MODELING RESIDUAL STRESSES IN WELDED STEEL PLATES USING A CONSTITUTIVE MODEL WITH PHASE TRANSFORMATION

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Abstract. *Welding is a process where localized intensive heat input is furnished to a piece promoting mechanical and metallurgical changes. Phenomenological aspects of welding process involve couplings among different physical processes and its description is unusually complex. Basically, three couplings are essential: thermal, phase transformation and mechanical phenomena. One important aspect associated with structural integrity of welded mechanical components is the presence of residual stresses. It is well known that residual stress plays a preponderant part on the structural integrity of a mechanical component, especially in nucleation and propagation of cracks. Residual stresses result directly from the thermal cycle associated with the welding process. High temperatures developed by the heat source induce phase transformation and plasticity that could promote significant mechanical and metallurgical changes near the welded area. The temperature gradients developed through the piece results in a nonhomogeneous plastic strain distribution, which promotes residual stresses fields when the piece reaches room temperature. This article is concerned with the modeling and simulation of welded steel plates using a constitutive anisothermal model that includes two phases (austenite and martensite microstructures). The model includes thermomechanical couplings in the energy equation associated with phase transformation, plasticity and hardening, allowing the investigation of the effects promoted by these couplings. A numerical procedure is developed based on the operator split technique associated with an iterative numerical scheme in order to deal with non-linearities in the formulation. Numerical simulations are carried out analyzing some aspects related to the welding process of steel plates during an underwater wet welding operation. Results suggest that the proposed model is capable of capturing the main behavior observed in experimental data.*

Keywords*: Residual Stresses, Welding, Thermomechanical Coupling, Modeling, Numerical Simulation.*

1. Introduction

Welding is a very complex process where localized intensive heat input is furnished to a piece promoting mechanical and metallurgical changes. There are several welding processes used in industry, each one with several different characteristics, advantages and disadvantages. Phenomenological aspects of welding involve couplings among different physical processes and its description is unusually complex. Basically, three couplings are essential: thermal, phase transformation and mechanical phenomena, meanwhile several authors have addressed these three aspects separately. Some authors considers only the thermomechanical coupling (Bang *et al.*, 2002; Fernandes *et al.*, 2003, 2004; Teng and Chang, 2004) but it is important to note that in many situations the phase transformation must be also considered (Zacharia *et al.*, 1995; Taljat *et al.*, 1998; Ronda and Oliver, 2000).

One important aspect associated with structural integrity of welded mechanical components is the presence of residual stresses (Almer *et al.*, 2000; Fernandes 2002; Fernandes *et al.*, 2003, 2004; Teng and Chang, 2004). Residual stresses result directly from the thermal cycle caused by the localized intensive heat input that promotes temperature gradients. High temperatures developed by the heat source promote phase transformation and plasticity. Mechanical properties present lower values at higher temperatures allowing the development considerable plastic strain. Moreover, phase transformation can promote phase transformation induced strain (Denis *et al.*, 1985; Pacheco *et al.*, 2001; Silva *et al.*, 2004; Ronda and Oliver, 2000). The temperature gradients developed through the piece results in a nonhomogeneous plastic strain distribution, which promotes residual stresses fields when the piece reaches room temperature. Due to the importance of estimate residual stresses in welding, several investigators had addressed this subject (Zacharia *et al.*, 1995; Taljat *et al.*, 1998; Bang *et al.*, 2002; Fernandes *et al.*, 2003, 2004).

It is well know that residual stress plays a preponderant part in the structural integrity of a mechanical component, especially in nucleation and propagation of cracks. Nevertheless, the presence of residual stress is not usually considered in traditional design of mechanical components. Traditional design methodologies assume a null stress state before the application of the operational loading in a mechanical component and the use of precise analytic and/or computational methods is not sufficient for a reliable structural integrity life prediction. The presence of tensile residual stresses can be especially dangerous to mechanical components submitted to fatigue loadings. In the presence of tensile stresses promoted by the operational loading conditions, both stresses are added resulting in much higher tensile stress levels than the ones predicted.

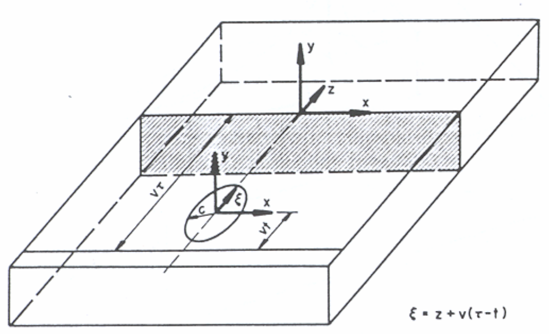
The present contribution regards on modeling and simulation of residual stress distributions in welded steel plates during an underwater wet welding operation using an anisothermal constitutive model with internal variables that considers the couplings of thermal, phase transformation and mechanical phenomena. The model was already applied to the study of other thermomechanical problems as the quenching of steel pieces (Pacheco *et al.*, 1997, 2001; Oliveira *et al.*, 2003; Silva *et al.*, 2004) and allows the investigation of the effects promoted by these couplings. In this article, the model is applied to the welding of long thin steel plates. A numerical procedure is developed based on the operator split technique associated with an iterative numerical scheme in order to deal with non-linearities in the formulation.

2. Moving Weld Heat Source

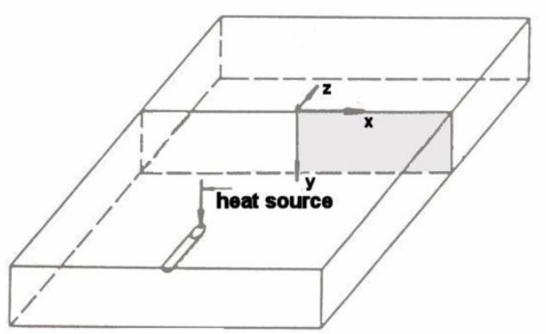
An accurate model for a moving weld heat source is essential in the analysis of the thermal cycle promoted by the welding process. Pavelic *et al.* (1969) suggested a Gaussian surface flux distribution disc. Fridman (1975) presents an alternative form for the Pavelic surface flux distribution disc expressed in a coordinate system (x,ζ) that moves with the heat source. The surface flux distribution of the source inside the disc becomes:

$$q(x,\zeta) = \frac{3Q}{\pi C^2} \mathrm{e}^{-3(x/c)^2} \; \mathrm{e}^{-3(\zeta / C)^2} \tag{1}$$

where $\zeta = v\,(\tau - t)$, v is the velocity of the heat source, t is the time, τ is lag factor needed to define the position of the source at time t = 0 and C is the radius of the surface flux Gaussian distribution in a disc with center at (0,0) and parallel to coordinate system, ζ, t. $Q = \eta\, V\, I$ is a power heat input source welding, where η is the heat source efficiency, V the voltage and I the current. Figure 1a shows the model for the surface flux distribution with the coordinate system.



(a)



(b)

Figure 1. Gaussian surface flux distribution heat source (a). Arrangement for the section bead on plate welds (b). (Goldak *et al.*, 1984).

As the object of study is a thin plate, heat flow through the thickness direction can be neglected. Also, for long plates, at a distance far from the edges, heat flow in the welding direction can be neglected (Goldak *et al.*, 1984). Considering these hypotheses, the model for a moving weld heat source of Eq. (1) and a symmetry condition at the plane yz (adiabatic condition), it is possible to reduce the analysis to the region of the gray area in Fig. 1b.

3. Constitutive Model

Constitutive equations may be formulated within the framework of continuum mechanics and the thermodynamics of irreversible processes, by considering thermodynamic forces, defined from the Helmholtz free energy, ψ, and thermodynamic fluxes, defined from the pseudo-potential of dissipation, ϕ (Lemaitre and Chaboche, 1990; Pacheco, 1994).

With this aim, a Helmholtz free energy is proposed as a function of observable variables, total strain, ε_{ij}, and temperature, T. Moreover, the following internal variables are considered: plastic strain, ε_{ij}^{p}, kinematic hardening, α_{ij}, volumetric fractions of different microstructures, represented by phases in a macroscopic point of view, $\beta = \beta_i$ (i = 1,...,N_{phases}), volumetric transformation strain, ε_{ij}^{tv}, and transformation plasticity strain, ε_{ij}^{tp}. The volumetric transformation strain is related to volumetric expansion associated with phase transformation from a parent phase and the transformation plasticity strain represents the result of several physical mechanisms related to local plastic strain promoted by the phase transformation (Denis *et al.*, 1985; Sjöström, 1985; Desalos *et al.*, 1982). It should be emphasized that this strain may be related to stress states that are inside the yield surface. Therefore, the following free energy is proposed, employing indicial notation where summation convention (i = 1,2,3) is evoked (Eringen, 1967), except when indicated:

$$\rho\psi(\varepsilon_{ij},\varepsilon_{ij}^{p},\varepsilon_{ij}^{tv},\varepsilon_{ij}^{tp},\alpha_{ij},\beta,T) = W(\varepsilon_{ij},\varepsilon_{ij}^{p},\varepsilon_{ij}^{tv},\varepsilon_{ij}^{tp},\alpha_{ij},\beta,T) = W_e(\varepsilon_{ij}-\varepsilon_{ij}^{p},\varepsilon_{ij}^{tv},\varepsilon_{ij}^{tp},\beta,T) + W_\alpha(\alpha_{ij}) + W_\beta(\beta) - W_T(T) \tag{2}$$

where ρ is the material density. The increment of elastic strain is defined as follows:

$$d\varepsilon_{ij}^{e} = d\varepsilon_{ij} - d\varepsilon_{ij}^{p} - d\varepsilon_{ij}^{tv} - d\varepsilon_{ij}^{tp} - \alpha_T dT\delta_{ij} \tag{3}$$

where the last term is associated with thermal expansion and the parameter α_T is the coefficient of linear thermal expansion.

This model was developed in a general formulation and was previously applied to the study of various related problems as the quenching of steel pieces (Pacheco *et al.*, 2001; Oliveira *et al.*, 2003; Oliveira, 2004; Silva *et al.*, 2004) and the thermomechanical coupling effects on low cycle fatigue life of metallic materials (Pacheco, 1994; Pacheco and Mattos, 1997), and allows one to identify different coupling phenomena, estimating the effect of each one in the process. A detailed description of this constitutive model may be obtained in the references cited in this paragraph.

This contribution considers the welding of long thin plates as an application of the proposed general formulation. With this assumption, heat transfer analysis may be reduced to a one-dimensional problem, as heat flow through the thickness and in the welding direction is neglected (respectively, directions y and z, in Fig. 1b). A null strain state is assumed in the welding direction ($\varepsilon_z = \varepsilon = 0$) to simulate the restriction associated with adjacent regions of the heated region, which are at lower temperatures. Also, null stresses are assumed in the thickness and transversal directions (respectively, directions y and x, in Fig. 1b). Therefore, a one-dimensional stress state is observed with the only non-null stress in the welding direction ($\sigma_z = \sigma$). Only two phases are considered: austenite and martensite. β represents the volumetric fraction of martensite. Under these assumptions, a one-dimensional model is formulated and the tensor quantities presented in the general formulation may be replaced by scalar quantities. For this situation the thermodynamics forces (σ, P, Q, R, X, B^β, s), associated with state variables $(\varepsilon,\varepsilon^{p},\varepsilon^{tv},\varepsilon^{tp}\alpha,\beta,T)$, are defined as follows:

$$\sigma = \frac{\partial W}{\partial\varepsilon} = E(\varepsilon - \varepsilon^{p} - \varepsilon^{tv} - \varepsilon^{tp}) - E\alpha_T(T-T_0) \quad ; \quad P = -\frac{\partial W}{\partial\varepsilon^{p}} = \sigma \quad ; \quad Q = -\frac{\partial W}{\partial\varepsilon^{tv}} \quad ; \quad R = -\frac{\partial W}{\partial\varepsilon^{tp}}$$
$$\mathrm{X} = \frac{\partial W}{\partial\alpha} = H\alpha \quad ; \quad \mathrm{B}^{\beta} = -\frac{\partial W}{\partial\beta} \quad ; \quad \mathrm{s} = -\frac{\partial W_T}{\partial T} \tag{4}$$

where T_0 is a reference temperature, E is the Young modulus and H is the plastic modulus.

In order to describe dissipation processes, it is necessary to introduce a potential of dissipation $\phi(\dot\varepsilon^{p},\dot\varepsilon^{tv},\dot\varepsilon^{tp},\dot\alpha\ ,\dot\beta,q\ )$, which can be split into two parts: $\phi(\dot\varepsilon^{p},\dot\varepsilon^{tv},\dot\varepsilon^{tp},\dot\alpha\ ,\dot\beta,q\ ) = \phi_I(\dot\varepsilon^{p},\dot\varepsilon^{tv},\dot\varepsilon^{tp},\dot\alpha\ ,\dot\beta) + \phi_T(q\ )$. Also, this potential can be written through its dual $\phi^*(P,Q,R,X,B^{\beta},g) = \phi_I^*(P,Q,R,X,B^{\beta}) + \phi_T^*(g)$:

$$\phi_I^* = I_f^*(P,X) + \varsigma_{A\to M}\dot\beta^{m}B^{\beta} \qquad \phi_T^* = \frac{T}{2}\Lambda\, g^2 \tag{5}$$

where $g = (1/T)\ \partial T/\partial x$ and Λ is the coefficient of thermal conductivity; $I_f^*(P,X)$ is the indicator function associated with elastic domain, related to the *von Mises* criterion (Lemaitre and Chaboche, 1990),

$$f(\sigma,X) = |\sigma - X| - S_Y \le 0 \tag{6}$$

where S_Y is the material yield stress. A set of evolution laws obtained from ϕ^* characterizes dissipative processes,

$$\dot\varepsilon^{p} = \frac{\partial\phi^*}{\partial P} = \lambda\,\mathrm{sign}(\sigma - H\,\alpha) \quad ; \quad \dot\varepsilon^{tv} = \frac{\partial\phi^*}{\partial Q} = \gamma\dot\beta \quad ; \quad \dot\varepsilon^{tp} = \frac{\partial\phi^*}{\partial R} = \left[2\kappa(1-\beta)\dot\beta\right]\sigma \quad ; \quad \dot\alpha = -\frac{\partial\phi^*}{\partial X} = \dot\varepsilon^{p}$$
$$\dot\beta = \frac{\partial\phi^*}{\partial B^{\beta}} = \varsigma_{A\to M}\dot\beta^{m} \quad ; \quad q = -\frac{\partial\phi^*}{\partial g} = -\Lambda T\ g = -\Lambda\frac{\partial T}{\partial x} \tag{7}$$

where λ is the plastic multiplier (Lemaitre and Chaboche, 1990) from the classical theory of plasticity, sign(x) = x / |x| and q is the heat flow. γ is a material phase property related to total expansion and κ is a material phase parameter related to transformation plasticity. The austenite-martensite phase transformation is described with the aid of the following condition:

$$\varsigma_{A\rightarrow M}\left(\dot{T},T\right)=\Gamma\left(-\dot{T}-rMs\right)\Gamma\left(M_s-T\right)\Gamma\left(T-M_f\right) \tag{8}$$

where rM_s is the critical cooling rate for the martensite formation, defined from the Continuous-Cooling-Transformation diagram (CCT) diagram; $\dot{T}$ is the cooling rate. Also, $\Gamma(x)$ is the Heaviside function. Therefore, the kinetics of martensitic phase transformation may be expressed by

$$\beta=\beta\left(T,\dot{T}\right)=\varsigma_{A\rightarrow M}\,\beta^m \tag{9}$$

where β^m is defined from the equation proposed by Koistinen and Marburger (1959):

$$\beta^m=1-\exp\left[-k\left(M_s-T\right)\right] \tag{10}$$

where, k is a material constant. M_s is the temperature where martensite starts to form in the stress-free state and M_f is the temperature where martensite finishes its formation in the stress-free state.

Assuming that the specific heat is $c=-(T/\rho)\,\partial^2 W/\partial T^2$ and the set of constitutive Eqs. (4) and (7), the energy equation can be written as (Pacheco, 1994):

$$\frac{\partial}{\partial x}\left(\Lambda\frac{\partial T}{\partial x}\right)-\rho c\dot{T}+\frac{Per}{A}\left[-h(T-T_\infty)+Q\right]=-a_I-a_T \qquad \text{where} \quad \begin{cases} a_I=\sigma\ \dot{\varepsilon}^p-X\dot{\alpha}+B^\beta\dot{\beta} \\ a_T=T\left(\dfrac{\partial\sigma}{\partial T}\left(\dot{\varepsilon}-\dot{\varepsilon}^p\right)+\dfrac{\partial X}{\partial T}\dot{\alpha}-\dfrac{\partial B^\beta}{\partial T}\dot{\beta}\right)\end{cases} \tag{11}$$

where h is the convection coefficient, T_∞ is the surrounding temperature, Per is the perimeter and A is the cross section. Terms a_I and a_T are, respectively, internal and thermal coupling. The first one appears in the right hand side of the energy equation and is called internal coupling. It is always positive and has a role in the energy equation similar to a heat source in the classical heat equation for rigid bodies. In this article, both terms are neglected and thermal problem is solved as a rigid body.

4. Numerical Procedure

The numerical procedure here proposed is based on the operator split technique (Ortiz *et al.*, 1983; Pacheco, 1994) associated with an iterative numerical scheme in order to deal with non-linearities in the formulation. With this assumption, coupled governing equations are solved from four uncoupled problems: thermal, phase transformation, thermo-elastic and elastoplastic.

Thermal Problem - Comprises a uniaxial conduction problem with convection and heat input generated by the weld heat source. Material properties depend on temperature, and therefore, the problem is governed by non-linear parabolic equations. An implicit finite difference predictor-corrector procedure is used for numerical solution (Ames, 1992; Pacheco, 1994).

Phase Transformation Problem – The volumetric fractions of the phases are determined in this problem. Evolution equations are integrated from a simple implicit Euler method (Ames, 1992; Nakamura, 1993).

Thermo-elastic Problem - Stress is evaluated from temperature distribution. Evolution equations are integrated from a simple implicit Euler method (Ames, 1992; Nakamura, 1993).

Elastoplastic Problem - Stress and strain fields are determined considering the plastic strain evolution in the process. Numerical solution is based on the classical return mapping algorithm (Simo and Miehe, 1992; Simo and Hughes, 1998).

5. Numerical Simulations

The proposed model is applied to the welding of a thin plate of AISI 4140H steel by manual shield metal arc welding (SMAW) process during an underwater wet welding operation. In this process a severe condition is established where high temperature rates and gradients are observed and some regions may experiment quenching. A butt-joint is considered for a plate with a thickness of 10 mm, a length (L) of 200 m, a width (w) of 100 mm and a root opening of 2.4 mm. Thermal and mechanical properties are temperature dependent and are fitted by polynomial equations from experimental data (Pacheco *et al.*, 2001; Oliveira *et al.*, 2003; Oliveira, 2004; Silva *et al.*, 2004). For the presented simulations the following parameters are used: Q = 4500 W, C = 10 mm, v = 0.5 mm/s, h = 7 kW/m^2, T_∞ = 20°C, ρ = 7800 kg/m^3 and initial temperature of 20 °C. For phase transformation parameters the following parameters are used: γ = 1.33 x 10^{-2}, k = 1.1 x 10^{-2} /C°, κ = 1.0 x 10^{-10}, M_s = 370 °C, M_f = 260 °C, rMs = 8 s and austenitization and solidification temperatures of 843 °C and 1400 °C, respectively.

As cited before the heat flow through the thickness and in the welding direction is neglected. These hypotheses are justified by the low thickness of the plate and the by the fact that the region of interest is located far from the edges. Also a symmetry condition at the plane yz (adiabatic condition) is considered. It is important to note that the proposed model presents some approximations that can influence the response and must be taken into account when analyzing the results. One important point is that in a real piece the surrounding material offers some degree of restriction in the welding direction (z) direction, but not completely as adopted in this model.

Numerical simulations are performed with a computational software developed in C programming language. A spatial discretization of 80 nodes is employed. The final mesh and the time step are defined after a convergence analysis. The analysis considers two stages: *welding process* and *cooling*. In the *welding process* stage, the weld heat source surface flux distribution represented by Eq. (1) is applied to the model during the heat source disc passage through the plane of the model (xy). The condition prior to operation, where a residual stress field develops, is achieved in the *cooling* stage where the plate changes heat with the surroundings by convection until it reaches thermal equilibrium with the surroundings. To simulate the absence of material at the root opening before the weld deposition, null values are attributed to the mechanical properties at this region until the solidification temperature is reached during the *cooling* stage. Figure 2 presents the spatial discretization mesh used in the numerical simulations developed.

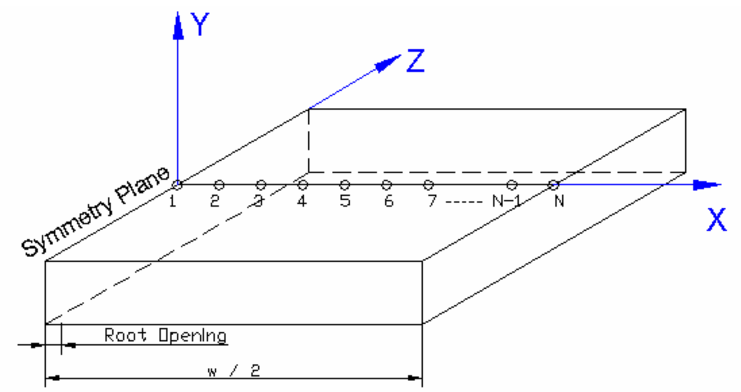
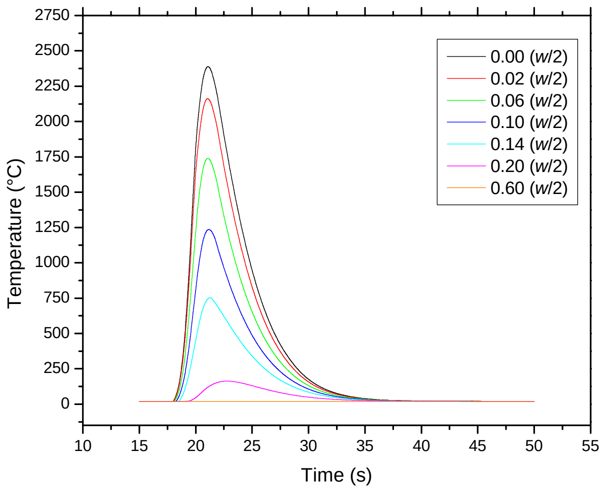
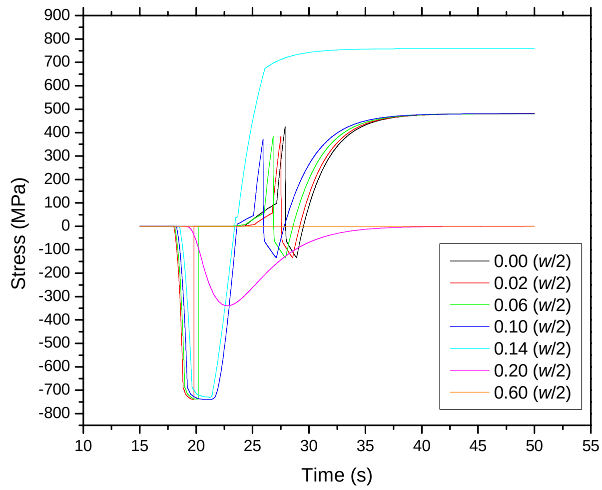


Figure 2. Spatial discretization.

Figure 3 presents the temperature and stress evolution for some points in the plate. It can be observed that during the process, high temperature gradients and rates develop through the piece. Figure 4 presents the plastic strain, kinematic hardening and martensite volumetric fraction evolution for some points in the plate. These figures show the complex nature of the process that involves the coupling between various phenomena. Figure 5 presents the residual stress and martensite volumetric fraction distributions through the plate for the final instant, when the plate is in thermal equilibrium with the surroundings.

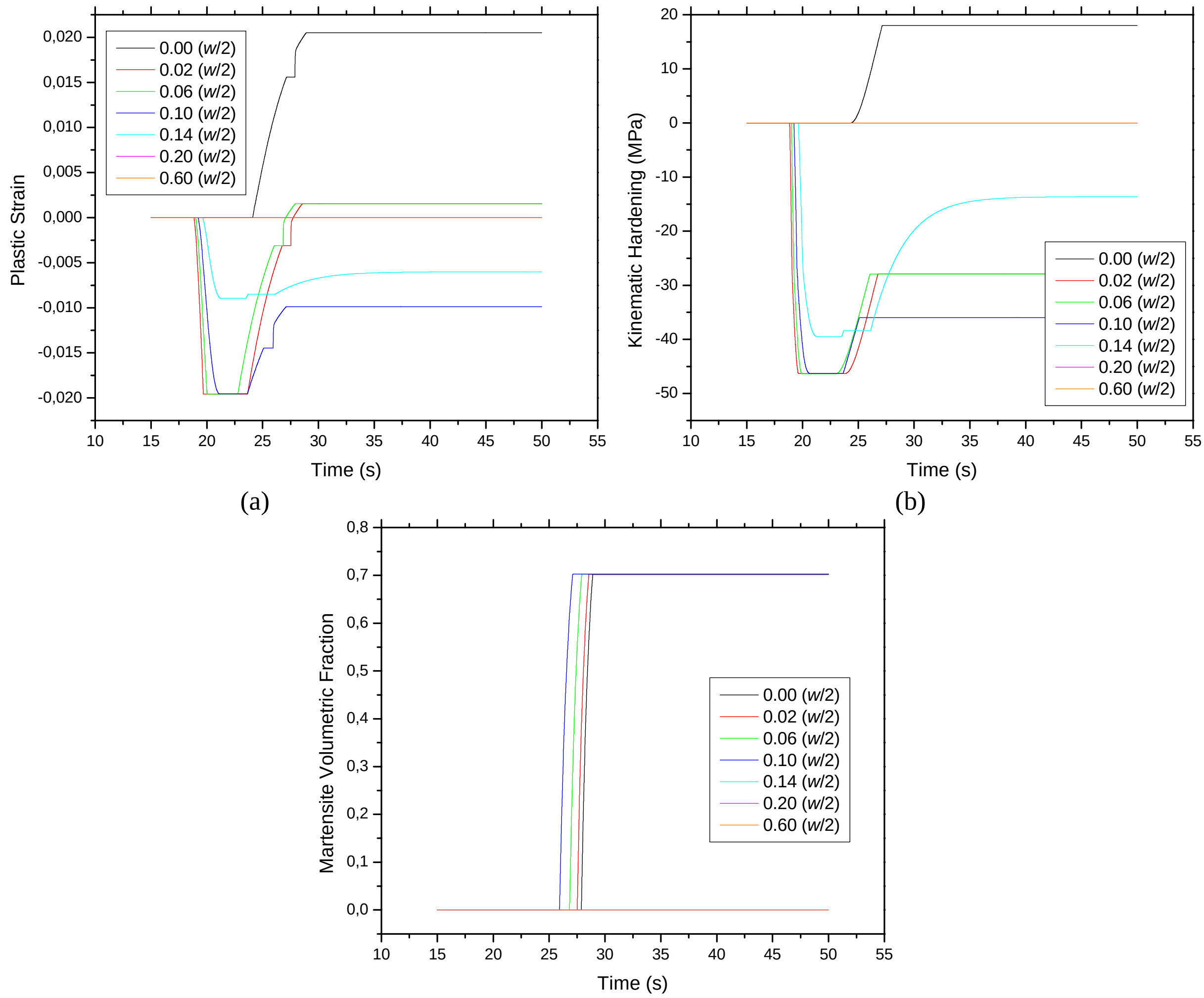


(a)



(b)

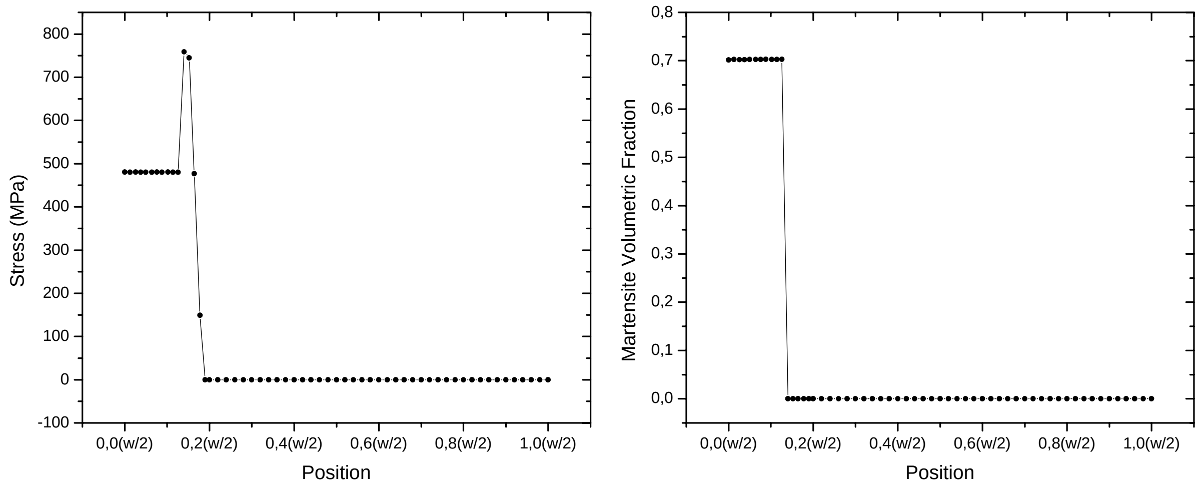
Figure 3. Temperature (a) and stress (b) evolution.



(a) (b)

(c)

Figure 4. Plastic strain (a), kinematic hardening (b) and martensite volumetric fraction (c) evolution.



(a) (b)

Figure 5. Residual stresses (a) and martensite volumetric fraction (b) distributions through the plate.

Figure 3a shows that the material that experiments fusion (T > 1400 °C) is concentrated in a region defined by x < 0.10 w/2 (5 mm) and that the temperature evolution for x > 0.60 w/2 (30 mm) is insignificant. In Fig. 3b can be observed a stress sign inversion, from compressive (during the heating) to tensile (during cooling). Note that the region near the heat source (x < 0.10 w/2) experiments a second stress sign inversion promoted by the phase transformation (austenite-martensite phase transformation results in a volumetric variation of ≅ +4%). For x <0.02 w/2 this stress sign inversion occurs at t ≅ 28 s and coincides with phase transformation, as can be observed from Fig 4c.

Note the peak stress that occurs in Fig. 5a at approximately $x = 0.14\ w/2$. The stress relief observed for lower values of x is promoted by the phase transformation. As pointed before, the austenite-martensite phase transformation results in a positive volumetric variation of approximately 4% that has the opposite sign of the thermal strain during cooling. Nevertheless, high values of residual stresses (of the initial yield strength magnitude) are observed at the end of welding process.

Figure 6 shows a comparison between the residual stress distribution prediction for two situations: *with phase transformation* and *without phase transformation*. The first one represents the complete proposed model. In the second one the phase transformation is not computed. The main difference occurs in the region where phase transformation develops. Higher residual stresses values are observed for the situation *without phase transformation*. The results indicate that it is important to consider phase transformation in the prediction of residual stresses in welded steel plates.

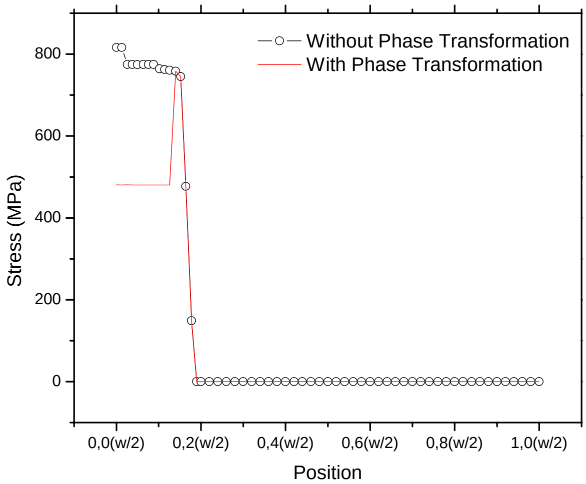


Figure 6. Residual stresses distribution through the plate for two situations: *with phase transformation* and *without phase transformation*.

6. Conclusion

This work presents a study of residual stresses in welded thin steel plates. A one-dimensional constitutive anisothermal model that includes two phases (austenite and martensite microstructures) is developed in order to estimate the residual stress distribution after the welding process. The hypothesis adopted in the proposed model results in a one-dimensional analysis that requires modest computational power. Nevertheless, the model considers the thermomechanical couplings in the energy equation associated with phase transformation, plasticity and hardening, and allows the investigation of the effects promoted by these couplings.

Numerical simulations show that there are high values of residual stresses (of the initial yield strength magnitude) at the end of welding process, and therefore before the mechanical component enters in operation. The results indicate that it is important to consider phase transformation in the prediction of residual stresses in welded steel plates as phase transformation may change considerably the residual stresses distribution.

The proposed methodology can be used as a powerful tool to study the effects of welding parameters, like heat input or welding velocity, in the residual stresses of welded mechanical components. Moreover, an experimental program to measure residual and operational stresses must be established.

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