

# NATURAL CONVECTION IN AN ARBITRARY INCLINED PLATE WITH PROTRUDING HEATED ELEMENTS

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*Abstract.* An experimental study was carried out on natural convection heat transfer to air from a vertical and arbitrarily inclined plate with protruding heated elements uniformly mounted on one surface. The heated surface faced downward. Heat loss by conduction and radiation were studied. A guard heater was used to keep the rear surface of the plate thermally insulated. Tests were performed for powers ranging from 30W to 70W, and inclinations ranging from 0° to 60° relative to the vertical position. The investigation showed that the Nusselt number of an inclined and protruding heated plate can be evaluated using a single correlation, since the gravitational acceleration is replaced by its component parallel to the plate in calculating the Rayleigh number. A correlation for the local Nusselt number for natural convection heat transfer is proposed.

**Keywords:** Natural Convection, Arbitrary Inclination, Protruding Elements.

## Nomenclature

$A$	area	[m <sup>2</sup> ]	$\beta$	thermal expansion coefficient, $1/T_\infty$	[K <sup>-1</sup> ]
$g$	gravitational acceleration	[m/s <sup>2</sup> ]	$\alpha$	thermal diffusivity	[m <sup>2</sup> /s]
$h_y$	heat transfer coefficient	[W/m <sup>2</sup> .K]	$\nu$	kinematic viscosity	[m <sup>2</sup> /s]
$k$	thermal conductivity	[W/m.K]	<b>Subscripts</b>		
$L$	height of plate	[m]	NC	natural convection	
$Q$	total power per plate	[W]	T	total rate	
$Ra_y^*$	local modified Rayleigh Number, $g \cos \phi \beta q_p y^4 / \alpha \nu k$		R	radiation	
$y, w$	spatial coordinates	[m]	C	conduction	
$Nu_y$	local Nusselt number, $h_y \cdot y / k$		e	protruding element	
$T$	temperature	[°C]	$\infty$	ambient value	
$\phi$	inclination angle	[deg]	p	plate	

## 1. Introduction

Among the cooling technologies for electronic equipment, natural convective air-cooling has received attention because of its inherent high reliability. It has been of high importance in many industrial applications.

Many studies have been published on natural convection along a vertical and inclined heated flat plate. Rich (1953) was the first to present the solution for the inclined flat plate. He suggested that the heat transfer coefficient in laminar flow for an inclined heated plate can be treated just as that of a vertical plate if the gravitational term in Grashof number is replaced by the component parallel to the inclined surface. Many other studies analyzed the effect of inclination of the plate on heat transfer coefficient, such as Lloyd and Sparrow (1970), Vliet (1969), Fujii and Imura (1972), and Shaukatullah and Gebhart (1978).

Ortega and Moffat (1985) performed an experimental analysis to describe the heat transfer by natural convection over a set of protruding heated elements mounted on a vertical plate. Wang et al. (1997) analyzed numerically the natural heat transfer in a vertical plate with five protruding heated elements. In a recent work, Silva et al. (2004) found the optimal distribution of the heat sources in vertical plate cooling by natural convection. However, studies on natural convection for inclined plates with protruding elements were not found in literature. In the present work, the natural

heat transfer from an arbitrarily inclined plate with protruding elements uniformly mounted on one surface was studied. The heated surface faced downward.

## 2. Experimental Apparatus and Procedure

**2.1 Test Plate and Protruding Heated Element.** The test plate was made of epoxy, and was 340 mm wide, 365 mm long and 1,6 mm thick. An array of seven aluminum protruding elements was uniformly mounted on one surface of the plate. Fig. 1 shows the protruding element. The element size was 12.2 x 12.25 x 340 mm. Each protruding element had a 10 ohm electric resistance embedded in its center. To minimize thermal contact resistance, the bottom surface of the protruding element received a thin layer of thermal paste before the element was screwed onto the plate. For tests with uniform heating of the test plate, the seven electric resistances were connected in series to an electric source. Fig. 2 shows the test plate.

In order to minimize heat loss by conduction, a guard heater was installed symmetrically to the test plate, and a foam plate, 20 mm thick, was placed between the two epoxy plates.

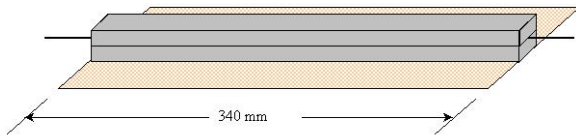


Fig.1: Schematic of the protruding element.

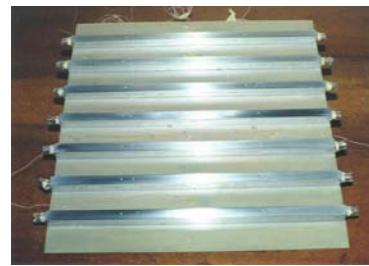


Fig. 2: Test plate.

**2.2 Guard Heater.** The rear side of the test plate was thermally insulated by means of a guard heater with the same dimensions as the test plate, and the same protruding elements, as illustrated in Fig. 3(a) and (b). Each electric resistance was connected to an independent power source to allow temperature control, in order to achieve maximum thermal insulation.

When the plate inclination was larger than zero degrees, the heated surface of the test plate faced downward, and the heated surface of the guard heater faced upward. Therefore, the flow pattern in the test plate was different from that in the guard heater. To avoid the influence of the upward flow from the guard heater on the main flow over the test plate, the whole guard heater surface was insulated with foam plates, as shown in Fig 3 (a), (b) and (c).

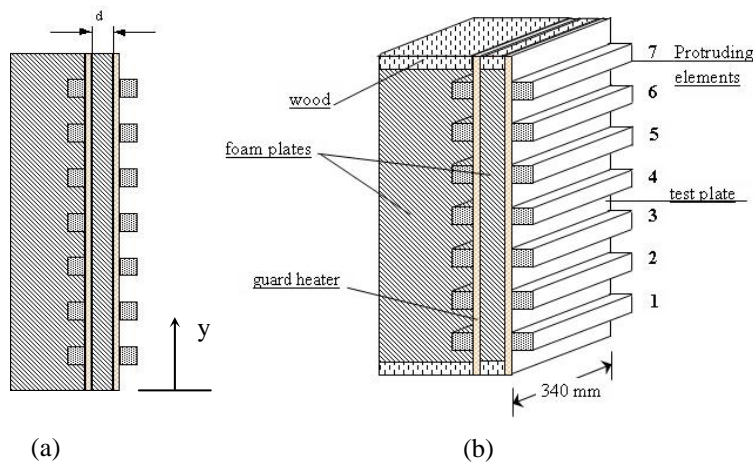


Fig. 3: (a) Side view of the test plate and the guard heater; (b) Schematic of the apparatus; (c) Experimental apparatus.

Finally, the test plate was attached to the front part of the wood structure, while the guard heater was attached inside the wood structure. To prevent lateral flow, an acrylic plate was placed on each side of the test plate (Fig. 3-c).

For the experimental tests with inclined plate, the apparatus was attached to an iron structure allowing the inclination to be changed. A protractor was used to adjust the angle of inclination. The experimental apparatus was placed 1.5 m above the ground. Power sources were placed in different rooms to prevent temperature changes of the environmental air.

**2.3 Procedure.** Each experimental test was performed according to the following steps: (1) Plate inclination was adjusted. (2) Power was adjusted. (3) Power in each guard heater element was adjusted. Guard heater temperature was kept very close to test plate temperature to minimize heat losses. (4) After the steady state was reached, it was checked whether temperatures at each guard heater and test plate element were close enough. If so, temperature data were acquired; otherwise the previous step was performed again.

This procedure was the same for all the experiments in the present work. Tests were carried out for total power dissipated per plate ( $Q$ ) varying from 30 to 70 W, and for inclination angles of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  from the vertical position. Temperature data were acquired after approximately 230 min.

**2.4 Data Reduction.** Considering  $Q_{T,e}$  the total power dissipated per element, the heat dissipated by natural convection  $Q_{NC,e}$  was calculated as

$$Q_{NC,e} = Q_{T,e} - Q_{C,e} - Q_{R,e} \quad (1)$$

The conduction heat loss through the epoxy plate ( $Q_{C,e}$ ) was calculated using an one-dimensional conduction model. The radiative heat loss ( $Q_{R,e}$ ) from the element to its surroundings was estimated using gray surface assumption and uniform radiosity for the cavity defined by two adjacent elements, the epoxy plate and environment at  $T_\infty$ .

**2.5 Uncertainty Analysis.** An uncertainty analysis was performed based on the methodology described by Moffat (1985). This required the uncertainty for each temperature measurement, as well as the uncertainty of the four parameters involved in the formula of the heat transfer coefficient. These values were either found experimentally or obtained from literature, such as the thermal conductivity (Fujii et al. (1996)) and emissivity (Sparrow and Cess (1978)). The overall uncertainty for the convective heat transfer coefficient was estimated at 5.5%.

### 3. Experimental Results and Discussion

**3.1 Protuberance Temperature.** Temperature distribution in the protruding array, for uniform heating and different angles of inclination, is presented in the figures below. The temperature profile is referenced to the ambient temperature. Results for minimum and maximum values of the total power per plate ( $Q$ ) are presented. Figs. 4 and 5 show that protuberance temperatures increase sharply until the third or fourth protuberance, and smoothly beyond that. The exception is the last protuberance, where the temperature decreases. This behavior will be discussed later. Ortega and Moffat (1985), in an experimental work with an array of ten rows of warm cubic protuberances mounted on a vertical plate and cooled by natural convection, also observed that after the fourth protuberance the temperature tends to remain constant.

Wang et al. (1997) analyzed numerically the natural convection in five two-dimensional discrete heated elements mounted on a vertical plate. They found that there is a large temperature difference between the first and second elements, and beyond them the increase is very moderate downstream.

The influence of inclination on protuberance temperature is small for  $Q=30$  W. For higher powers, however, its influence is more marked, and the temperature difference referenced to the vertical plate increases along the plate.

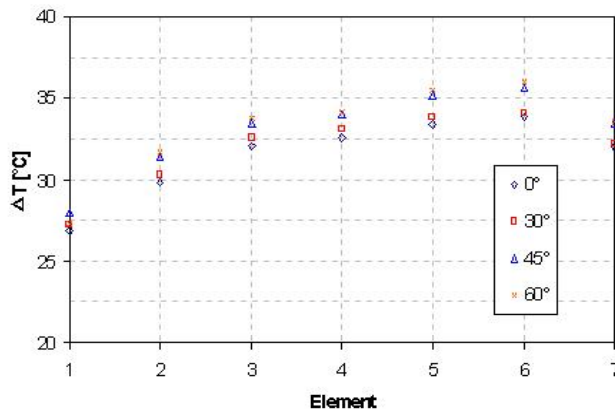


Fig. 4: Protuberance temperature for  $Q=30$  W.

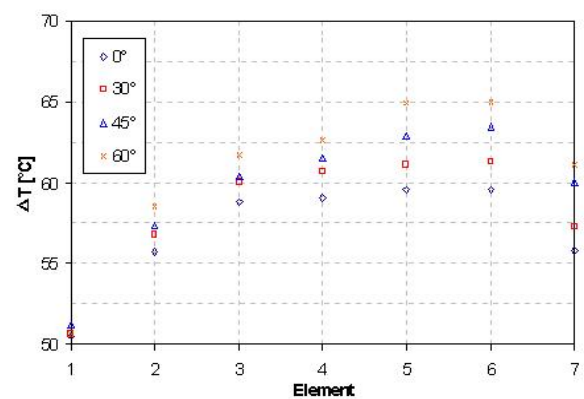


Fig. 5: Protuberance temperature for  $Q=70$  W.

**3.2 Conduction and Radiation Heat Losses.** Fig. 6 shows the average values of the computed heat losses. Note that conduction and radiation heat losses do not differ much. This behavior was observed for all experimental data. The average conduction heat loss was about 17%, and radiative heat loss about 12%, of the total power dissipated per element. Thus, natural convection removes around 71% of the total power input in each protruding element.

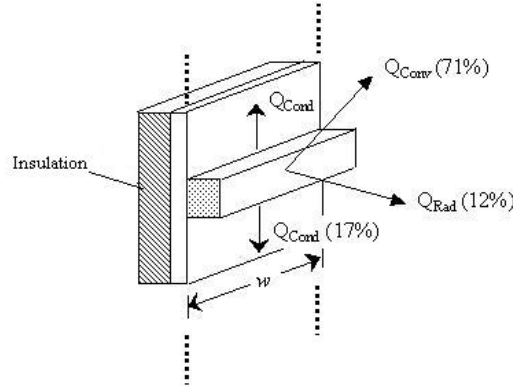


Fig. 6: Average heat losses.

**3.3 Local Heat Transfer Coefficient.** Local heat transfer coefficients along the array of protruding elements, for uniform heating and different angles of inclination, are presented below. The local heat transfer coefficients were computed as

$$h_y = \frac{Q_{NC,e}}{A_e(T_e - T_\infty)} \quad (2)$$

Figs. 7 and 8 show that, starting from the third or fourth protuberance, the heat transfer coefficient decreases smoothly. Notice that the heat transfer coefficient increases in the last protruding element. This behavior is attributed to the change in flow pattern in this region. Avelar (2001) analyzed numerically and experimentally the natural convection in vertical channels with protruding elements under uniform and non-uniform plate heating. The plates and protruding elements had the same dimensions as those in the present work. Results showed the occurrence of recirculation between intermediate protuberances and, consequently, a different flow pattern after the last protuberance. It is possible that this flow enhances the heat transfer coefficient for the superior surface of the last protruding element, causing its temperature to decrease.

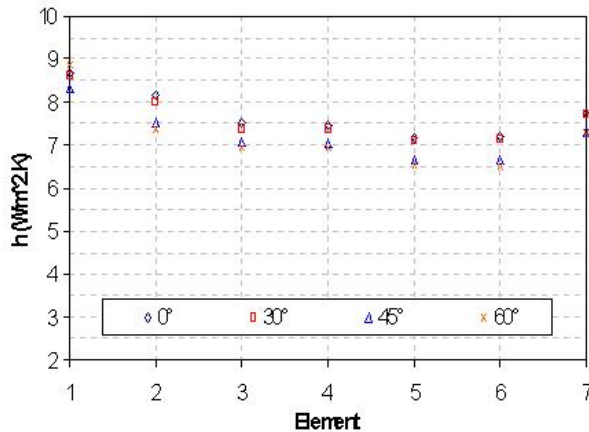


Fig. 7: Heat transfer coefficient for 30W.

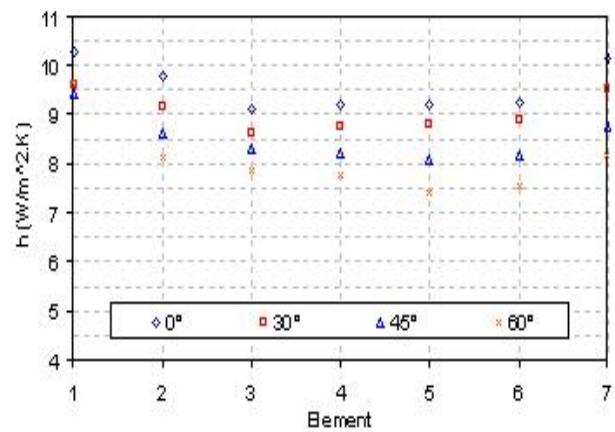


Fig. 8: Heat transfer coefficient for 70W.

**3.4 Correlation for Nusselt number.** The local Nusselt number as a function of the modified Rayleigh number is presented here. In the modified Rayleigh number, the heat flux is defined as  $Q/A_p$  and the component parallel to the plate of gravitational acceleration ( $g \cos \phi$ ) is used.

Rich (1953) suggested that the heat transfer coefficient for an inclined heated plate can be treated just as for a vertical plate, provided that the gravitational term in the Rayleigh number is replaced by its component parallel to the inclined surface.

Assuming that the same hypothesis is valid for a plate with protruding elements, the local Nusselt is plotted as a function of the modified Rayleigh. The results are shown in Fig. 9. Properties were evaluated for the mean value between maximum plate temperature and ambient temperature.

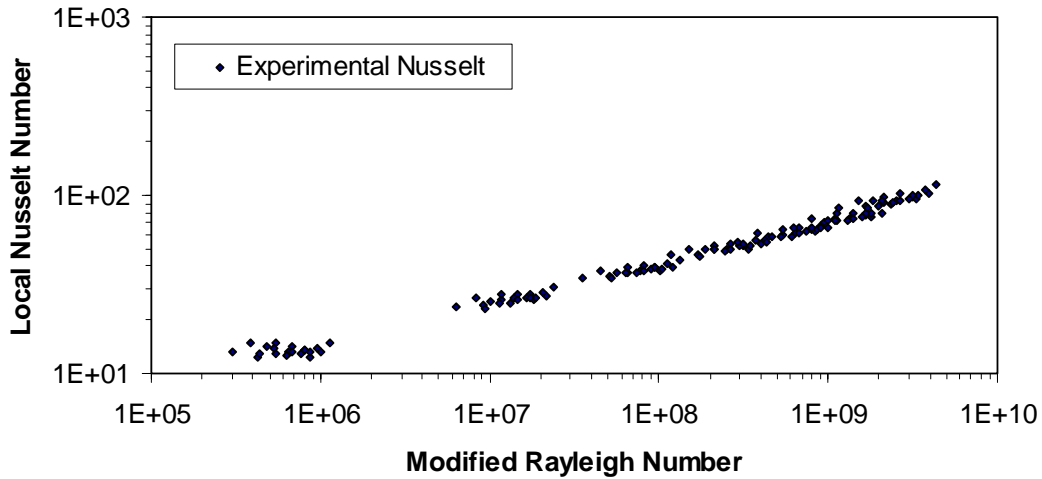


Fig. 9: Experimental Nusselt number.

It can be seen that all experimental results collapse into a single curve over the range of variation of the Rayleigh number covered in the present work. A power law correlation for the local Nusselt number, illustrated in Fig. 10, is proposed as a function of the modified Rayleigh number as

$$Nu_y = 0.6Ra_y^{0.23} \quad \text{for } 3 \times 10^5 \leq Ra_y \leq 4.3 \times 10^9 \text{ and } 0^\circ \leq \varphi \leq 60^\circ \quad (3)$$

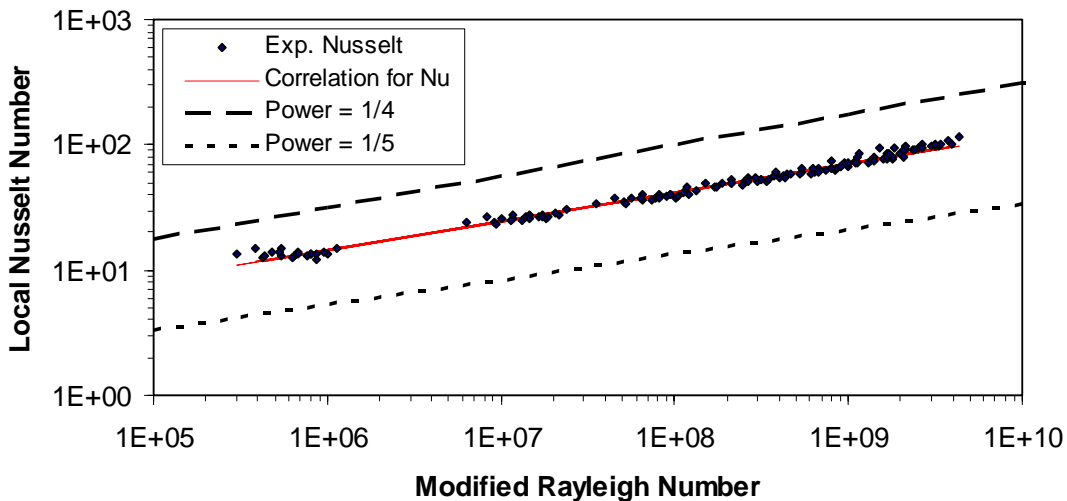


Fig. 10: Comparison between power laws.

The dashed lines in Fig 10 illustrate the power laws ( $Nu \propto Ra^n$ ) corresponding to laminar ( $n=1/5$ ) and turbulent ( $n=1/4$ ) Nusselt number correlations for a uniformly heated flat plate. The power law expressed in Eq. (3) is approximately the arithmetic mean of these values. Ortega and Moffat (1985) noticed that the heated protrusions on the smooth wall introduce sufficient disturbance into the flow, through production of turbulence behind the elements, to cause turbulent-like behavior even at low Rayleigh numbers.

The proposed correlation fits the experimental Nusselt number very well, indicating that a single correlation can be used to evaluate the local Nusselt number for natural convection in a arbitrarily inclined plate with protruding heated elements, provided that  $(g)$  is replaced by  $(g \cos \phi)$  to compute the Rayleigh number.

#### 4. Conclusions

The heat transfer by natural convection over protruding elements uniformly mounted on one surface of a plate with arbitrary inclination was experimentally studied. Radiation and conduction heat losses were computed to calculate the heat dissipated by natural convection only. Uncertainty analysis was performed and the overall uncertainty for the convective heat transfer coefficient was estimated to be 5.5%. The plate faced downward, and further investigation is necessary to check whether the present conclusion could also be applied to a plate facing upward. In the case of laminar natural convection over a uniformly heated flat plate, Vliet (1969) concluded that the experimental data correlate well when the component of gravity parallel to the heated surface is used. For turbulent flow, however, the actual gravitational acceleration rather than the parallel component is used to correlate the data. In the present work, over the investigated Rayleigh number range ( $3 \times 10^5 < Ra < 4,3 \times 10^9$ ), the main conclusion is that if the gravitational acceleration component parallel to the plate ( $g \cos \phi$ ) is used to evaluate the Rayleigh number, the natural convection Nusselt number in an inclined plate with protruding heated elements can be computed by a single correlation. A power law correlation based on the modified Rayleigh was proposed for the local Nusselt number.

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