designed such that $||W_u||_\infty > ||\Delta||_\infty$ and the robust stability condition becomes:

$$||W_u(s)T_{z1d}(s)||_\infty < 1 \tag{2}$$

which takes care of the first objective, to avoid spillover, assured that $W_u(s)$ presents a frequency distribution guaranteed over the uncertainties.

Similarly, to achieve vibration attenuation the following condition is considered:

$$||W_p(s)T_{z2d}(s)||_\infty < 1 \tag{3}$$

It is possible to combine the result of H_∞ sub-optimal problem and the small-gain theorem in the form:

$$||W_u||_\infty ||T_{z1d}||_\infty < \gamma \Rightarrow ||T_{z1d}||_\infty < \gamma ||W_u^{-1}||_\infty, \tag{4}$$

$$||W_p||_\infty ||T_{z2d}||_\infty < \gamma \Rightarrow ||T_{z2d}||_\infty < \gamma ||W_p^{-1}||_\infty, \quad (5)$$

Finally, the filter $W_n(s)$ is considered here, for the sake of simplicity, a constant gain of 0.08, in order to scale a measurement noise appropriately.

Since the filters were designed related to the bandwidth including the considered modes, the H_∞ control problem can be solved to obtain the controller $K(s)$. The LMI-based approach to a regional pole placement is conducted simply including the additional constraint inequations to the problem.

3. Flexible structure model

In order to verify the applicability of the concepts of H_∞ with LMI region constraints to a flexible structure problem, a cantilever beam was used as shown in Figure 2.

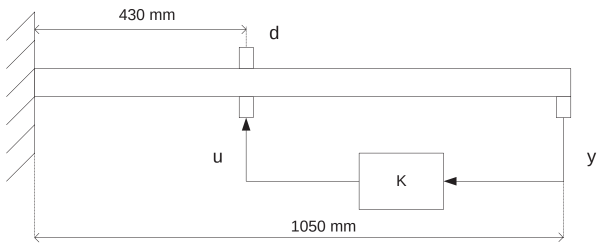

Figure 2. Sensor and actuators positions on the cantilever beam

The dynamic equation that describes the beam vibration is:

$$M\ddot{y} + C\dot{y} + Ky = u(t) + d(t), \quad (6)$$

where M, C and K are respectively the mass, damping and stiffness matrices; $u(t)$ and $d(t)$ are the control and disturbance forces acting on the structure.

The matrices M and K were obtained in this work by the finite element discretization (Desai, 1979). A proportional damping matrix C was considered in order to simplify the model.

The major difficulty associated to the vibration problem of flexible structures is that the real structure has infinity number of frequencies, and a reduced model, used as a nominal plant for the controller design, has to be able to permit an effective controller design for the real structure.

The flexible structure transfer function can be represented as a infinite summation of modes in the form:

$$P_r(s) = \sum_{i=1}^{\infty} \frac{F_i}{s^2 + 2\zeta_i w_i s + w_i}, \quad (7)$$

where w_i is the natural frequency, ζ_i is the modal damping factor, and F_i reflects the scaling related to the positions of the sensor and actuator.

The transfer function $P(s)$ is usually truncated to n modes of interest that reflects the operational conditions and that defines the nominal plant P_n:

$$P_n(s) = \frac{Y(s)}{U(s)} = \sum_{i=1}^{n} \frac{F_i}{s^2 + 2\zeta_i w_i s + w_i}, \quad (8)$$

and the residual modes can be considered the dynamical uncertainties as:

$$\Delta(s) = \sum_{i=n+1}^{\infty} \frac{F_i}{s^2 + 2\zeta_i w_i s + w_i}. \quad (9)$$

Note that the real plant can be considered in modal basis as:

$$P_r(s) = P_n(s) + \Delta(s). \quad (10)$$

An aluminum flexible cantilever beam (density=$2700 kg/m^3$, Young modulus=$69e9 N/m^2$, cross section: $0.003m \times 0.032m$, lenght=$1.05m$) is used to investigate the active vibration control problem. It is modelled by the finite element method with 44 degrees of freedom (the axial DOFs were not considered). This model was diagonalized in order to be treated in a modal basis (Ewins, 1984).

For simulation purposes, the nominal plant is based on the 3 first vibration modes and the "real" plant based on the 6 first modes, i.e., 3 uncertain vibration modes were used to define the dynamical uncertainty for simulation of the real plant. Figure (3) shows the bode plots of nominal and real plants.

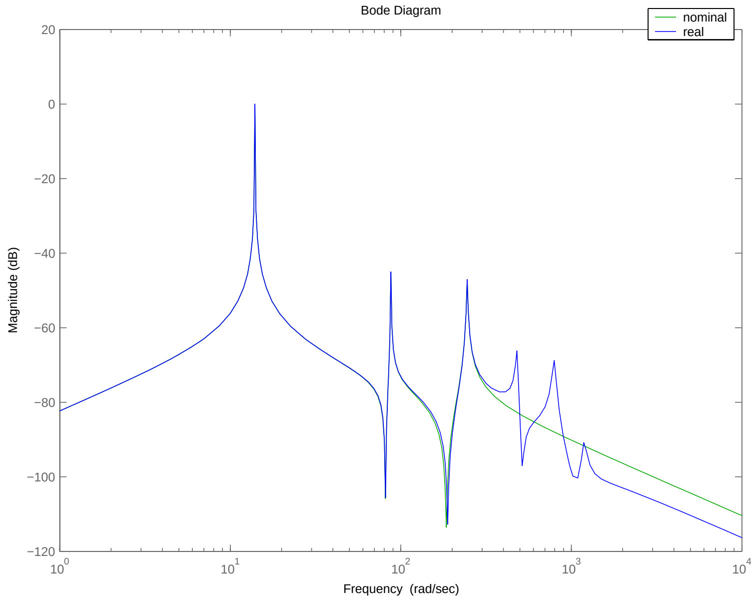

Figure 3. Bode plots of nominal and "real" plants

4. Controller design and simulated results

4.1 Regular H_∞ controller

The framework depicted in Figure 1 may be used to design a regular H_∞ and also to include the LMI-regions to be considered. As such, the filter design is the most judgemental step in the process. According to the nominal plant P_n frequency distribution it is possible to define the filters W_u and W_p. These filters can be interpreted in a similar form of R and Q matrices of the LQR analysis but with the advantage that they consider the frequency distribution, resulting in a concentration of the control effort. $W_u(s)$ needs to be small in the low frequency range to allow high control signal levels to attenuate the modelled vibration modes and high in the out-of-the-bandwidth frequency range to avoid to excite the residual modes. $W_p(s)$ needs to be high in the low bandwidth frequency range to ensure good attenuation for the controlled modes. These filters were designed according to previous frequency requirements in order to satisfy the small-gain theorem. The chosen filters are:

$$W_u(s) = \frac{0.1s + 10}{s + 10000} \;\; and \;\; W_p(s) = \frac{s + 5}{s + 0.5}. \tag{11}$$

The MATLAB software with the LMI toolbox and Simulink was used to generate the following results. A regular H_∞ controller was designed based on the above filters, resulting in a γ-value of 0.085. Figures 4 and 5 shows the bode plots of the system transfer functions and γ multiplied by the respective inverse filters (γW^{-1}). It is possible to see in the figures that the filters satisfy the small-gain theorem.

The plant response and the closed-loop response to a disturbance represented by a chirp signal with frequencies in the range of 0.1 to 200 Hz are shown in Figure 6. It may be seen that a good attenuation of the vibration was achieved. This result was obtained using the "real" plant in the simulation.

4.2 Regional H_∞ pole-placement

It is possible to formulate the regular H_∞ problem as a convex mathematical programming problem of the form:

$$\begin{array}{ll} minimize & \gamma \\ subjected\ to: & A(P_n, K) < 0, \end{array} \tag{12}$$

where $A(P_n, K) < 0$ is a linear matrix inequality constraint and means that matrix A, which is an affine function of the parameters of the plant P_n and of the controller K, is negative-definite. For the H_∞ problem this LMI is in general obtained through some congruence transformations over the Lyapunov stability equation (see Boyd et. al., 1994 for details).

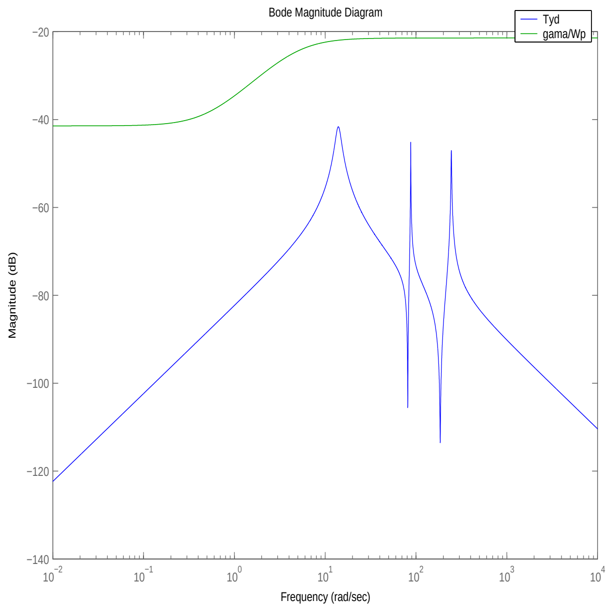

Figure 4. Bode plots of T_{yd} and γW_p^{-1}

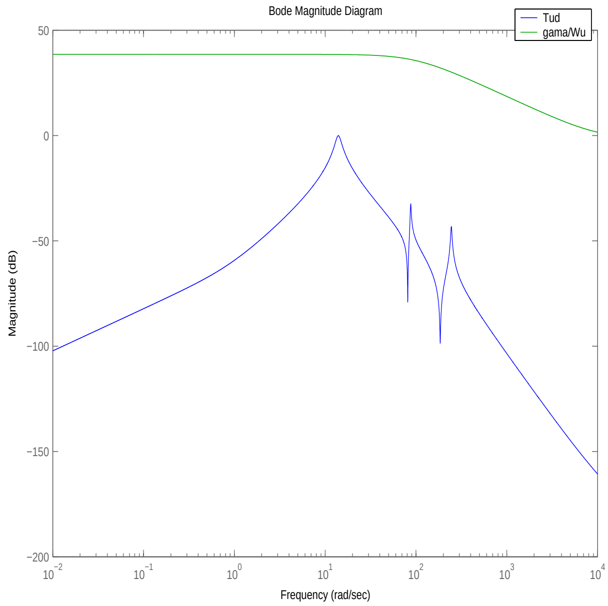

Figure 5. Bode plots of T_{ud} and γW_u^{-1}

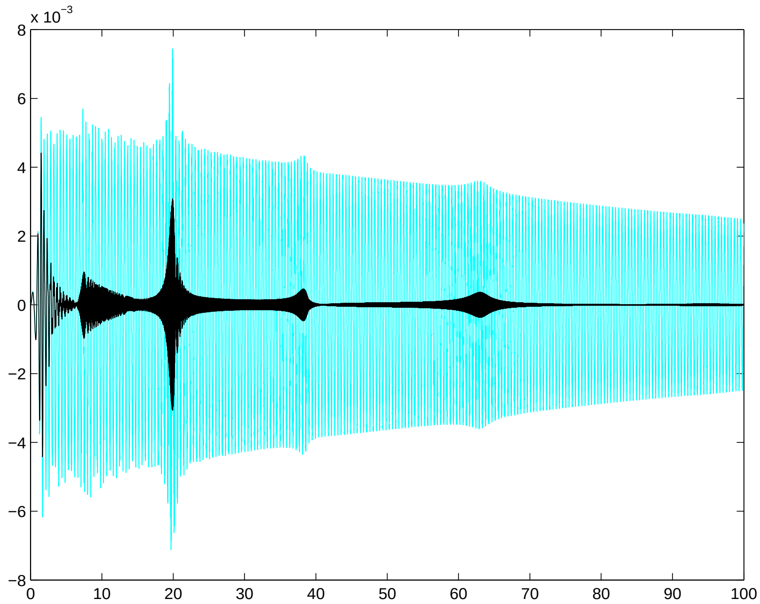

Figure 6. Plant and controlled disturbance response for the regular H_∞ controller

The LMI regional pole placement includes other LMI constraints to the minimization problem that becomes:

$$\begin{array}{ll} minimize & \gamma \\ subjected\ to: & A(P_n, K) < 0 \\ & B(P_n, K, r) < 0 \end{array} \tag{13}$$

where $B(P_n, K, r) < 0$ is an LMI that depends of P and K parameters, and also of the region r of the complex-plane in which the close-loop poles should be located. The most usual regions are the vertical strip, the cone sector and the circle. Gutman and Jury (1981) and Chilali and Gahinet (1996) discuss the mathematical procedures to write the these regions in matrix linear inequalities form.

The main advantage of this kind of formulation is that the final minimization problem keeps its convexity and can be solved by efficient mathematical programming interior point methods (Nesterov and Nemirovskii, 1994). Although the mathematical theoretical foundations are very elegant, some practical difficulties to find feasible solutions is common in real cases where the number of decision variables is not small.

To include a regional pole placement constraint, a vertical strip was initially considered, implying that all closed-loop poles would be positioned at the left of the strip. The best achieved value for the vertical strip, considering the previous filters, was $-0.4s^{-1}$. The Bode plots of the transfer functions are presented in figures 7 and 8. It may be seen that they both satisfy the small-gain theorem. A γ-value of 0.089 was achieved, and the result may be seen in Figure 9, where the disturbance response for the plant and controlled "real" system are represented. The vibration attenuation is similar to the first design, however it is clearly visible in Figure 9 better results for the in-the-bandwidth first and second modes.

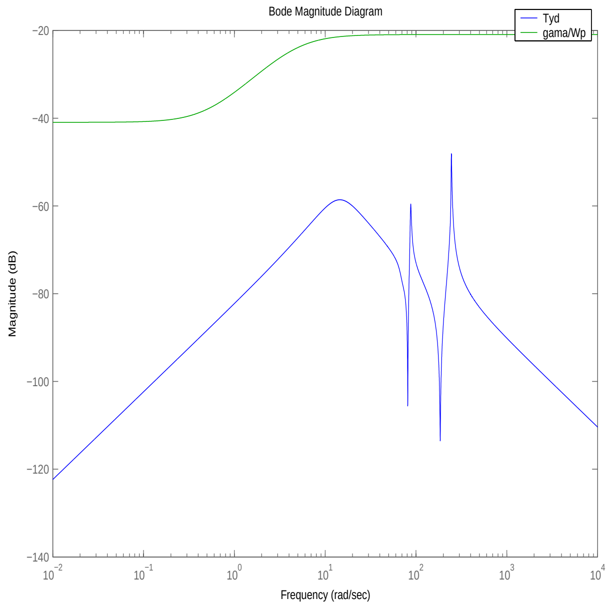

Figure 7. Bode plots of T_{yd} and γW_p^{-1} for the vertical strip

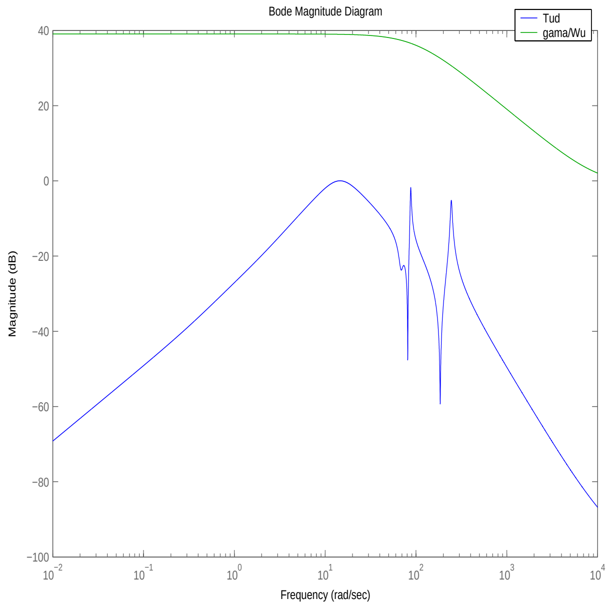

Figure 8. Bode plots of T_{ud} and γW_u^{-1} for the vertical strip

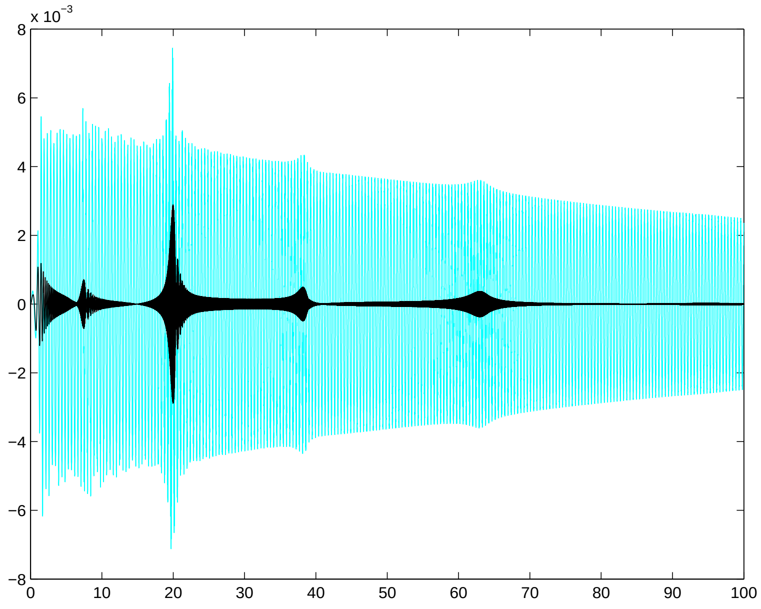

Figure 9. Plant and controlled disturbance response for the vertical strip

A second region was considered for another controller design, in a attempt to improve the results. Expecting to increase the damping factor of the closed-loop poles, another region, merging the previous vertical strip with a conic sector centered at the origin and an aperture of 0.9π, was considered. The new controller design achieved a γ-value of 2.43, and the Bode plots are shown in Figure 10 and 11. However, when this controller was applied to the real plant, unstable closed-loop poles due to the non-modelled dynamics were found.

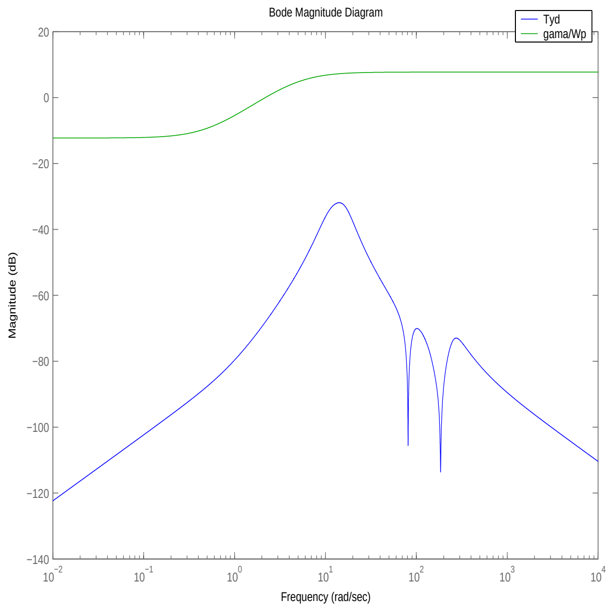

Figure 10. Bode plots of T_{yd} and γW_p^{-1} for the conic sector

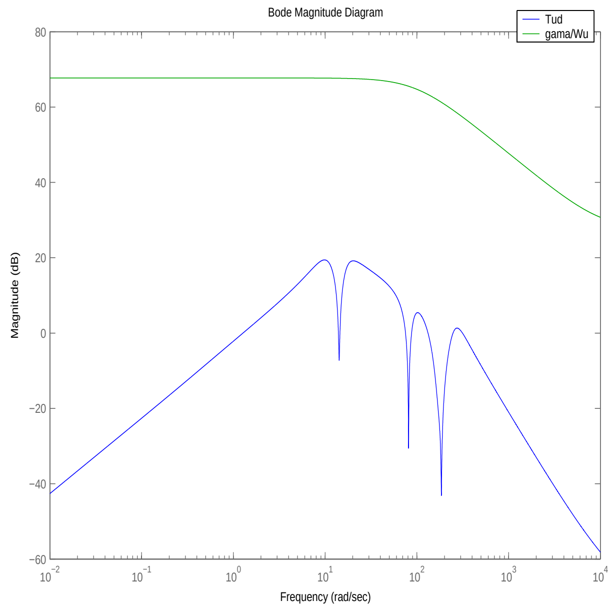

Figure 11. Bode plots of T_{ud} and γW_u^{-1} for the conic sector

5. Conclusions

Some known difficulties to effectively control mechanical structures vibrations make this a challenging problem. An LMI-based approach to regional H_∞ pole-placement is here proposed as a means to provide robustness to uncertainties necessarily present in every example of vibration controller design.

To assess the applicability of the proposed method, initially a regular H_∞ control was designed and numerically tested against a cantilever beam, presenting good results. Then, the LMI-based regional pole placement method was considered, adding new constraints expecting to improve the previous H_∞ vibration controller. Two regions were tested. The first region conducted the closed-loop system to a better performance, but the second region, with higher constraints on the performance, resulted in a infeasible problem.

The proposed method is very straightforward, but despite its theoretical appeal, the achieved results are not significantly better than the regular H_∞ controller design. The main difficulty to obtain a good solution seems to be related to the very low damping factors usually encountered in mechanical structures, which renders an infeasible problem if regions with high damping values are selected. New investigations to improve the design method are necessary, in order to become possible to design feasible constraint regions with better closed-loop characteristics. On the other hand, considering the unstable closed-loop poles due to non-modelled modes, a problem yet to be considered not adequately solved for any vibration control method, it is possible to constrain also the uncertainty to the selected region, implying that new uncertainty representation are necessary. As a general conclusion, the proposed method in this initial investigation presented good results, compared to regular controllers, and it deserves further theoretical and experimental studies as a real possibility to implement better active control of mechanical structure vibrations.

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