Stability Prediction and Handling Qualities Analysis of Hypersonic Flight Vehicles During Acceleration.

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Abstract:
A new method for the analysis of the stability and dynamics of hypersonic flight vehicles is developed. The novel approach is based on the Generalized Multiple Scales technique, by which the rapid and slow aspects of the nonautonomous dynamics are systematically separated by using fast and slow time scales. It is essential that the scales are, in general, nonlinear functions of real time. This enables us to derive accurate approximations to represent the stability and dynamics of the system. The approach is illustrated by application to the Generic Hypersonic Aerodynamic Model Example (GHAME) developed by NASA, which flies along the Space Shuttle trajectory, which is designed to minimize the weight of the thermal protection system. Also presented are new visual aids to the pilot of the hypersonic aircraft in terms of moving widows, showing potential degradation possibilities in vehicle stability to warn the pilot and to avert catastrophic consequences. The theory is extended to develop a new theory of aircraft handling qualities during acceleration. This approach subsumes the standard handling qualities criteria of conventional aircraft. Further advantages are: accuracy, ease of computation and enhanced insight. This approach also enables us to derive error estimates and, if necessary, to improve the accuracy of the representation systematically, by means of the fast and slow time scales.

Key Words: Flight Stability, Handling Qualities, Multiple Scales,

Introduction.
In the analysis of high speed aircraft dynamics, one of the important goals is to predict the stability of the vehicle through variable flight conditions in the atmosphere. The stability behavior of a hypervelocity vehicle as it travels along a prescribed trajectory is highly complex. The difficulty stems mainly from the fact that the vehicle experiences variable flight conditions rendering the system nonautonomous. The usual approach in stability and control analysis of such flight vehicles is first to ``freeze'' the system at a particular operating point. This is based on the assumption that the system is ``slowly'' varying. The resulting constant linear system is analyzed from the point of stability and control, using the standard methods. However, in general, such a simplistic approach can lead to a total misrepresentation of the temporal behavior, both in regard to stability and control. It is, therefore, of primary importance that a proper and rigorous stability analysis is carried out.

Nonautonomous systems often exhibit behaviors that can be totally counter-intuitive. . Ramnath [2] has distilled a comprehensive enumeration of these effects from the standpoint of systems theory. Therefore, it is clear that in general, the methods of constant linear systems cannot be applied directly. For example, it is possible, in general, that the characteristic roots of a linear time-varying (LTV) system have *negative real parts*, but the system response may be *unstable!* Or, even when the roots have *nonzero imaginary parts*, the response might be *nonoscillatory!* A careful analysis is required to interpret the dynamics properly. Identification and prediction of these effects demand a fundamental understanding of such complex nonautonomous behaviors. Often such an insight into system behavior is possible only through exact mathematical solutions which constitute a rare event. As a consequence, such counter-intuitive effects are generally unfamiliar to applied engineers.

However, the complex dynamic phenomena are rendered transparent by the asymptotic technique of Multiple Time Scales [see references, esp. 1-4].

Method of Approach:
Such a system usually exhibits dynamics on various time scales (fast and slow). Usually the approach used in the analysis of such systems usually involves overly simplistic analytical approach or numerical methods. However,

results obtained using numerical methods are limited in scope and are valid only for the values evaluated. Moreover, numerical results do not easily give insights on how certain parameters influence the solutions.

A clear understanding of the complex dynamics of aircraft during acceleration and deceleration is the key for developing a good theory of stability and handling qualities. Although such an approach is generally more difficult, the potential gain in the deeper system understanding is greater. A promising method to obtain approximate solutions to physical problems involving a mixture of rapid and slow variations is the Multiple Time Scales (MTS) method largely developed by Ramnath[see references]. This method is potentially useful to decompose a complex dynamic system into subsystems based on the time scales. The MTS approach is based on the concept of extension. The fundamental idea of the concept of extension is to enlarge the domain of the independent variable,(which is time in the context of dynamic systems), to a space of higher dimension (Fig. 1).

This extension yields a set of new independent time-like variables, which are called time scales. These time scales are normally function of real time t and a small parameter ε, which is usually present in a system containing rapid and slow dynamics. The new time scales are *independent* and each time scale captures a certain behavior of the system. Mathematically the extension leads to a set of partial differential equations with respect to the fast and slow time scales. They are solved asymptotically, order by order. For example, the fast time scale captures only the fast behavior of the system. With this extension, the dependent variable is also extended as a function of the fast and slow time scales. This is not a limitation, however, because the extended equation is usually simpler than the original equation.

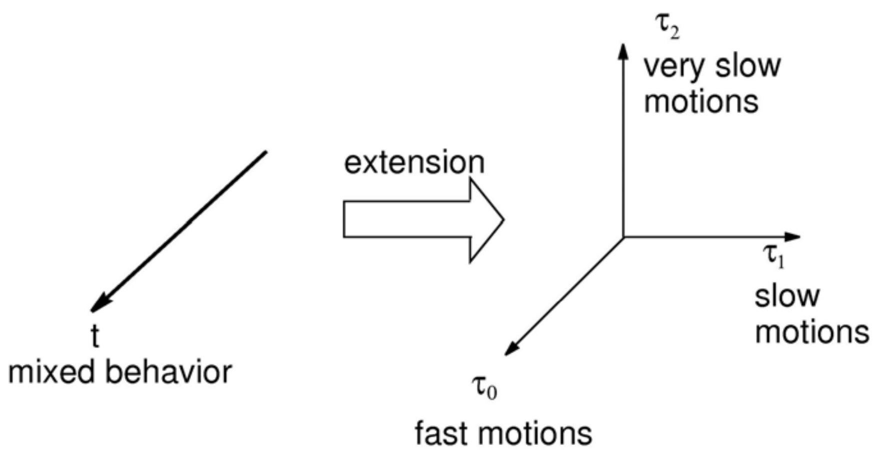

Fig. 1 The concept of extension in MTS method.

With the MTS approach, the dynamics of the system are naturally separated in accordance with the speed of the variation. Moreover, the MTS method potentially yields solutions that show the interrelationship of the system parameters. Hence, the MTS approach leads to significant insights into the dynamic properties of the system. The MTS method has been generalized by Ramnath[3] to include *nonlinear* and *complex time scales*. The Generalized Multiple Scales (GMS) method has been applied successfully to solve a large number of complex aircraft dynamics problems[see references].

The GMS method enables us to analytical solutions of the aircraft dynamics in terms of the fast and slow time scales. Based on these, simple criteria for aircraft stability and handling qualities prediction can be developed[1,2]. From the GMS stability criterion, an estimate can be of the critical altitude, $h_{c,}$ i.e., at which the angle-of-attack oscillations could potentially become unstable, leading to amplitude divergence. These criteria are to be used mainly as guidelines. Further, although the simple criteria have been demonstrated for ballistic trajectories, the GMS theory is itself valid for other general trajectories, and lead to variations on the form of the stability criteria and critical altitudes. Such criteria, which are derived from GMS theory, provide a useful predictor of the system behavior.

It turns out from the application to GHAME vehicle [1], that the stability parameter never becomes negative as the vehicle travels along the trajectory. The second order longitudinal dynamics remain stable for the entire trajectory. At approximately 60,000 vehicle lengths or 225 seconds into the trajectory, the stability parameter is at a minimum and the GHAME vehicle is close to becoming longitudinally unstable. Thus the second order longitudinal dynamics are most affected by changes in the above three important aerodynamic coefficients at approximately the same time that the vehicle is closest to becoming longitudinally unstable. Clearly, careful attention is required in traversing this particular area of the re-entry trajectory.

Aid to Flight Tests:

In order to alert the flight crew of such critical sections in the trajectory, it may be desirable to present the stability information in the form of a graphical display in the cockpit. The stability prediction approach developed in this research effort has useful applications in providing aids to the flight test program of the hypersonic vehicle. Indeed, because of the complex nonautonomous behavior of the accelerating flight vehicle, a predictive capability for vehicle stability would be extremely useful to the pilot. Otherwise, the onset of instability, which could occur without warning, could potentially lead to great difficulties in controlling the vehicle, catching the pilot by surprise. A preliminary analysis into this problem has led to some useful approaches[1].

If it is assumed that all of the trajectory and vehicle information is known prior to the flight, then the stability parameter P can be displayed in a manner such that the flight crew is presented with its immediate past, present and future values. The pilot is provided with a moving ``window" in time at any particular instant, which will display a projection into the near-term future. This display will show a degradation or improvement in the vehicle stability, as time progresses. It will thus provide a warning to the pilot in case of impending stability problems, so that the pilot can take corrective actions. The display itself may be in the form of instantaneous *bar graphs*. This concept is utilized to show the values of the stability parameter in the neighborhood of a particular instant along the trajectory. Stability values are displayed for up to 60 seconds into the immediate future and for 30 seconds of the immediate past. The display is updated continuously and the elapsed time readout provides the flight crew with their relative location along the entire trajectory. In [1] it is shown that the display for which the vehicle has been flying along the trajectory for 200 seconds, shows the region discussed above, where the longitudinal stability parameter is at its minimum. Using displays such as the ones shown, it is possible to effectively inform the flight crew as to when the vehicle is traversing sensitive and critical sections of the flight trajectory.

Handling Qualities:

Handling qualities of advanced aircraft constitute an area of particular interest and importance, to the designer, analyst and the pilot. Since the early years of powered flight, a great deal of work has been done on this problem. For example, when a particular aircraft was designed, it was mainly motivated by the designer's technical perspective, involving such areas as aerodynamics, power plant, mission requirements, and performance. However, it was ultimately left to the pilots to evaluate the aircraft from the standpoint of flying qualities. In an effort to bring these two apparently disparate approaches together, several criteria have been developed describing the handling qualities both in the frequency domain and the time domain. This is possible essentially because we can develop analytical expressions to describe the transient response of the aircraft, as the equations of motion can be solved exactly, as the equations are linear and time-invariant (LTI). To date, the description of aircraft handling qualities is based on an analysis of the dynamic equations of motion at constant flight conditions. The parameters used to define the acceptable handling qualities (natural frequency, damping, bandwidth, etc.) are derived by means of classical methods of LTI systems. When applicable, these handling qualities criteria are useful and have found wide acceptance and are in general use.

Novel analytical approaches need to be developed and blended into a software package to enable the analyst to perform comparison of the old and new approaches.

Background:

However, the standard theory of handling qualities of conventional aircraft in steady flight is predicated on the assumption that the flight conditions are constant. Indeed, most high performance aircraft operate in situations where the flight conditions are not constant. The variation can be due to changes in velocity, density or inertial properties. For such situations, the standard theory is inadequate to rigorously describe the handling qualities. The difficulty stems from the fact that the equations of motion are time-varying and nonlinear in general. For such equations, it is impossible to develop exact analytical solutions in general. Thus there is a great need for developing a new handling qualities approach that would be applicable to flight vehicles through variable flight conditions.

Such an approach is now possible through the asymptotic theory of Generalized Multiple Scales (GMS) theory. This theory enables us to develop analytical asymptotic solutions to the time-varying equations describing the aircraft motion through variable flight conditions. Further, the GMS theory separates the rapid and slow aspects of the aircraft dynamics, which in turn could lead to the development of a handling qualities theory of fast and slow dynamics.

In the following, we present a new approach to describe the handling qualities of a high performance flight vehicle through variable flight conditions, based on the GMS theory. Initially, a preliminary study was undertaken on a generic system of low order, to examine the effects of acceleration and deceleration.

Application to Flight Vehicles.

We address the problem of describing the handling qualities of a flight vehicle as it flies along a trajectory. This is accomplished by comparing the values of certain vehicle parameters with simplified specifications designed to differentiate levels of handling qualities. The specifications are based on the flying qualities standards for piloted vehicles [8,9]. Further, GMS theory is employed to briefly consider the handling qualities of a generic second order time-varying system. By comparing the responses of systems with differing characteristic root behavior, general conclusions are drawn regarding the relationship between the system root movement and the handling qualities. For purposes of the military specifications document, the generic vehicle may include a variety of configurations such as a high speed aircraft, or a Class III aircraft, which is large, heavy, and of low-to-medium maneuverability. Further, the optimal trajectory is assumed to be of flight phase Category B. This part of the mission is described as a phase normally accomplished using gradual maneuvers without precision tracking although accurate flight path control may be required.

The specifications to meet the flying qualities requirements are presented such that the handling qualities are separated into three levels. Level 1 represents flying qualities which are clearly adequate for accomplishing a particular flight phase. If the handling qualities allow the completion of a flight phase but only after some increase in pilot workload, then they are of Level 2. Finally, Level 3 flying qualities represent dynamics which allow the vehicle to be controlled safely but only after excessive pilot workload or with inadequate mission effectiveness. In terms of a well-known subjective rating system, Level 1 corresponds to Cooper-Harper ratings of 1 through 3, while Level 2 represents ratings of 4 through 6. A Cooper-Harper rating between 6 and 9 corresponds to handling quality Level 3 [8,9].

Requirements necessary for classification into one of these levels are made on each of the longitudinal and lateral-directional modes. The time responses of these modes of the aircraft are characterized in terms of the damping ratio and the natural frequency of the modes. The handling qualities are, therefore, related to these two quantities. Accordingly, a set of simplified requirements for the Short Period and the lateral modes may be described as in Table 1. For example, the handling quality requirements for a particular class of flight vehicles in a specific category flight phase are shown as follows [1].

Table 1. Short Period Requirements

Level	Min. ζ	Max. ζ
Level 1	0.3	2.00
Level 2	0.2	2.00
Level 3	0.15	-

Table 2. Dutch Roll Requirements

Level	Min. ζ	Min. $\zeta\,\omega_n$	Min. ω_n
Level 1	0.08	0.15	0.40
Level 2	0.02	0.1	0.40
Level 3	0	-	0.40

Table 3. Roll and Yaw Mode Requirements

Level	Roll Convergence (Max. Time Constant)	Spiral Divergence (Min. time to double amplitude)
Level 1	1.4 sec.	20 sec.
Level 2	3.0 sec.	8 sec.
Level 3	10 sec.	4 sec.

Variable Flight Conditions:

In case of flight at variable speeds, the problem of characterizing the handling qualities becomes considerably more complex. As the vehicle accelerates or decelerates, there are no simple ways to represent the natural motions in terms of analytical expressions, in stark contrast to the steady flight case. In spite of these difficulties, the asymptotic theory of Multiple Time Scales (MTS), indeed its generalization, Generalized Multiple Scales (GMS) enables us to solve the problems and develop a useful extension of standard handling qualities theory applicable to variable flight conditions [1].

The GMS method was developed by Ramnath in a series of papers and books [see references]. Basically, the approach consists of extending the independent variable, time, into a number of new time scales, each of which counts time at a different rate. Thus, there are fast, slow, very slow times. This is tantamount to employing a number of observers, each of whom performs readings of a dynamic phenomenon using the fast and slow clocks. The representation of the complex phenomenon becomes tractable. The generalization involves the use of nonlinear time scales, which corresponds to accelerating and decelerating clocks. Ramnath has developed this technique and applied it successfully to many aerospace problems (see references), including re-entry and high speed aircraft and also VTOL aircraft dynamics and control during transition from hover to forward flight [1,11].

In accordance with GMS theory, the damping ratios and natural frequencies of the Dutch Roll and Short Period modes of the vehicle during flight are calculated from their respective characteristic roots (i.e. clock functions). As the flight conditions are changing, the damping ratios and natural frequencies are functions of time (or distance along the trajectory. If the complex clocks are written in terms of their real and imaginary parts, then the damping ratio and natural frequency also become variable functions. If the natural frequencies are plotted against the corresponding damping ratio for the Short Period and the Dutch Roll modes, it is interesting to note that except for a scaling factor, these two plots are quite similar. This is to be expected since the clocks also exhibit similar behaviors.

It is useful to consider a plausible approach which has been applied to general flight vehicles, presented as follows, for illustrative purposes. The information contained in these plots is compared to the handling quality requirements of Tables to determine whether the Short Period and Dutch Roll modes of the flight vehicle exhibit adequate handling qualities during the flight. As an example, a lifting re-entry vehicle may be considered. Upon analysis, the Short Period behavior was found to be quite inadequate in terms of handling qualities. It is not until the vehicle is at the end of the trajectory that the Short Period damping ratio satisfies minimum Level 3 requirements. The Dutch Roll re-entry behavior also has poor implications on the handling qualities of the vehicle. It turns out that although the Dutch Roll natural frequency by itself satisfies Level 1 requirements, ζ and $\zeta\ \omega_n$ satisfy only Level 3 specifications. Therefore, Dutch Roll behavior merits a handling quality rating of Level 3 during this phase (for e.g. re-entry). Thus, neither of the two oscillatory modes of the flight vehicle presents satisfactory handling qualities during the flight. At Level 3, the Dutch Roll behavior allows the vehicle to be controlled safely, but only after excessive workload on the pilot. The Short Period re-entry behavior does not even qualify for Level 3 status, and renders the vehicle unsatisfactory during the flight.

Continuing this example, the handling quality specifications for Roll Convergence and Spiral divergence modes can also be expressed in terms of time parameters concerning the amplitudes of their respective GMS responses. The Roll Convergence requirement is placed on the time constant which is defined as the time required for the amplitude of a response to decay to exp(-1) times its original value. From the characteristic response, the time constant of the Roll

Convergence mode is approximately 750 seconds. This does not even satisfy Level 3 requirement. For this example, the Spiral divergence, however, does exhibit favorable behavior in handling.
The time for Spiral Divergence characteristic response to double its amplitude is approximately 300 seconds. Clearly this satisfies Level 1 handling quality requirements detailed in Table 5. Although the Spiral Divergence behavior is favorable to good handling, the other three modes represent ratings of Level 3 or worse.

Handling Qualities of a Generic System.

In this section, a generic time-varying system of low order is considered in order to understand the effects of time variation on the handling qualities. As discussed earlier, conventional theory of LTI systems does not lead to a connection between the system equations and the response, when the system is time-varying. Such a connection is essential to develop a handling qualities theory of variable systems. The Generalized Multiple Scales approach leads to exactly such a connection between the mathematical model and the response. We will utilize the GMS solutions in order to gain some insight into the desirable responses of a generic linear time-varying (LTV) system of second order.

If the coefficients are constant, the response is governed by the characteristic roots, which remain constant. However, if the coefficients vary with time, one cannot even develop the characteristic roots by conventional methods. The GMS theory enables us to develop the characteristic roots in terms of *clock* functions, which, of course, do not remain stationary. The path and speed of the clocks in the complex plane are determined by the nature of the coefficients. The effects of simple variations in the path and speed of the clocks on the response and handling qualities are examined in [12,13]. In order to accomplish this, systems having the same locations of the roots at the initial and final times are considered. The path and speed with which the roots move from the initial point to the final location are varied and their responses are compared using the GMS method.

From a handling qualities perspective, the initial and final points of the root migration are chosen so as to represent two different levels of handling qualities ratings. Writing the clocks k as complex conjugates, it turns out that

$K_r(0)=-0.054$; $k_i(0)=0.8984$

$k_r(T)=-0.225$; $k_i(T)=1.483$

where T is the final time. Therefore, it follows that:

$\omega_n(0)=0.9$ rads/sec; $\zeta(0)=0.06$

$\omega_n(T)=1.5$ rads/sec; $\zeta(T)=0.15$

If the generic system represents a Dutch Roll mode, the initial point represents a Level 2 rating for handling qualities, as per the natural frequency and damping ratio. Similarly, the system exhibits Level 1 handling qualities at the final time. The roots are now allowed to move from Level 2 to Level 1 in different ways. The responses of these differing root behaviors are compared to each other as well as to those of two constant systems where the roots remain fixed at Level 1 and Level 2 points. Initially, systems with roots moving from Level 2 point to Level 1 point in a straight line with constant root speed are considered. The total time allowed for the roots travel from initial to the final point is varied in order to determine how the speed of root movement affects the response. The specifications of the systems considered as well as the two constant systems are shown in Table 4. The solutions to the systems are approximated using GMS theory and are plotted. Based on these, we can determine the effect of the different root variations on the handling qualities. A detailed discussion is beyond the current scope, but can be found in [8-10].

Table 4: Straight Line Root Variations at Constant Root Speed.

Case	Root Trajectory	Initial Root Position	Final Root Position	Total Time
1	-	Level 1	Level 1	-
2	-	Level 2	Level 2	-
3	Straight Line	Level 2	Level 1	800 sec.
4	Straight Line	Level 2	Level 1	250 sec.
5	Straight Line	Level 2	Level 1	125 sec.
6	Straight Line	Level 2	Level 1	50 sec.
7	Straight Line	Level 2	Level 1	25 sec.
8	Straight Line	Level 2	Level 1	5 sec.
9	Straight Line	Level 2	Level 1	1 sec.

The use of the GMS method to analyze the problem will be attempted first, as this method is promising and potentially leads to significant insight of the dynamic properties of the system. It is envisioned that such an approach will clarify the interrelationships of the important parameters of the system, which in turn could lead to a better handling qualities and control approach. Special considerations are needed to analyze the handling qualities of flight vehicles through variable flight conditions. Such effects and variations of other parameters have to be studied carefully.

Summary:

The asymptotic theory of Generalized Multiple Scales is very useful in predicting the dynamics of advanced aircraft during acceleration. The counter-intuitive behavior can be represented in terms of simply calculable functions by means of the GMS approach. Further, the technique enables us to develop a comprehensive theory of aircraft handling qualities during acceleration. The advantages include simplicity of representations, accuracy and enhanced insight, along with computational facility.

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