A COMPARATIVE STUDY OF NEURAL NETWORKS AND SENSITIVITY MODAL ANALYSIS FOR DAMAGE ASSESMENT

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Abstract. *Nowadays, the advances in neural networks have opened new sights in structural engineering. The robustness and ability in dealing with incomplete and noisy data make neural networks the best alternative for this purpose. Associated to vibration signature analysis, this technique has shown a robust behavior. The assessment of structural damage and identification of the damaged place in a large and complex structure is a hard task. It is well known that from variations in structural natural frequency measured "in situ" and a well-calibrated structural model, it is possible to detect position as well as intensity of damaged states. Some new advances have been made in this area by means of neural networks. A brief review in literature in the application of neural networks in the last decade is outlined. Emphasis is set to the application of neural networks with radial basis functions for damage detection in a civil structure. A numerical example is presented highlighting the main features of neural networks for detection and evaluation of damage. This example shows that the use of neural networks is very promising, indicating great potential in damaged detection tasks for civil structures.*

Keywords*: Structural Damage, Damage Identification, Neural Networks*

1. Introduction

There are several ways to deal with the structural integrity evaluation. There is a consensus that it is necessary establish inspection procedures which systematically evaluate the structural integrity. The goal is to find and quantify the damage on the structure. The main techniques may be separated as nondestructive or destructive testing.

Basically, an evaluation through the dynamic behavior begins determining perceptible variations in the modal characteristics (natural frequencies, vibration modes and damping), starting from observations accomplished regularly in the structure. In other words, if the structure presents variations in its dynamic behavior along regular periods it will mean that there is some imminent problem. This obviously happens because the degradation or the structural damage unavoidably causes a degradation of the local stiffness of the structure, mass loss or changes in the material damping ratio that affect its dynamic characteristics or even support conditions. All these facts have intrinsic relationship with the dynamic characteristics of the structure and, consequently, with its dynamic behavior.

In this work, the technique of damage detection is investigated through the modal analysis, specifically with the variations of natural frequencies of the damaged structure. This technique is well established in the literature. It will be also investigated the use of artificial neural networks with radial basis function for training sets of damage patterns and posterior prediction of the damages levels and locations.

2. Bibliographical review

A great number of nondestructive evaluation techniques have been developed based on the changes of the dynamic parameters. Cawley and Adams (1979) have used the changes in the natural frequency together with a Finite Element model to locate the damage. Penny (1993) has observed that this method was susceptible to measurements uncertainties and then has suggested forms of improving the location task through a statistical analysis. Biswas (1990) has accomplished experiments in a traffic bridge and has demonstrated that the changes of the natural frequencies by itself were enough to detect the damage. Loland (1976) has led to similar conclusions.

Through the inclusion of the vibration mode shapes, many authors have developed methods that are able to detect the damage position as well as damage extent. Pandey and Biswas (1990) have used the vibration mode shapes of damaged and not damaged structures to localize and evaluate extent of the fault through the solution of a system of linear equations, differently from Zimmerman and Kaouk´s (1994) algorithm which need two steps, one for the detection and another to the damage extent evaluation.

Messina (1996) has proposed an uncertainty approach for damage detection that was later extended by Contursi and Messina (1998) to identify the damage extent in several sites. The data validation was accomplished through numerical tests free of noise. This approach, however, can involve a significant computational effort when dealing with large structures with many degrees of freedom.

A robust methodology for the evaluation of the structural damage should be able to recognize patterns of the structural behavior whose components present damage and also to determine the extension (intensity) of this damage. A general paradigm found in the damage detection was presented by Pandey and Barai (1993) when working for steel bridges. The difficulties they found were due to the fact that the available data of the *in situ* measures were frequently inaccurate and noisily. This problem can be outlined by the use of Artificial Neural Networks which has been demonstrated (Wu et al., 1992; Szewczyk et al., 1992) the feature of immunity to random noise.

In order to use effectively an Artificial Neural Network for the damage diagnosis it should be trained with well-known and already diagnosed damages sceneries as training samples. In an analytic study, Wu et al. (1992) has used the power spectral density of displacements, generated by a numerical model of a simple plane frame to detect the damage. It was used this parameter as input values of an artificial neural network to capture patterns changes. Elkordy et al. (1993) has used the percentile changes of the natural frequencies, obtained by an experimental study of a five floor plane frame as input for the neural network. They demonstrated that using the relative percentile changes in the vibration frequencies instead of its absolute value, indeed would distinguish among patterns corresponding to different damage states. Using this approach, typical cases of damage were used to train the artificial neural networks and as result, it was found the correct diagnose of the damage state with its corresponding percentile variation in the natural frequencies.

An investigative and introductory work accomplished by Mukherjee (1997) has evaluated the application of artificial neural networks. The ability of a self-organized neural network to identifying the natural mode shapes of plane frames with multiple floors was demonstrated. The first four vibration mode shapes for several frames of multiple floors were used as input for the neural network. The neural network was able to classify the data set correctly on four groups. The ability of the neural network in predicting new patterns was demonstrated presenting new mode shapes of a plane frame to a neural network trained for the just first two mode shapes. If the displacement of a building are measured *in situ*, the data will certainly contain noise in the form of imprecise measurements. They conclude that the property of the neural network to be tolerant to noises has demonstrated to be very useful in these circumstances.

2.1. Damage detection by modal sensitivity analysis

The dynamic behavior of linear elastic systems with n degrees of freedom can be described by Eq. (1):

$$\mathbf{M}\ddot{y}(t) + \mathbf{C}\dot{y}(t) + \mathbf{K}y(t) = \mathbf{F}(t) \tag{1}$$

where **M**, **K,** and **C** are the n x n mass, stiffness and damping matrix, $\mathbf{F}(t)$ means the external force vector, and $\ddot{y}$, $\dot{y}$ and y means the acceleration, velocity and displacements vectors respectively. When the system has small damping ratio, the frequencies and mode shapes may be obtained through an eigenvalue problem as described by Eq. (2):

$$\left(\mathbf{K} - \mathbf{\Omega}\mathbf{M}\right)\mathbf{\Phi} = 0 \tag{2}$$

where $\mathbf{\Omega}$ is a diagonal n x n matrix that contains the squares of natural frequencies ω_i^2 and $\mathbf{\Phi}$ is an n x n matrix that contains the respective vibration mode shapes, where the i-th column corresponds to the group of displacements for the i-th vibration mode shape $\mathbf{\Phi}_i$. If a small variation is applied to Eq. (2) one has Eq. (3), as follows:

$$(\mathbf{K} + \delta\mathbf{K})\ (\mathbf{\Phi} + \delta\mathbf{\Phi}) = (\mathbf{\Omega} + \delta\,\mathbf{\Omega})(\mathbf{M} + \delta\mathbf{M})(\mathbf{\Phi} + \delta\mathbf{\Phi}) \tag{3}$$

Frequently, the damage occurrence generates on one side significant reduction on stiffness and on the other and a small mass reduction. In the following equation (Eq. (4)) the effect of the mass reduction is ignored as well as variation second order terms (δ^2):

$$(\mathbf{K} - \mathbf{\Omega}\mathbf{M})\ \mathbf{\Phi} + \delta\mathbf{K}\mathbf{\Phi} - \delta\,\mathbf{\Omega}\mathbf{M}\mathbf{\Phi} + (\mathbf{K} - \mathbf{\Omega}\mathbf{M})\ \delta\mathbf{\Phi} = 0 \tag{4}$$

Hence, after some algebraic operations, yields:

$$\delta\mathbf{\Omega} = \frac{\mathbf{\Phi}^T\delta\mathbf{K}\mathbf{\Phi}}{\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}} \tag{5}$$

and particularly, for a single mode shape $\mathbf{\Phi}_i$, yields

$$\delta\omega_i^2 = \frac{\mathbf{\Phi}_i{}^T\delta\mathbf{K}\mathbf{\Phi}_i}{\mathbf{\Phi}_i{}^T\mathbf{M}\mathbf{\Phi}_i} \tag{6}$$

which represents the changes in the *i*-th natural frequency as consequence in a small variation of the global stiffness matrix. Through the adoption of a finite element model that represents the structural system it is possible to obtain a relationship between the damage at an individual element and the variations in the global natural frequencies. Thus, the global stiffness $\mathbf{K}$ matrix and the mode shape vector $\mathbf{\Phi}_i$ can be decomposed:

$$\mathbf{\Phi}_i^T \mathbf{K} \mathbf{\Phi}_i = \sum_{e=1}^{n} u(\mathbf{\Phi}_i)_e^T \mathbf{k}_e u(\mathbf{\Phi}_i)_e \tag{7}$$

where $\mathbf{k}_e$ and $u(\mathbf{\Phi}_i)_e$ are the element stiffness matrix and the corresponding displacements of the *i*-th mode shape of the *e*-th element. For example, the above mentioned matrix and vectors for a planar bar element with six degrees of freedom (Fig. 1) are indicated by Eq. (8):

$$u(\mathbf{\Phi}_i)_e = \{u_1, u_2, ..., u_6\}^T \qquad \mathbf{k}_e = \begin{bmatrix} k_{11} \dots k_{16} \\ \vdots \quad \ddots \quad \vdots \\ k_{61} \dots k_{66} \end{bmatrix} \tag{8}$$

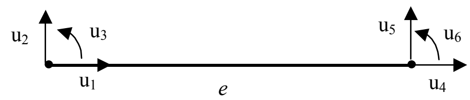

Figure 1 – Beam Finite Element with six degrees of freedom.

Following the same procedure for $\delta \mathbf{K}$, one obtains:

$$\mathbf{\Phi}_i^T \delta \mathbf{K} \mathbf{\Phi}_i = \sum_{e=1}^{n} u(\mathbf{\Phi}_i)_e^T \delta \mathbf{k}_e u(\mathbf{\Phi}_i)_e \tag{9}$$

and substituting this equation in the Eq. (6), yields:

$$\delta \omega_i^2 = \frac{\sum_{e=1}^{n} u(\mathbf{\Phi}_i)_e^T \delta \mathbf{k}_e u(\mathbf{\Phi}_i)_e}{\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i} \tag{10}$$

which represents the changes in the *i*-th natural frequency as consequence of a small variation of the element local stiffness. Particularizing the damage for an element *m:*

$$\delta \omega_{m,i}^2 = \frac{u(\mathbf{\Phi}_i)_m^T \delta \mathbf{k}_m u(\mathbf{\Phi}_i)_m}{\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i} \tag{11}$$

this represents the changes in the *i*-th natural frequency as consequence of a small variation of the local stiffness of the element *m*. Assuming that there is a direct relationship between the variation of the element stiffness and the damage extent, yields:

$$\delta \mathbf{k}_m = \delta D_m \mathbf{k}_m \tag{12}$$

where δD_m is a real representing the damage. Hence, we can substitute this last equation on the Eq. (11) and obtain:

$$\delta \omega_{m,i}^2 = \frac{\delta D_m \; u(\mathbf{\Phi}_i)_m^T \mathbf{k}_m u(\mathbf{\Phi}_i)_m}{\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i} \tag{13}$$

which represents the variation of natural frequency of the structure as function of the location and damage extent. Normalizing this last equation with regard to the largest element local frequencies variations in the structure (at least for the factor $\delta D_m / \delta D_n$ since in this point we are not interested in the absolute value of the damage but just on its location), one obtains:

$$\delta \omega_{m,i}^2 / \delta \omega_{n,j}^2 = \frac{u(\mathbf{\Phi}_i)_m^T \mathbf{k}_m u(\mathbf{\Phi}_i)_m}{\mathbf{\Phi}_i^T \mathbf{M} \mathbf{\Phi}_i} / \frac{u(\mathbf{\Phi}_j)_n^T \mathbf{k}_n u(\mathbf{\Phi}_j)_n}{\mathbf{\Phi}_j^T \mathbf{M} \mathbf{\Phi}_j} \tag{14}$$

which is used to evaluate the location of the damage through the evaluation of the damage index location (Jm). This index estimates, for all elements, the inverse of the standard deviation ($1/\sigma_m$) of the differences between numerical and experimental values of the changes in frequency, as it indicates by Eq. (15). Index values close to 1 will indicate the matching of the numerical and experimental patterns and therefore the presence of the damage in those elements.

$$\sigma_m = (1/n)\sum_{i=1}^{s}\left[(\Delta\omega_i^2/\Delta\omega_f^2) - (\delta\omega_{m,i}^2/\delta\omega_{m,j}^2)\right]^2 \quad \text{and} \quad J_m = 1/\sigma_m.[\sum_{e=1}^{n} 1/\sigma_i] \tag{15}$$

A simple approach to evaluate the damage extent without using the structural mode shapes is to use the previously evaluated damage location index. Once the elements with possible damage are located, the index can be used jointly an inverse analysis (for example, with the singular value decomposition method), since a fewer number of frequencies can be measured. This way, the contributions for the square variations of natural frequencies of an element ${}_m\delta\varpi_i^2$ are beforehand weighted by the damage location index as indicated by Eq. (16).

$$\begin{Bmatrix} \Delta\omega_1^2 \\ \Delta\omega_2^2 \\ \vdots \\ \Delta\omega_i^2 \end{Bmatrix} = \begin{bmatrix} \delta\omega_{1,1}^2 & \delta\omega_{2,1}^2 & \cdots & \delta\omega_{n,1}^2 \\ \delta\omega_{1,2}^2 & \delta\omega_{2,2}^2 & \cdots & \delta\omega_{n,2}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \delta\omega_{1,i}^2 & \delta\omega_{2,i}^2 & \cdots & \delta\omega_{n,i}^2 \end{bmatrix} . \begin{Bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_n \end{Bmatrix} . \begin{Bmatrix} \delta D_1 \\ \delta D_2 \\ \vdots \\ \delta D_n \end{Bmatrix} \tag{16}$$

3. Neural networks for damage detection

The project of a neural network may be understood as a hyper-surface adjustment in a multidimensional space. Thus, the neural network training is equivalent to find a multidimensional surface that better fits some training data. A neural network with radial basis functions is composed by three layers, which are entirely different to each other. The input layer is composed by input neurons (sensors). The second layer is a hidden layer of enough dimension. And the third is an output layer of neurons. Figure 2 presents an outline of a single neuron with radial basis function.

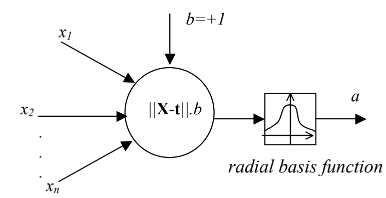

Figure 2 – Single neuron with a radial basis function.

The neuron with radial basis function receives the input vector **X**={x_1, x_2,..., x_n}. At the neuron's kernel it is calculated the Euclidian norm of the distance between the input vector and the fixed center *t* of the neuron. Then this result is multiplied by the threshold *b* which has a constant value input of *+1*. The value of the threshold *b* allows that the sensitivity of the neuron. The resulting product is passed to a non-linear function of radial basis G (activation function, for example, Gaussian G(x)=exp(-x2/2)) that outputs a value that is maximum if the neuron is tuned with the input pattern (input vectors that are far from its center vector has outputs lower than those closer to the center vector). The whole process can be summarized as indicated by equation (17):

$$a = G(|\mathbf{x} - \mathbf{t}|b) \tag{17}$$

It can be demonstrated that, being $\boldsymbol{X}_1$, $\boldsymbol{X}_2$,..., $\boldsymbol{X}_n$, *n* different points in R^p, then the fitting matrix *n* by *n*, which elements are $\varphi_{i,j}=G(\|\boldsymbol{X}_i - \boldsymbol{X}_j\|)$, where *G* is a radial basis function, it should be positive defined matrix and the inverse problem can be solved. In this work the inverse problem is solved with the Singular Value Decomposition Method and the center vectors "***t***" is chosen in a self-organized fashion. Neural Networks with radial basis functions on its traditional form have an input layer (that receives the input data), a hidden layer composed by neurons with radial basis functions in amount of the number of input patterns used for training and an exit layer with neurons of the linear type in amount of the output vector data. More details of the algorithm implementation can be found in Haykin (1994).

An important detail for the successful use of Neural Networks is the correct choice of the parameters or patterns used for training the ANN. In this work the input parameters are the square variations of the natural frequencies as shown previously for the modal sensitivity analysis, and as output parameter it could be used the damage indexes at each element of the structure. A database of the behavior of the structure is created in terms of its modal characteristics through a well calibrated model in finite elements. It was opted to limit the learning area for values of interest of the damage between 0% and 40%. The percentile of the damage was swept between those limits for each element, as well as randomly variations of the damaged between elements together. After ANN training with the relationships of the changes in the squares of the natural frequencies variation and respective location and damage intensity, obtained by a numerical model, these relations are kept at the connections of the ANN. The following step is to test the ANN to verify its learning. This should be made presenting to the ANN new patterns of damage of the structure, in other words, new frequency variations different from those used for the training. The damage pattern obtained as output from the ANN is verified for matching the corresponding numerically damage scenery.

4. Numerical example

A simple supported beam with length L=2,4m, Young modulus $E=2,5x10^{10}$ N/m2, material density ρ=2500 kg/m^3 and rectangular cross section with width b=0,14 m and height h=0,24 m was investigated for damage assessment. The beam is indicated in Fig. 3 below.

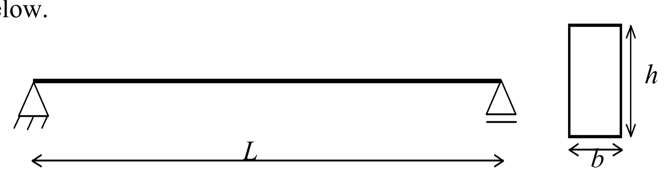

Figure 3 - Outline of the analyzed beam and dimensions.

This beam was discretized with 24 beam elements. A finite element code was used to simulate the damage states just induced through reduction of the element inertia. Several damage sceneries were simulated as indicated on Table 2 below:

Table 2 - Sceneries for analyzed damage states.

Scenery	1	2	3	4	5	6	7	8
Damaged Element	9	3	13	22	16	2,13,22	9	9
Damage%	10%	10%	10%	10%	10%	10%	5%	1%

These sceneries were tested with the detection algorithm with modal analysis using just the square of frequency variations (undamaged and damaged states) and also with the trained ANN with radial basis functions. Using the ANN the training set was generated modifying the damage intensity from 0% to 40% on equally spaced intervals. The total number of samples used for the training data set was of (24 elements x 6 damage levels), evaluated once a time for each element plus 2 x (24 x 6) samples evaluated randomly in any element (damage in more than one element and with different levels) + 1 sample of the undamaged structure, with a total amount of 433 samples. The ANN´s architecture was defined in function of the number of used natural frequencies: number of samples: number of finite elements used to discretize the domain (5-433-24).

Figure 4 shows the damage index location J_m for the Sensitivity Modal Analysis with just first five natural frequencies. Only for scenery 6 (stiffness reduction of 10% on elements 3, 13 and 22), the damage index location predict wrong damaged elements as elements 4, 11, 14 and 21. In all other sceneries the location was satisfactory.

Figure 5, shows for the same damage sceneries, the inverse analysis used to evaluate damage extent with the proposed algorithm and just first five natural frequencies. As expected, just for low values of the damage extent, (1% and 5%) the methodology has a good correlation (predicted values of 2% on element 9 and 4% on element 9 respectively). For multiple element damage scenery (scenery 6) and great extent damage (10%), the prediction is harmed (predicted values of 13% on the element 3, 25% on the element 14 and 42% on element 24).

Figure 6 indicates the same inertia percentile reduction on Fig. 5, using now the first 10 natural frequencies (difficult task for real and complex structures). In this case, the predicted values of the inertia reduction were much closer to the expected ones, except for scenery 2 (10% of damage in the element 2), whose damage estimate was 15% on element 2.

It could be observed in the previous Figures that the more natural frequencies are used, best are the stiffness reduction prediction. It could also be observed that the more the inertia reduction, worst is its prediction.

In Fig. 7 stiffness reduction percentile with the first five natural frequencies and using the ANN is shown. It is observed that for ANN predictions, except for scenery 6, where multiple elements are damaged, all other sceneries are detected accurately when compared with those obtained by Modal Sensitivity Analysis.

In order to observe the robustness to noise presence (always present in experimental measurements), it was simulated noise in the data set multiplying the measured values of the square of frequency variation (input values for the

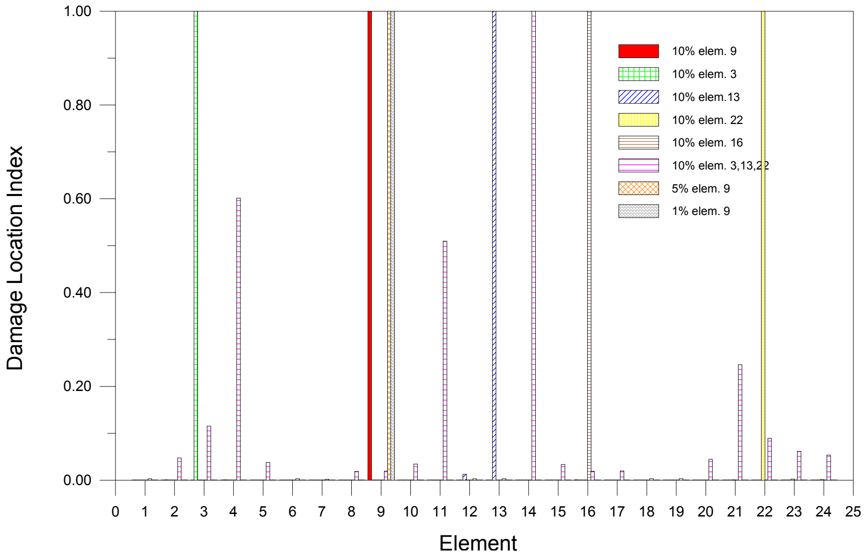

Figure 4 – Damage location index with Modal Sensitivity Analysis (5 natural frequencies used).

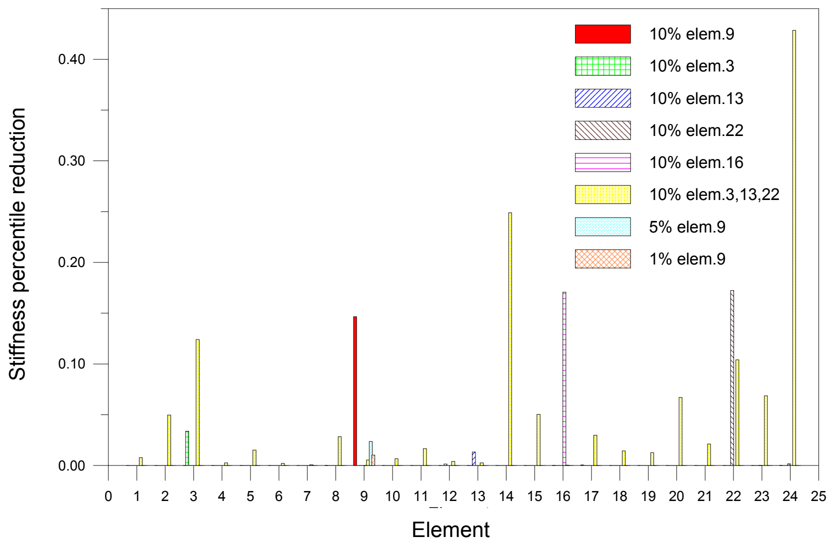

Figure 5 – Stiffness percentile reduction predicted by Modal Sensitivity Analysis (5 natural frequencies used).

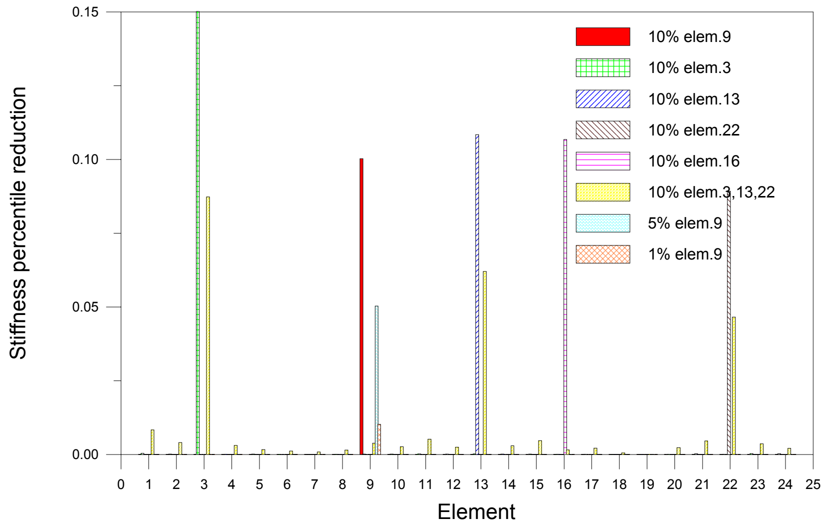

Figure 6 – Stiffness percentile reduction predicted by Modal Sensitivity Analysis (10 natural frequencies used).

ANN) for the scenery 3 (10% of stiffness reduction on element 13) by a random noise, so that providing increasing noise levels (percentile random fluctuations around nominal values).

Figure 8 shows the inertia reduction percentile obtained with the first five natural frequencies and by ANN with increasing noises. It could be observed clearly that the ANN results get worse with increasing noise.

Finally, damage of 40%, was simulated on element 13 with 10% of noise in the natural frequencies evaluated numerically using the first 5 natural frequencies. For the Modal Sensitivity Analysis, the damage location index, as expected, doesn't indicate the damaged element, besides it supplies wrong values for the percentile of damage on several elements. The ANN, indicated with relative precision, the damage location; however the predicted extent is 50% lower than the expected. These results are shown in Fig. 9.

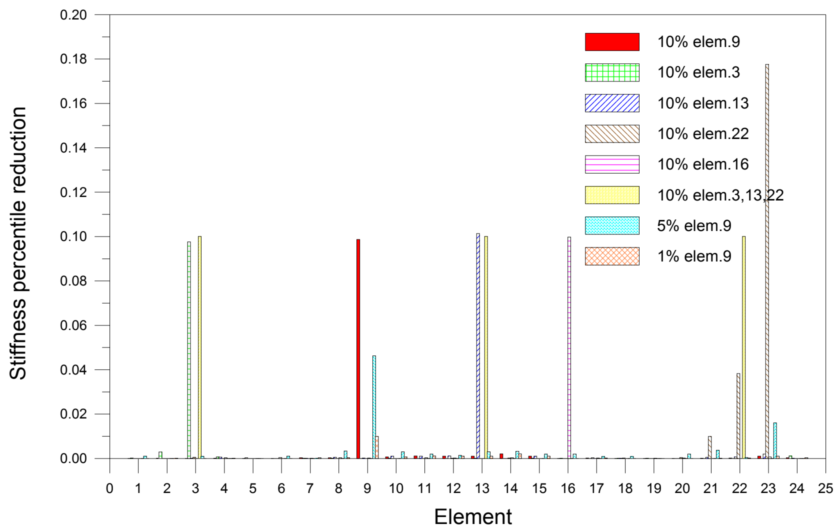

Figure 7 – Stiffness reduction percentile with ANN with radial basis function (5 natural frequencies used).

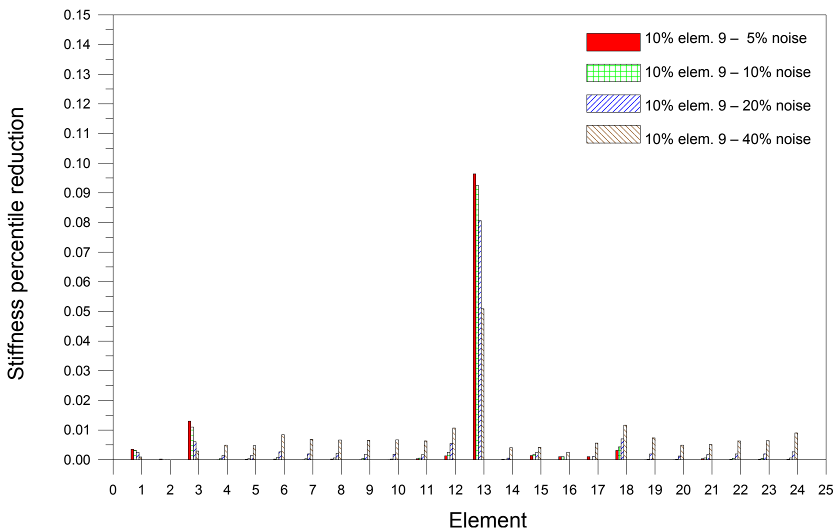

Figure 8 - Stiffness reduction percentile with ANN with radial basis function for damage scenery 3 (5 natural frequencies used) and increasing noises.

5. Conclusions

It was noticed with the examples that the training data set is a critical task for satisfactory results with ANN. The samples should represent appropriately the structural variability behavior so reliable prediction could be traced by the trained ANN. Structures with large number of elements would need a larger volume of training data and this could lead a time consuming training stage. Rough estimates could be used by allowing damage variation on some critical structural areas instead at all elements. The ANN shows quite robust with regard to the presence of some level of noise. In the presence of high damages extents (>30%) and some noise type (10%), the modal sensitivity analysis shown inefficient in damage site prediction as well as in damage extent evaluation.

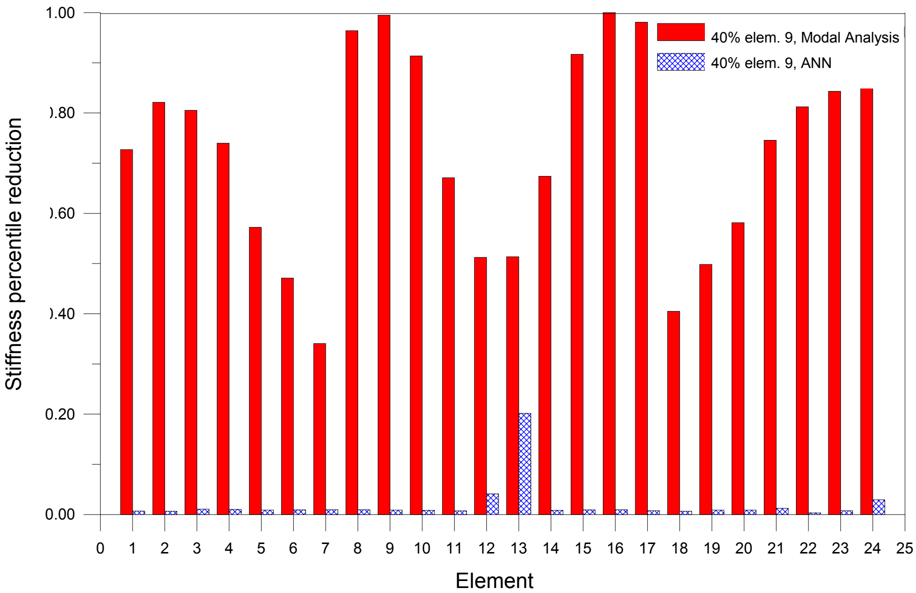

Figure 9 – Damage location index for 40% damage extent on element 13 and 10% simulated noise using Modal Sensitivity Analysis and ANN with radial basis functions (5 natural frequencies used).

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