Trajectory Modeling of CaPaMan (Cassino Parallel Manipulator) by using4th order B-splines .

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Abstract *In this paper a general procedure devoted to trajectory optimization is presented. This procedure is used for optimizing the trajectory of a parallel manipulator named CaPaMan (Cassino Parallel Manipulator) by using a performance criterion that includes the mechanical energy and the total traveling time. The resulting scalar objective function is then minimized. The trajectory is represented by 4th order B-splines, which produce quadratic acceleration curves and linear jerk. Numerical results from the optimization procedure are presented to illustrate the efficiency of the proposed formulation. Genetic algorithms are used in the optimizer for obtaining these results.*
Keyword: *Optimization, Parallel Manipulators, Dynamics, Path Planning.*

1.Introduction.

The parallel architectures show advantages such as higher stiffness and accuracy positioning when compared to the serial architectures, moreover they can operate at high velocities and accelerations. These characteristics permit their use for many applications: assembly and disassembly processes, packing, manipulation, milling machines and motion simulation. Thus, several new parallel mechanisms have been conceived, designed and built together with the development of theoretical and practical investigations like those presented in (Ceccarelli, 1997 and Merlet, 2000). CaPaMan (Cassino Parallel Manipulator) is a parallel manipulator, having three degrees of freedom, which has been conceived at LARM: Laboratory of Robotics and Mechatronics at Cassino, Italy. A prototype has been built and the performance and suitable formulation for kinematics, statics and dynamics have been investigated and results are reported in Carvalho and Ceccarelli (2001).

In general parallel robots get workspaces smaller than those produced by serial robots; this fact influences the length of the trajectories of robots with parallel structure, which are generally very small. Venkataraman (1997) solve this problem type building parallel robots on mobile platforms and with flexible legs.

When repetitive processes are imposed, it is possible to develop a methodology to move a robot along a specified optimum path. This can be achieved with minimum cost through an optimization problem formulation. Considerable research has been done to obtain optimal robot path and the literature is very rich. For example, in Saramago and Steffen (2001) have proposed a procedure to construct a cubic polynomial joint trajectory and the algorithm for minimizing the travelling time subject to the physical constraints on joint velocities, accelerations and jerks. Shin and Mckay (1986) have presented a solution to the problem of minimizing the cost of moving a serial robotic manipulator along a specified geometric path subject to input torque/force constraint, taking into account the dynamics of the manipulator. Other optimum path planning methods can be found in (Chen, 1991) or recently, Saramago and Ceccarelli (2004).

In this work, a general formulation has been proposed for optimum path planning for parallel manipulators by taking into account the mechanical energy of the actuators and the total travelling time for the formulation of an multi-objective function. Feasible trajectories have been defined by using 4th order B-spline functions and then they have been obtained through off-line computation. This procedure has been applied to CaPaMan as a practical example, also in order to further improve its dynamic performance by reducing its power consumption through a minimization of the needed actuator energy and the total travelling time. The importance of using B-splines of fourth order is due to the fact to produce derivatives of second order as being quadratics polynomials, which allow smooth curves to acceleration. The derivatives of third order are linear functions and allow linear jerks, these facts enable control operations and motion of robots. The velocities are smooth curves too because its derivatives are cubic polynomials functions. The velocities are smooth curves too because its derivatives are polynomials functions of third order.

A genetic algorithm has been used for solving the numerical optimization procedure. The genetic algorithms (G.A.) are computational search methods based on the mechanisms of natural evolution and genetics. In the G.A., a population of possible solutions for the problem in subject develops in agreement with probabilistic operators conceived from biological metaphors, so that there is a tendency that, in the average, the individuals represent better solutions as the evolutionary process continues (Michalewicz, 1995).

A numerical example of optimum path planning for CaPaMan is presented in order to show the numerical efficiency and feasibility of the proposed methodology.

2. CaPaMan Prototype.

CaPaMan architecture has been conceived at LARM Laboratory of Robotics and Mechatronics in Cassino, where a prototype has been built for experimental activity. Indeed, by using the existent prototype, simulations have been carried out to validate the proposed optimum design by considering several guess solutions and imposing workspace and stiffness characteristics of the built prototype. The numerical results of the tests have been obtained according to the dimensions of the prototype.

A schematic representation of the CaPaMan manipulator is shown in Fig.1, where the fixed platform is FP and the moving platform is MP. MP is connected to FP through three identical leg mechanisms and is driven by the corresponding articulation points H_1, H_2, H_3. A built prototype is shown in Fig.2. An articulated parallelogram AP, a prismatic joint SJ and a connecting bar CB compose each leg mechanism. AP's coupler carries the SJ and CB transmits the motion from AP to MP through SJ; CB is connected to the MP by a spherical joint BJ, which is installed on MP. CB may translate along the prismatic guide of SJ keeping its vertical posture and BJ allows MP to rotate in the space. Each plane, which contains AP, is rotated of $\pi/3$ with respect to the neighbor one.

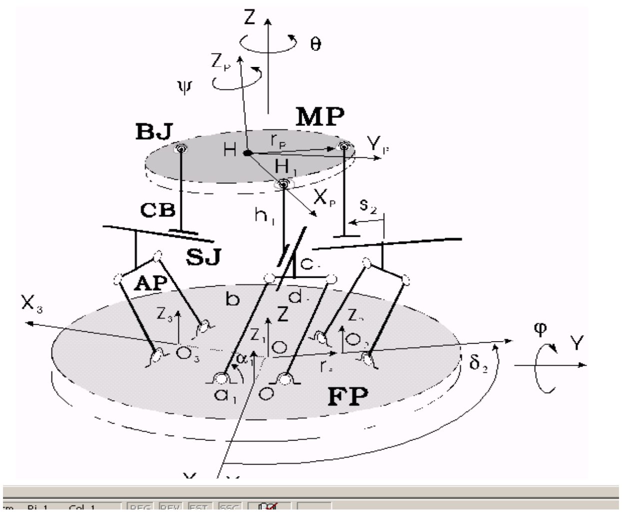

Fig.1 - Kinematic chain and design parameters of CaPaMan (Cassino Parallel Manipulator).

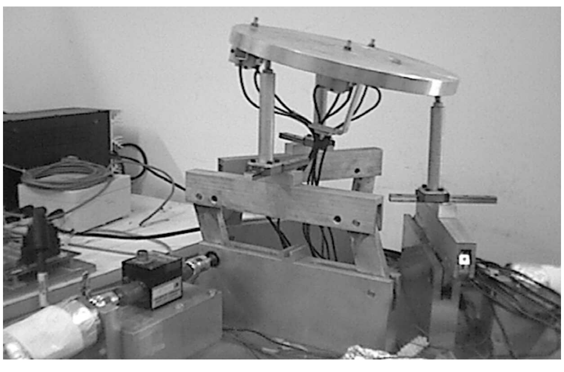

Fig.2 - A prototype of CaPaMan with accelerometers and a dynamic torsion meter at LARM in Cassino.

Table 1 - Sizes and motion parameters of the built prototype for CaPaMan, Figs.1 and 2.

$a_k = c_k$ [mm]	$b_k = d_k$ [mm]	H_k [mm]	$r_P = r_f$ [mm]	α_k [deg]	s_k [mm]
200	80	100	109.5	45 ; 135	-50 ; 50

Particularly, design parameters of the k-th leg are identified through: a_k, which is the length of the frame link; b_k, which is the length of the input crank; c_k, which is the length of the coupler link; d_k, which is the length of the follower crank; h_k, which is the length of the connecting bar. The kinematic variables are: α_k, which is the input crank angle; s_k , which is the stroke of the prismatic joint. Finally, the size of MP and FP are given by r_p and r_f, respectively, H is the center point of MP, O is the center point of FP, H_k is the center point of the k-th BJ and O_k is the middle point of the frame link a_k, Fig.1. The motion of MP with respect to FP can be described by considering a world frame O-XYZ, which is fixed to FP, and a moving frame H-$X_pY_pZ_p$, which is fixed to MP.

The symmetry characteristics of CaPaMan architecture have been useful to formulate analytical dynamic equations to compute the input torques which are necessary for a given motion trajectory of the movable platform. Considerations have been made in order to simplify the equations: the effects of link elasticity and viscous damping of the joints have been neglected; links are assumed to be rigid bodies and the joints are frictionless and have no clearance. It has been considered, in a first way, only the inertial effects of the movable platform, since the legs of parallel architectures are lighter than the movable plate. Successively, the inertial effects of the legs and prismatic joint have been superposed.
By neglecting the friction on prismatic and spherical joints, the only forces applied to the rods CB by the mobile platform are those which are contained in the plane of the articulated parallelogram i.e. F_{ky} and F_{kz} as shown in Fig.3. The F_{kx} component, when it is not zero, determines the sliding of the prismatic joint to an equilibrium position. This consideration allow to solve the inverse dynamics problem for the movable plate of CaPaMan in a closed form in order to obtain an explicit expression for the torques τ_{Pk} (k=1,2,3) on the input crank shaft of each articulated parallelogram only due to the inertial effects of the movable platform as

$$\tau_{Pk} = \frac{F_{kz}\, b_k \operatorname{sen}(2\alpha_k)}{2\operatorname{sen}\alpha_k} - F_{iy}\, b_i \left(\frac{h_k}{c_k \tan\alpha_k} + 1\right)\left(1 - \frac{h_k}{h_k\cos\alpha_k + c_k \operatorname{sen}\alpha_k}\right)\operatorname{sen}\alpha_k \tag{1}$$

where b_k, c_k and h_k are the geometrical dimensions shown in Fig.1; F_{ky} and F_{kz} are the reaction forces in the spherical joints H_k. The details of the formulation for obtaining F_{ky} and F_{kz}, are reported in (Carvalho and Ceccarelli, 2001).
The contribution of the legs to the inverse dynamics of the CaPaMan has been determined by the kinematic analysis of the articulated parallelograms. The centers of mass of the links have been defined has shown in Fig.3.
A kinetostatic analysis and superposition principle can be used for computing separately the inertial and gravitational effects of the links b_k, c_k, and d_k and then we have combined them into the input torque τ_{Mk} as

$$\tau_{Mk} = 2 l_{bk} F_{inbk} \operatorname{sen}(\alpha_k - \beta_k + \pi) + F_{23k}\, b \operatorname{sen}(\alpha_k + \pi - \gamma_k) + b\left[m_{bk} \cos\alpha_k + \frac{(m_{ck} + m h_k)\operatorname{sen}2\alpha_k}{2\operatorname{sen}\alpha_k}\right] g \tag{2}$$

The angle β_k in Eq.(2) defines the direction of the acceleration of the mass center of the k-th link with respect to the horizontal axis. It is assumed to be positive counter-clockwise. Similarly, the angle γ_k in Eq.(2) defines the direction of the reaction force vector acting on the ground pivot of link d_k and it is also assumed to be positive counter-clockwise. The terms l_{bk}, F_{23k} and γ_k in Eq.(2) can be written as :

$$l_{bk} = \frac{b}{2} + \frac{I_{Gbk}\,\ddot{\alpha}_k}{F_{inbk}} \frac{1}{\sin(\alpha_k - \beta_k + \pi)} \tag{3}$$

$$F_{23k} = \sqrt{\left\{ F_{inck}\left[\cos(\beta_k + \pi) + \frac{\sin(\pi - \beta_k)}{2\tan\alpha_k}\right]\right\}^2 + \left[\frac{F_{inck}\sin(\beta_k + \pi)}{2}\right]^2} \tag{4}$$

$$\gamma_k = tg^{-1}\left\{ \frac{\left[\dfrac{F_{inck}\sin(\beta_k + \pi)}{2}\right]}{F_{inck}\left[\cos(\beta_k + \pi) + \dfrac{\sin(\pi - \beta_k)}{2\tan\alpha_k}\right]} \right\} \tag{5}$$

The details of derivations of the terms in Eqs.(3) to (5) are reported in (Carvalho and Ceccarelli, 2001).

The superposition principle can be used again in order to obtain the total torque τ_k on the input shaft of the articulated parallelogram as the sum of the torques computed in Eqs.(1) and (2) in the form

$$\tau_k = \tau_{Pk} + \tau_{Mk} \tag{6}$$

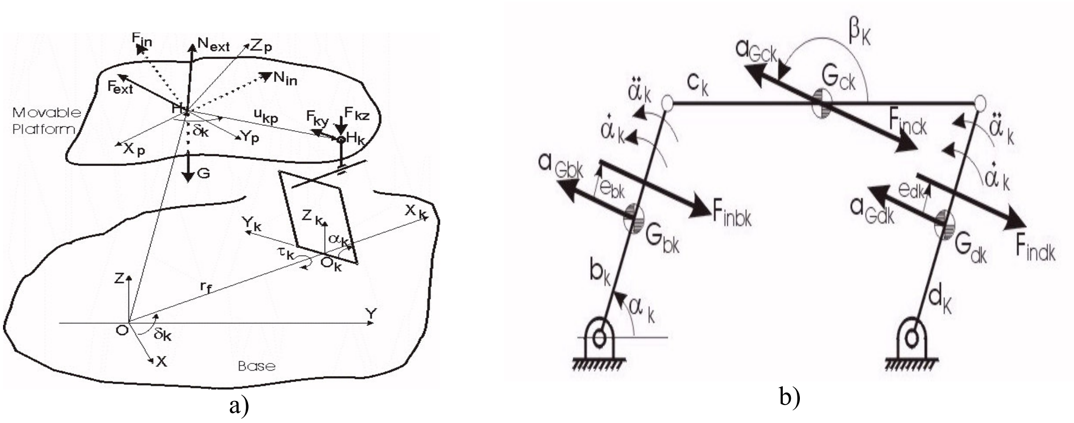

Fig.3. a)Forces acting on the mobile platform; b) Inertia forces on each articulated parallelogram of CaPaMan legs.

3. A Formulation For Optimal Path Planning.

The path planning task for a manipulator with n degrees-of-freedom (d.o.f.) can be described using m knots for the trajectory for each k-th joint of a manipulator. The prescribed task can be given by the initial and final points P_0 and P_m of the trajectory. The movement of the manipulator can be obtained by the simultaneous motion of the n joints in order to perform the prescribed task.

Among the many available criteria, one can assume the energy aspect as one of the most significant performance in order to optimize the manipulator operation (Saramago and Ceccarelli, 2004), since the energy formulation can consider simultaneously dynamics and kinematics characteristics of the performing motion. It should also be considered that the maximization of the operation speed of the manipulator corresponds to the minimization of the total travelling time. Thus, the optimal travelling time and the minimum mechanical energy of the actuators have been considered together to build a multi-objective function such as the optimization problem was defined as follows:

$$Minimize \quad f = w_1 \frac{E}{E_0} + w_2 \frac{Tt}{T_0} \tag{7}$$

subject to

$$\alpha_k^l \le [\alpha_k(t)] \le \alpha_k^u \quad (k=1,2,3), \quad Tt^l \le Tt \le Tt^u \tag{8}$$

in which the total energy of the manipulator can be expressed as

$$E = \int_0^{Tt} \sum_{k=1}^{n} [\tau_k(t)\dot{\alpha}_k(t)]dt \qquad \text{with} \qquad \tau_k^l \le \tau_k \le \tau_k^u \tag{9}$$

where k=1,2,3; t_k is the actuator torque on the k-th input shaft; a_k is the k-th joint variable; t is the time variable in the interval [0, Tt] for the path between P_o and P_m; the time derivatives of a_k are indicated as dot a_k; Tt is the total travelling time at the end point P_m when t=0 is assumed at the initial point P_0. The side constraints have been formulated in Eqs. (8) given by lower and upper limits for each crank angle (a^l_k and a^u_k) and the lower and upper limits for the total travelling time (Tt^l and Tt^u). E_0 is the initial energy spends to travel the initial trajectory and T_0 is the time of travelling for this trajectory. In the Eq. (7) w_1 and w_2 are the parameters to the multi-objective function f. The proposed formulation, Eqs. (7) to (9), requires the computation and consideration of the manipulator kinematics and dynamics.

The nature of this optimization problem is such that, in general, several local minima exist and can be observed throughout the numerical procedure. Genetic Algorithm (Michalewicz, 1995) is a probabilistic search algorithm capable of finding the global minimum amongst many local minima, particularly in the case where traditional techniques fail. For this reason, a genetic algorithm has been used for solving the numerical optimization procedure.

4. A Formulation For trajectory Modelling.

In order to determine the joint trajectories, one can use the given initial and final points P_0 and P_m in the Cartesian coordinates. These given points can be transformed into joint coordinates by solving the inverse kinematics. B-splines are frequently used as interpolating functions to represent the trajectory of mechanical systems. An important characteristic is that they allow controlling the degree of continuity between two adjacent segments. This fact is important because smooth transition is required for path planning in many applications such as in robotics. Another important characteristic of the fourth order B-splines is that they satisfy the convex hull property which allows the refinement of the trajectory. Thus, each trajectory $\alpha_k(t)$ is modeled by a uniform B-spline of fourth order given by,

$$\alpha_k(t) = \sum_{i=0}^{m} p_i^k \; B_{i,d}^k(t), \quad m \geq 4, \quad k = 1,2,3. \tag{10}$$

where p^k_i are the m control points corresponding to the trajectory α_k (t), and $B_{i,d}$ are functions defined by the Cox deBoor recurrence formulas (Foley et al. , 1990), with d= 5 for B-spline of fourth order

$$B_{i,1}(t) = \begin{cases} 1 & se \;\; t_i \leq t \leq t_{i+1} \\ 0 & out \end{cases}, \qquad B_{i,d}(t) = \frac{t - t_i}{t_{i+d-1} - t_i} B_{i,d-1}(t) + \frac{t_{i+d} - t}{t_{i+d} - t_{i+1}} B_{i+1,d-1}(t) \tag{11}$$

Since $\alpha_k(t)$ is a polynomial, its j-th derivatives with respect to t can be straightforward computed as

$$\frac{d^j \alpha_k(t)}{dt^j} = \sum_{i=0}^{m} p_i^k \; \frac{d^j B_{i,d}^k(t)}{dt^j} \tag{12}$$

the equations of the segments of curves for B-splines of fourth order are deduced using recurrence formulas given by Eq. (11) being given for:

$$P(u) = B_{0,4}(u)P_0 + B_{1,4}(u)P_1 + B_{2,4}(u)P_2 + B_{3,4}(u)P_3 + B_{4,4}(u)P_4 \quad \text{where,} \tag{13}$$

$$B_{0,4}(u) = \frac{1}{24}\left(u^4 - 4u^3 + 6u^2 - 4u + 1\right); \qquad B_{1,4}(u) = \frac{1}{24}\left(-4u^4 + 12u^3 - 6u^2 - 12u + 11\right)$$

$$B_{2,4}(u) = \frac{1}{24}\left(6u^4 - 12u^3 - 6u^2 + 12u + 11\right); \quad B_{3,4}(u) = \frac{1}{24}\left(-4u^4 + 4u^3 + 6u^2 + 4u + 1\right) \tag{14}$$

$$B_{4,4}(u) = \frac{1}{24}u^4$$

Eqs. (14) are the concordance functions for B-splines of fourth order, with $0 \leq u \leq 1$.

The optimization design variables are the control points p^i_k of each trajectory and the total time of traveling. Thus for n=3 mechanisms the total number of design variables is (n x m)+1.

5. A Numerical Example of Path Planning.

A numerical example with CaPaMan been carried out by assuming a starting position given by α_1=60, α_2=50, α_3=80 [deg] and a final position given by α_1=90, α_2=120, α_3=100 [deg]. Data of CaPaMan are reported in Table 1. The robot is at rest initially and comes to a full stop at the end of the trajectory. Thus, $\dot{\alpha}_k(0) = \dot{\alpha}_k(Tt) = 0$ for all mechanism.

The program GAOT (Genetic Algorithms Optimization Toolbox) (Houck et al.,1995) has been used to perform the genetic algorithms. For optimization purposes a general analysis code developed by the authors (involving the dynamical model and the trajectory planning of the robot) has been coupled to the optimization program. The initial and final values are reported in Table 2 showing that there is a significant improvement of the performances index by using the trajectories shown in Fig. 6.

Table 2 - Optimum values obtained from the optimization process.

		Multi-objective Function f	Needed Energy E [Nm/s^2]	Total Time [s]
w_1=0.8, w_2=0.2	Initial value	1.0	36.50	2.0
	Optimum value	0.81	23.50	3.0
	Performance index	19.0%	35.6%	Increase
w_1=0.5, w_2=0.5	Initial value	1.0	36.50	2.0
	Optimum value	0.98	35.77	2.0
	Performance index	2.0 %	2.0 %	constant
w_1=0.2, w_2=0.8	Initial value	1.0	36.50	2.0
	Optimum value	0.81	71.30	1.0
	Performance index	19.0%	Increase	50.0%

Figure 5 shows a 3D plot of the initial and optimum path planning of the center of the movable plate of CaPaMan. In particular, the guess path between the given points Po and Pm has been chosen as a fourth order polynomial curve. We can observe that the optimal trajectory is smooth and obey the initial and final points of the path.

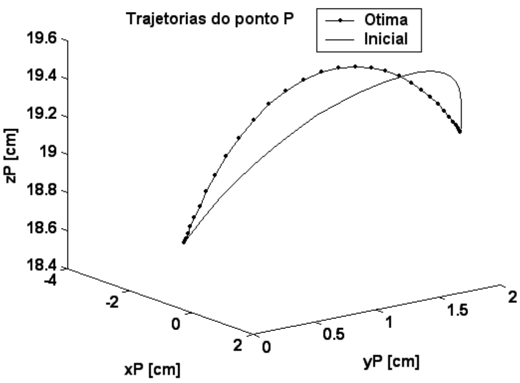

Fig.5. A 3D plot of the position of the center of the movable plate for CaPaMan as function of time (w_1=0.8, w_2=0.2).

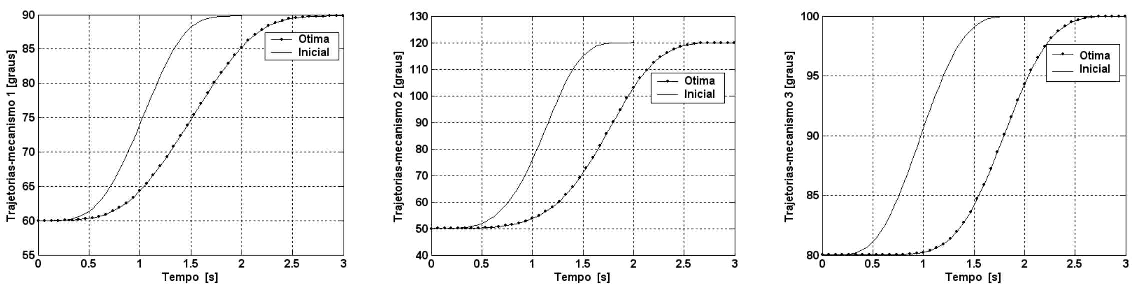

Fig.6. A plot of the input crank angles for CaPaMan as function of time obtained as result of the optimum path planning (w_1=0.8, w_2=0.2).

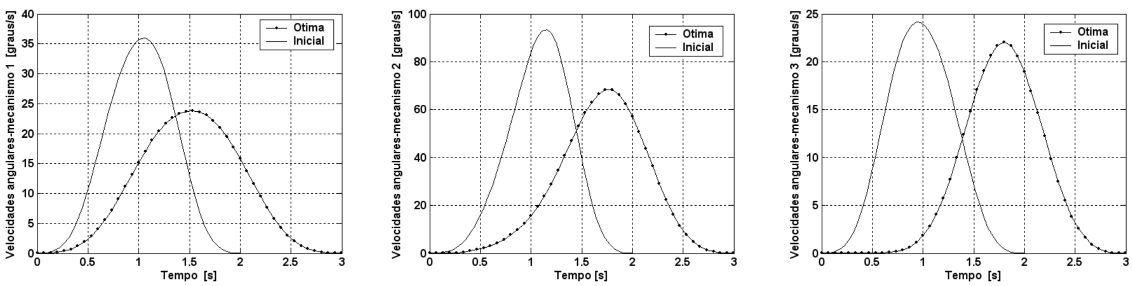

Fig. 7. A plot of the input velocity crank angles for CaPaMan as function of time obtained as result of the optimum path planning (w_1=0.8, w_2=0.2).

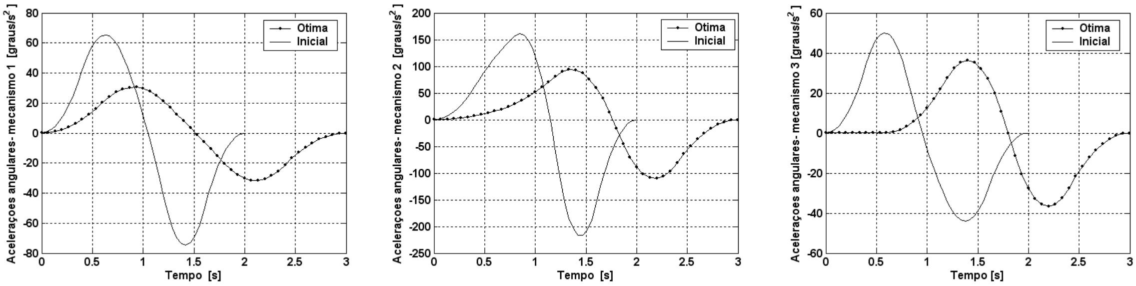

Fig. 8. A plot of the input acceleration crank angles for CaPaMan as function of time obtained as result of the optimum path planning (w_1=0.8, w_2=0.2).

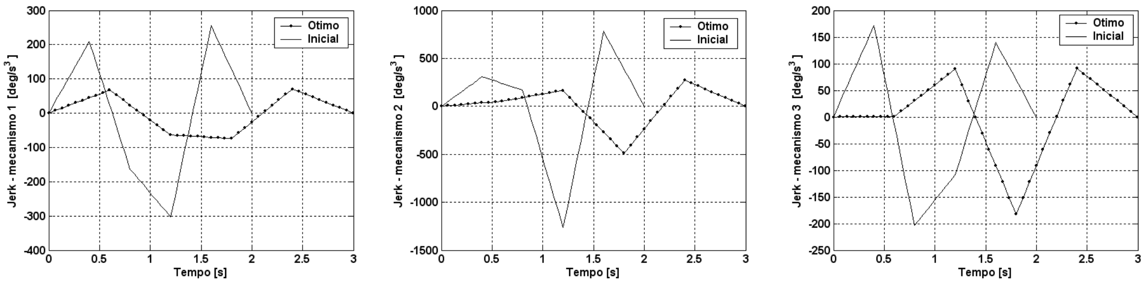

Fig. 9. A plot of the input jerk crank angles for CaPaMan as function of time obtained as result of the optimum path planning (w_1=0.8, w_2=0.2).

Figures 5 to 10 show results of the optimum path planning procedure for the case w_1=0.8, w_2=0.2. In particular, Fig. 5 shows a 3D plot of the optimized trajectory of the center of the movable plate of CaPaMan. Figure 6 shows the plots of the optimized trajectories for the input angles. Figure 7 shows the plot of the input velocity crank angles for CaPaMan as function of time obtained as result of the optimum path planning. Figures 8 and 9 show the plots of the acceleration and jerk , respectively. Figure 10 shows the actuator torque on the input shafts for CaPaMan as function of time obtained as result of the optimum path planning.

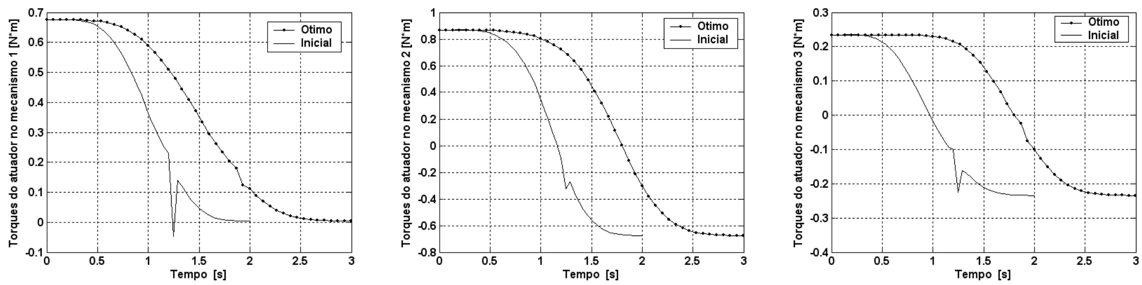

Fig.10. A plot of the actuator torque on the input shafts for CaPaMan as function of time obtained as result of the optimum path planning (w_1=0.8, w_2=0.2).

The optimum result represents a commitments solution among the functions that compose the multi-objective function given by for Eq. (7), we can see, this fact, observing the behaviour of the time and of the energy when w_1=0. 8 and w_2=0. 2, Tab. 2, or w_1=0. 5 and w_2=0. 5, or yet w_1=0. 2 and w_2 = 0. 8. It is important to observe that even in the case w1=0. 5 and w2=0. 5 the energy is minimized. The percentile of just 2% in this case is significant, because robots with parallel structures have generally very small trajectories and operate on high velocities . The Tab. 2 show that pays himself an price to minimize the time, because the energy increases.

6. Conclusions.

The results of the optimization procedure show that the proposed formulation is efficient in the sense that the optimal configuration of the system improves the dynamics of a given parallel manipulator with respect to its original design. Consequently, it was possible to reduce energy consumption and to limit jerks to acceptable values along the motion of the manipulator. The importance of using fourth order B-splines is related to the fact that smooth curves are obtained for the acceleration and linear jerks. This means that control operations and motion characteristics become more feasible for the optimal configuration of the system. Finally, it should be pointed out that a new method for trajectory optimization of parallel manipulators was presented.

7. References.

Carvalho, J.C.M., Ceccarelli, M., 2001, "A Closed Form Formulation for the Inverse Dynamics of Cassino Parallel Manipulator", J. Multibody System Dynamics, Vol.5, No.2.

Chen, Y.C., 1991, "Solving Robot Trajectory Planning Problems with uniform Cubic B-Splines", *Optimal Control Applications & Methods*, 12, 247-262.

Foley, J.D., Van.Dam, A., Feiner, S. K and Hughes J. F., 1990, " Computer Graphics: Principles and Practice" , Second 2nd ed, Addison-Wesley Publishing Company, ISBN 0-201-12110-7.

Goldberg, D.E. 1989, "Genetic Algorithms in Search", Optimization, and Machine Learning; Addison-Wesley.

Houck, C.R., Joinez J.A., Kay, M. G., 1995, " A Genetic Algorithms for Function Optimization: a Matlab Implementation" NCSU-IE Technical Reported.

Lin, C.S., Chang, P.R. and Luh , J.Y.S., 1983, "Formulation and Optimization of Cubic Polynomial Joint Trajectories for Industrial Robots", *IEEE Trans. Automat. Contr.*, 28, 1066-107.

Merlet, J.P., 2000, "Parallel Robots", Kluwer, Dordrecht

Michalewicz,Z., 1995, "Genetic Algorithms + Data Structures = Evolution Programs"; Springer-Verlag.

Saramago, S.F.P., Ceccarelli, M., 2004. "Effect of Some Numerical Parameters on a Path Planning of Robots Taking Into Account Actuating Energy", Mechanism And Machine Theory, Elsevier Science, v. 39, n. 3, p. 247-260.

S.F.P. Saramago, Steffen Jr, V., 2001, "Trajectory Modeling of Robots Manipulators in the Presence of Obstacles", Journal of Optimization Theory and Applications, Kluwer Academic, v. 110, n. 1, p. 17-34.

Shin, K.G., Mckay, N.D., 1986, "A Dynamic Programming Approach to Trajectory Planning of Robotic Manipulators", *IEEE Trans. Automat. Contr.,* AC-31, No. 6, 491-500.

Venkataraman, S.T., 1997, "A Simple Legged Locomotion Gait Model", Robot an Automonous Systems, no. 22, pp. 75-85.