NUMERICAL SCHEMES FOR SIMULATION OF NON LINEAR COUPLED AXIAL/TORSIONAL VIBRATIONS OF DRILL-STRINGS

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Abstract. *The present work deals with qualitative and quantitative studies on different schemes of solution of mathematical models which describe the non-linear dynamics of drill strings. The literature reports for these systems instabilities such as stick-slip, bit-bouncing, etc. Moreover the numerical models are usually stiff, hence hard to integrate. For the particular problem of axial/torsional vibrations of a drill-string, represented by means of a geometrically non-linear beam model developed by the authors, the characterization of the boundary conditions and/or reactive forcing terms, both non-linear in essence, plays an important role in the performance of the solution schemes in order to obtain an appropriate dynamic response. The qualitative and quantitative performances of dynamic responses are related to the behavior of integration methods in connection with variation of parameters in the modeling of the non-linear problem. In this article a study of the behavior of the solution performance of Matlab solvers in the axial/torsional dynamics of drill-strings is carried out. The Matlab solvers offer a good alternative to the solvers like Newmark methods, which can be very expensive in computational cost. This kind of analysis is devoted not only to compare computational costs and effectiveness, but to give qualitative and quantitative assessment to the numerical methods.*

Keywords: *Drill-strings, Non-linear Dynamics, Integration Methods*

1. Introduction

The appropriate characterization of the dynamical behavior of drill-strings is quite important in the drilling process, not only for the proper understanding of the problem (inherently complex) but for the obtention of results of practical interest. The dynamics of drill-strings, as slender structures, can be studied employing linear or non-linear models both described by means of lumped parameters (Yigit and Christoforou, 2000 and Richard et al, 2004) or with a one-dimensional continuum (Trindade et al, 2005 and Sampaio et al, 2005). In order to distinguish the modeling abilities of lumped parameters and continuous models, it is interesting to note that the use of a continuous model (by means of its discretization) gives a scheme of approximation. That is, for an allowed error tolerance the number of degrees of freedom of the discrete model can be computed, and then the dynamics of the structure can be described.

The literature reports that the non-linear dynamics of drill-strings -described by non-linear beam models- manifests different instabilities such as stick-slip or bit bouncing which can affect the drilling process. These instability patters can appear depending on the values of operating conditions as well as features of the structure. In the particular case of coupled extensional/torsional vibrations of a drill-string, it was shown (Sampaio et al. 2005) that the use of a non-linear beam model (instead of a linear) is crucial in the simulation of long-time stick-slip patterns. The non-linearities of the model are considered in the geometric stiffness terms. These terms as well as the reactive forces due to rock formation (specially reactive torques) in the boundaries play an important role in the schemes of numerical approximation such as the finite element method.

In this work an effort is made to characterize quantitatively and qualitatively the influence of different spatial discretization alternatives and different models of reaction torques, among others, in the solution performance of the integration methods. It is well known that the numerical solution of non-linear systems of equations is a time-demanding task. Moreover the numerical approaches of the non-linear beam models are usually stiff and hence hard to integrate. In these context is it important to provide certain reduction in the modeling without the sacrifice of accuracy and representativeness of the non-linear response. Consequently, a study of the influence of the several nonlinearities in the dynamical response of the beam model is also performed.

The Matlab platform offers many first order solvers that can be employed to face the numerical integration of the non-linear finite element models presented in this article. However, as it was expressed in the previous paragraph, the numerical approaches of non-linear models are normally stiff. This heighten the influence of the integration method performance in order to give an appropriate assessment to the numerical approximation. In this sense qualitative and quantitative comparisons of the Matlab ode solvers are also carried out.

2. Description of the non-linear model

2.1 Theoretical Introduction

This section briefly introduces the non-linear beam model developed in a recent work of the authors. For particular descriptions and definitions, the readers are encouraged to follow the deduction of the model in Sampaio et al. (2005). The beam model only accounts for the coupling between extensional and torsional deformations in the dynamics of drill-strings. The Fig. 1 shows an initially straight slender beam with annular cross-section (R_o and R_i are the outer and inner radii). The beam -which has a length L in the undeformed state- undergoes large displacements and small strains, and it is subjected to its own weight.

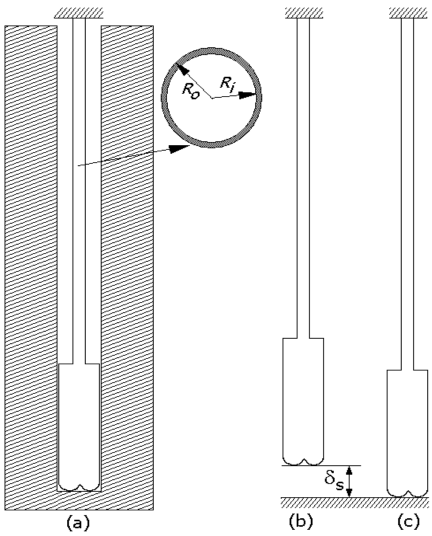

Figure 1. Drill String Scheme. (a) Description (b) Undeformed Configuration (c) Deformed Configuration

The variational form of the strain energy accounting only for extensional and torsional effects, can be decomposed in linear and non-linear contributions, which are given by:

$$\delta H = \delta H_L + \delta H_{NL} \tag{1}$$

where

$$\delta H_L = \int_0^L \left[\delta u' \; (EA\, u') + \delta\theta' \; (GI_0\, \theta')\right] dx \tag{2}$$

$$\begin{aligned}\delta H_{NL} = \; & \int_0^L \delta u' \left[\frac{EA}{2}\left(3\,u'^2 + u'^3\right) + \frac{EI_0}{2}\left(\theta'^2 + u'\theta'^2\right)\right] dx + \\ & \int_0^L \delta\theta' \left[\frac{EI_0}{2}\left(2\,u'\theta' + u'^2\theta'\right) + \frac{EI_{02}}{2}\theta'^3\right] dx\end{aligned} \tag{3}$$

where, E is the longitudinal modulus of elasticity and G is the transverse modulus of elasticity. A and I_0 stand for the cross sectional area and polar moment of inertia, whereas I_{02} is a generalized cross-sectional constant.

The virtual work done by inertial forces, damping forces and the own weight of the beam, can be expressed as:

$$\delta T = \int_0^L \left[\delta u\big(\rho A\ddot{u}\big) + \delta\theta\big(\rho I_0\ddot{\theta}\big)\right] dx \tag{4}$$

$$\delta D = \int_0^L \left[\delta u\big(C_u\dot{u}\big) + \delta\theta\big(C_\theta\dot{\theta}\big)\right] dx \tag{5}$$

$$\delta W = \int_0^L \delta u\big(\rho g A\big)dx \tag{6}$$

where, ρ is the material density, g is the gravity acceleration and, C_u and C_θ are the axial and torsional damping constants, which can be calculated from the considerations of Spanos et all (1995) among others.

The general virtual work equation accounting for all the aforementioned contributions can be written as:

$$\delta H + \delta T - \delta D - \delta W = 0 \tag{7}$$

and the equilibrium equations can be obtained performing in Eq. (7) the conventional steps of variational calculus.

2.2 Non-Linear Finite Element Formulation

A Finite Element model can be constructed through discretization of virtual work components of strain, inertia, damping and applied forces. The discretization is carried out using isoparametric elements with linear shape functions for both axial displacements and torsional rotations. Then the displacements can be defined by the following matrix form:

$$\begin{aligned} u &= \mathbf{N}_u \mathbf{q}_e \\ \theta &= \mathbf{N}_\theta \mathbf{q}_e \end{aligned} \tag{8}$$

The shape functions and generalized displacements vector can be defined by Eq. (9) for linear finite elements.

$$\begin{aligned} \mathbf{N}_u &= \{1-\xi, 0, \xi, 0\} \\ \mathbf{N}_\theta &= \{0, 1-\xi, 0, \xi\} \\ \mathbf{q}_e^T &= \{u_1, \theta_1, u_2, \theta_2\} \end{aligned} \tag{9}$$

where, $\xi = x/l_e$ is the non-dimensional element variable and l_e is the element length.

Now employing the usual procedures of domain discretization and matrix assembling (see, Sampaio et al. 2005), one gets the following finite element equation:

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{D}\,\dot{\mathbf{q}} + \left[\mathbf{K}_e + \mathbf{K}_g\left(\mathbf{q}\right)\right]\mathbf{q} = \mathbf{F}_g \tag{10}$$

where $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}_e$ and $\mathbf{K}_g$ are the global matrices of mass, damping, elastic stiffness and geometric stiffness, respectively, whereas $\mathbf{F}_g$ is the global vector of gravity forces. The force vector in the discrete expression (10) can be extended to account for other contributions, like impacts, etc.

2.3 Analysis about an initially deformed configuration

In order to perform a dynamic analysis, the discretized non-linear model is linearized with respect to a initially deformed configuration as it is shown in Sampaio et al (2005). The Drill-string is lowered being subjected by its own weight, when drill-bit reaches the rock formation, acts a reaction, which is considered time-invariant in this work. At this stage the drill-string starts its operational motion. Fig 1(b) and Fig 1(c) represent, respectively, the idealized undeformed and deformed configurations of the drill-string. In these circumstances two "a posteriori" forces are incorporated in the finite element model defined in Eq. (10). Then, in addition to the gravity force vector $\mathbf{F}_g$, in the bottom node is applied a time-independent force $\mathbf{F}_f$ to simulate the axial reaction due to rock formation, and a reactive torque T_{bit} is applied through the external generalized force vector $\mathbf{F}_T$. This reactive torque is applied at the bottom node N, i.e. in the $(2N)^{th}$ degree of freedom, and it can be defined combining different interaction models (Kreuzer and Kust, 1996 and Yigit and Christoforou, 2003), in the following form:

$$T_{bit} = \mathbf{F}_{T_{2N}} = \mu W_{ob} f_i(\theta_{bn}) \left[Tanh[\dot{\theta}_{bn}] + \tfrac{\alpha_1 \dot{\theta}_{bn}}{1+\alpha_2 \dot{\theta}_{bn}^2}\right] \quad with\, f_i(\theta_{bn}) = \begin{cases} f_1(\theta_{bn}) = \frac{1}{2}\left(1 + Cos[\theta_{bn}]\right) \\ f_2(\theta_{bn}) = 1 \end{cases} \tag{11}$$

where W_{ob} is the axial reaction of the rock formation, μ is a factor depending on the drill cutter characteristics, α_1 and α_2 are constants depending on rock properties. θ_{bn} and $\dot{\theta}_{bn}$ are the rotational angle and speed at the drill bit respectively. In order to take advantage of different modeling options, $f_i(\theta_{bn})$ may be one or a function depending on the rotation angle at the bit.

In this work, it is supposed that after the quasi-static lowering and when the reaction force reaches the prescribed value, the axial displacement of the drill bit is prescribed as it is suggested in Fig 1(c). Then further motions take place around this initial deformed configuration, which is obtained from the following equation:

$$\mathbf{q}_s = \mathbf{K}_e^{-1}\left(\mathbf{F}_g + \mathbf{F}_f\right) \tag{12}$$

Now, defining a new displacement vector $\bar{\mathbf{q}}$ relative to the static or initial deformed configuration $\mathbf{q}_s$, as

$$\bar{\mathbf{q}} = \mathbf{q} - \mathbf{q}_s \tag{13}$$

By linearization of Eq. (10) with respect the initially deformed configuration (12), one gets (Sampaio et al. 2005) the following equations of motion in terms of the relative displacement vector

$$\mathbf{M}\,\ddot{\bar{\mathbf{q}}} + \mathbf{D}\,\dot{\bar{\mathbf{q}}} + [\mathbf{K}_e + \mathbf{K}_g\,(\bar{\mathbf{q}} + \mathbf{q}_s)]\,\bar{\mathbf{q}} = \mathbf{F}_T \tag{14}$$

Then, the axial displacement of the drill bit it locked into its static value, i.e. $\bar{u}^L = 0$ or $u = u_s^L$, and the top position of the drill-string is subjected to a constant forcing rotary speed ω.

3. Numerical Analysis

In this section, the qualitative modeling of non-linear coupled extensional/torsional vibrations of drill-string is carried out. This analysis is based on a drill-string whose geometrical and material properties are the ones summarized in the Tab. 1 of Sampaio et al. (2005). The drill string consists of two segments. The upper segment is composed of slender drill pipes normally subjected to large traction forces due to their own weight. On the other hand, the lower segment is subjected to compressive forces due to the action of own weight of the upper part and the reactive forces, consequently the lower pipes of the drill-bit have larger diameters. The rock-bit interaction parameters $\alpha_1 = \alpha_2 = 1$ are employed in Eq. (11). The drill-bit parameter has the value $\mu = 0.04$. The drill-string is subjected to a forcing rotary speed of 10 rad/seg at the top. The function $f(\theta_{bn})$ in the torque, has the form varying with the rotation angle except in the last example where the responses of the non-linear model are compared for different torsional excitations.

3.1 Convergence Studies

In this section a convergence study is carried out. The Matlab solver ode15s was employed to integrate the models of finite elements. A relative tolerance error of $\epsilon_r = 10^{-8}$ was adopted in the solver.

A measure of convergence can be obtained by means of the relative error (between a coarse and the finest discretizations) calculated by expressions (15) and (16) for angular velocity and rotation angles, respectively. These expressions are computed in a given point x_g and at given integration instant t_g.

$$\epsilon\% = 100\left|\frac{\dot{\theta}(x_g,t_g)_{coarse} - \dot{\theta}(x_g,t_g)_{finest}}{\dot{\theta}(x_g,t_g)_{finest}}\right| \tag{15}$$

$$e\% = 100\left|\frac{\theta(x_g,t_g)_{coarse} - \theta(x_g,t_g)_{finest}}{\theta(x_g,t_g)_{finest}}\right| \tag{16}$$

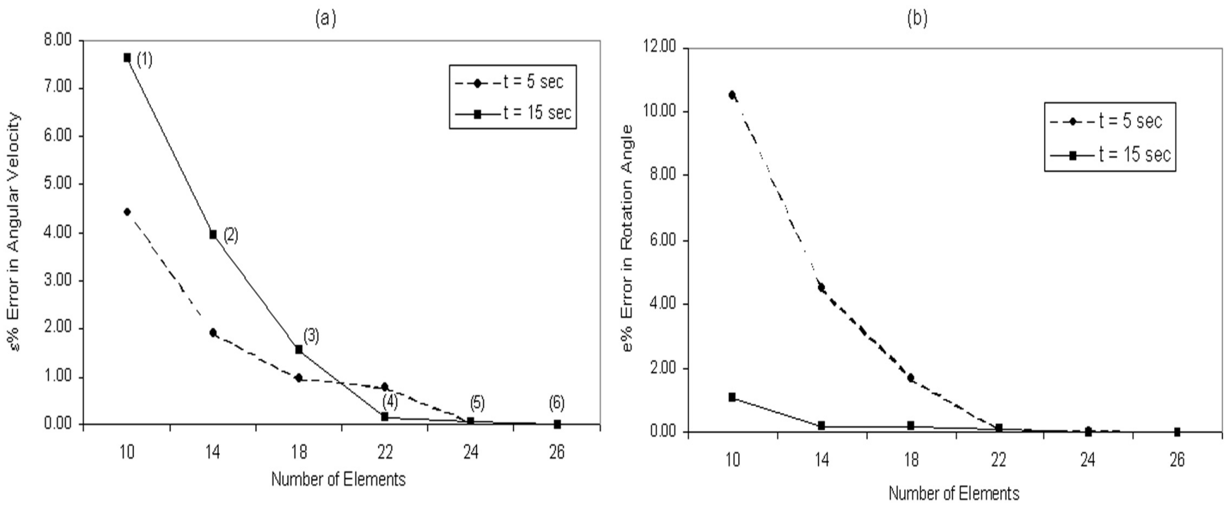

Figure 2. Convergence of approximation at drill-bit (a) angular velocity (b) rotation angle

Figure 2(a) shows a convergence of the angular velocity in the drill-bit calculated at instants $t_g = 5$ seconds and $t_g = 15$ seconds for models with different number of elements. Figure 2(b) shows the relative error for the rotation angle in the drill-bit at the same instants.

In Fig. 2(b), one observes a fast convergence of the rotation angle measured at the instant $t_g = 15$ seconds in comparison to the case measured at instant $t_g = 5$ seconds. This is reasonable due to the fact that as the drill-bit evolves the rotation angle becomes larger, whose effect is to reduce the relative error (see for example Fig. 3(a) for a measure of the magnitude of rotation angle). Consequently, coarser models reach acceptable convergence for the rotation angle. However, in practical drilling process there is more interest in the behavior of angular velocity than rotation angles. Now, in Fig. 2(a), it is possible to see a slow convergence of angular velocity for the coarser models (10 and 18 elements) at the beginning instants (i.e. $t_g = 5$ seconds). Models with more than 22 elements give better responses and a faster convergence for $t_g = 5$ seconds. But, to reach a good convergence at instant $t_g = 15$ seconds one needs at least models with more than 18 elements, and to reach reasonable convergence at instants $t_g \geq 15$ seconds it is imperative to employ the finest mesh possible. However, the use of finer meshes demands greater computational effort. In other words, the total calculation time required for the discrete equations at points (1) and (3) in Fig. 2(a) were 420 and 660 seconds, respectively; however the calculation time required at points (4), (5) and (6) in the same Figure were 1100, 3220 and 8900 seconds, respectively. The calculations were carried out on a Pentium M (1.7 GHz) with RAM of 1.0 Gb.

3.2 Comparison of Matlab solvers

As it was mentioned in the previous subsection the integration was carried out employing the Matlab solver ode15s, which is an implicit solver for "stiff" systems. However, with the aim to evaluate the efficiency and performance of other solvers available in Matlab, a comparison between implicit and explicit solvers is presented in this section. Models with 22 elements were tested employing two implicit solvers for "stiff systems" (ode15s and ode23s) and other two for "non-stiff systems" (ode113 and ode45). Figure 3(a) shows the variation of the drill-bit angular rotation calculated with the four solvers with a relative error tolerance of $\epsilon_r = 10^{-8}$. Figure 3(b) shows the difference of the responses of ode45, ode113 and ode23s with respect to ode15s. Figure 4 shows the same results but for the rotation velocity (where $\partial_t \theta(t)$ means the rotary speed, i.e derivation with respect to the time). The absolute error of solver-responses presented in Figs. 3(b) and 4(b) are calculated with expressions (17) and (18) respectively.

$$\epsilon_\theta = \theta(x_g, t)_{odeXX} - \theta(x_g, t)_{ode15s} \tag{17}$$

$$\epsilon_{\dot\theta} = \dot\theta(x_g, t)_{odeXX} - \dot\theta(x_g, t)_{ode15s} \tag{18}$$

One observes that although in Figs. 3(b) and 4(b) there are absolute differences between two solvers, the relative errors are insignificant in comparison to the values of the rotation angle and angular velocity.

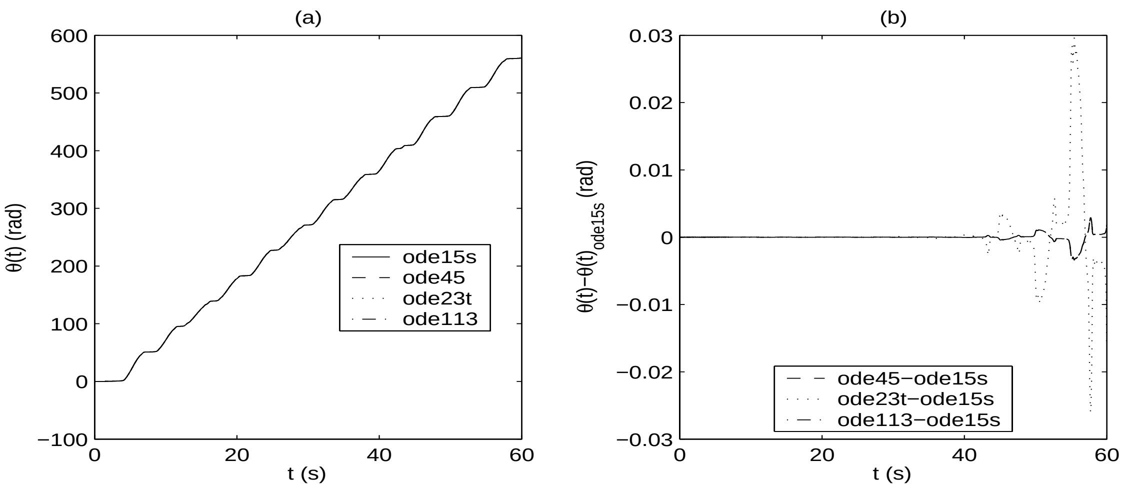

Figure 3. Performance of Matlab Solvers (a) rotation angle at drill-bit (b) difference with respect to ode15s

The Table 1 presents the time demanded for each solver to calculate the response of a 60 seconds period, for different cases of internal error tolerance adopted in the Ode solvers. One can see huge amount of time demanded to obtain the response for internal error tolerance of $\epsilon_r = 10^{-12}$ instead of $\epsilon_r = 10^{-8}$. As it can be inferred the selection of a particular solver involves the decision of the computing cost which, for this problem may vary between few minutes and more than seven hours.

The selection of Matlab solvers instead of the Newmark's family of solvers to integrate the finite element equations has two connected reasons. The first reason is related to the numerical performance of the Newmark's method, which depends on the selection of certain parameters that can lead to an unstable numerical behavior. Although, it is possible

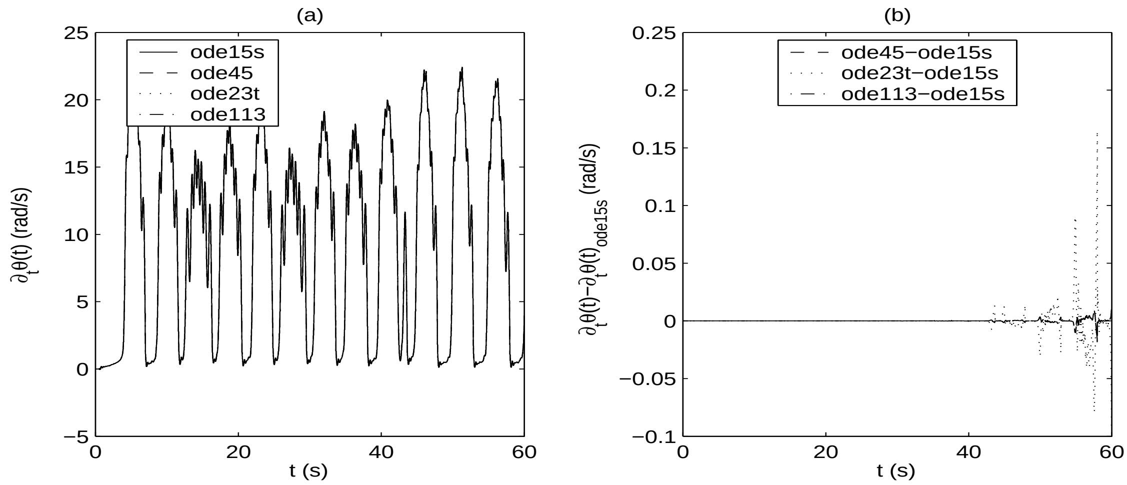

Figure 4. Performance of Matlab Solvers (a) angular velocity at drill-bit (b) difference with respect to ode15s

Table 1. Computing-cost (in seconds) of Matlab solvers for different relative error tolerances

Solver	$\epsilon_r = 10^{-8}$	$\epsilon_r = 10^{-12}$
ode15s	1692	6956
ode23s	7123	27456
ode113	359	1508
ode45	1270	4953

to add numerical damping with the scope to stabilize the method, not always this alternative gives good results. The second reason is connected with the fact that for strong non-linear problems (specially when the excitation force directly depends on the speed and displacement) the time-step has to smaller, consequently demanding more computational effort. The authors also have carried out a calculation -not shown here- with the Newmark's Method (with 20 finite element, parameters $\gamma = 0.5$ and $\beta = 0.25$) for the problem studied in this section, that demanded more than twelve hours to calculate a period of thirty seconds without numerical instabilities.

3.3 Comparison of Torque Modeling

The Eq. (11) offers two alternatives to model the perturbation torque. Kreuzer and Kust (1996) employed a form which depends on the rotation angle and the rotary speed at the drill-bit, other authors (Yigit and Christoforou (2000) among other) employed a torque on bit which only depends on the rotary speed at the drill-bit. The ode15s solver was used to integrate a model with 22 finite elements.

Figure 5(a) shows the angular velocity at the drill-bit for both perturbation torques expressed in (11). Figure 5(b) offers the absolute difference between the angular velocity due to both torques, i.e. $\dot{\theta}(t, T_{n_{f_1(\theta_{bn})}}) - \dot{\theta}(t, T_{n_{f_2(\theta_{bn})}})$. The stick-slip pattern is obviously modified. The use of $f_i(\theta_{bn}) = f_2(\theta_{bn})$ instead of $f_i(\theta_{bn}) = f_1(\theta_{bn})$ in Eq. (11), leads to a reduction of the quantity of stick-slip periods, giving uniform angular velocity peaks, as it is possible to infer observing Eq. (11).

4. Conclusion

In the present article a qualitative study on numerical modeling of non-linear extensional/torsional vibrations of drill-string was presented. The aim of this work was to characterize and identify the features of the integration schemes for the discretized equations of the non-linear model developed recently by the authors. In these circumstances four solvers of the Matlab software were tested. These results suggest that for the case of extensional/torsional vibrations of drill-strings the use of a particular integration method may lead to a sensible spare of time preserving the accuracy of the numerical approximation. The use of Matlab solvers offers a good alternative to well known solvers of Newmark's family methods which can have numerical instabilities requiring a lot of computing effort. The effect of torque modeling was also analyzed. The perturbation torque depending only on the angular velocity (i.e. with $f_i(\theta_{bn}) = f_2(\theta_{bn})$) has the effect to reduce the quantity of stick-slip periods in the drill-bit.

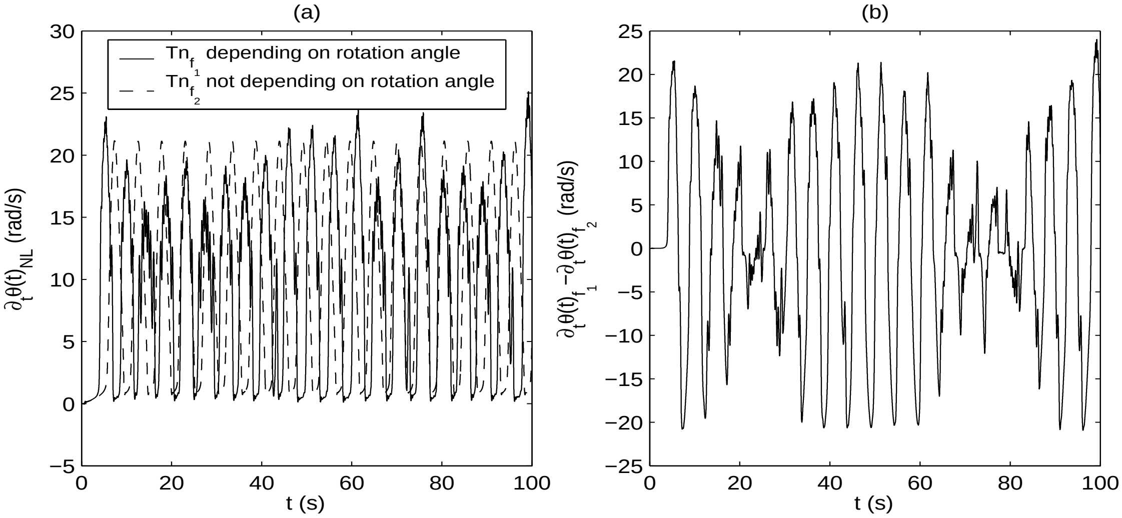

Figure 5. Effect of perturbation torques (a) angular velocity at drill-bit (b) difference of responses

5. Acknowledgements

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