# FATIGUE LIFE MODELING FOR INDUSTRIAL COMPOSITES MATERIALS

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**Abstract:** The objective of this work is to analyze the fatigue behavior of two FGRP (fiber glass reinforced plastic) laminate configurations used for industrial purposes, as well as to model **S-N** curves and Goodman Diagrams. With this objective fatigue tests were carried out in both laminates for six values of stress ratio, as follows: 1.43, 10, -1.57, -1, 0.1 and 0.7. The laminates were industrially manufactured using orthopthalic polyester resin and E-glass fiber reinforcement in the form of mat and (bidirectional) woven textile fabric. Based on the experimental data the fatigue behavior was analyzed for both laminates through a comparative study to observe the applicability of the mathematical model developed in this work using the correlation coefficient as the parameter analysis.

**Keywords:** S-N curves, Goodman Diagram, Composite Materials, Fatigue

## 1. Introduction

Due to their unique properties, in the last decades composite materials have been widely used for industrial purposes ranging from the manufacturing of aerospace and military equipments to recreation apparatus (Heracovich, 1977). However, in order to use these materials it is of the utmost importance to know their mechanical properties and especially the behavior of these materials under the action of cyclic loadings since their structural elements are submitted to this type of loading. As it is well known, repeated stress on a given structure or component can lead to their fatigue failure.

Due to fatigue failure, any structural project or machine elements which are subjected to the action of cyclic loads should be designed considering the material fatigue life. Fatigue life could be evaluated by the material lifetime or the number of cycles to failure, being the latter more reliable. For example, airplane components should endure at least a million cycles ($10^6$) before they start showing any failure, helicopters should endure hundred million cycles ($10^8$), and for design of structures supposed to last thirty years, the number of cycles should be five billions ($5x10^9$) (Sutherland, 1999). Consequently, prior knowledge of the material fatigue behavior is a very important factor for any structural project. Thus, it is whished a mathematical model that can clearly and accurately define this behavior.

According to literature survey on this subject (Subramanian et al, 1995, 2002, Wahl et al, 2002, Harris, 2003, Adam et al, 1989) the existence of various empirical models is already known which are used as a preventive measure concerning the fatigue behavior of composite materials, either related to *S-N* curves or failure diagrams, such as the Goodman Diagram. These models present advantages and disadvantages in terms of their applicability and should be used having in mind the statistical factors, such as the correlation coefficient (*r*).

The objective of this work is to analyze the fatigue behavior of industrial composite laminates used for manufacturing of activated carbon reservoirs. These laminates are made of orthopthalic polyester resin reinforced with E-glass fiber in the form of mat and (bidirectional) woven textile fabric. This study was developed using two different laminate configurations with variation on the number of layers. Fatigue tests were carried out for six stress ratio values, namely $R$ = 1.43, 10, -1.57, -1.0, 1 and 0.7 ($R$ is defined as the ratio between the $\sigma_{min}$ minimum tension and the $\sigma_{max}$ maximum tension values). These values were chosen with the aim to cover the widest data interval relating fatigue behavior of these materials. These tests were carried for high cycle fatigue (above $10^3$) and constant amplitude for each stress ratio. Mathematical modelling of these data was carried firstly analyzing the *S-N* curves and following building the Goodman Diagram through the equation proposed by Adam (Adam et al, 1989). Besides the fatigue tests other tests were accomplished including uniaxial tension and compression tests.

## 2. Experimental Procedures

The industrial laminates used in this work were manufactured as 1.0 $m^2$ plates using the hand lay-up process. The raw-materials used were orthopthalic unsaturated polyester resin and as reinforcement E-glass mats (5 cm, $450^o g/m^2$), and (bidirectional) woven fabric textile (450 $g/m^2$). Two laminates were manufactured, one of them with 10 layers and the other one with 12 layers of 7.0 and 10.0 mm thickness, respectively, with the following configurations.

$$[M/T/M/T/M]_s \quad \text{Staking sequences of laminate with 10 layers (C10)}$$

$$[M/T/M/T/M/M/T/M/T/M/T/M] \quad \text{Staking sequences of laminate with 12 layers (C12)}$$

*M* and *T* refer to E- glass fiber mats and (bidirectional) woven fabric textile, respectively. *C10* and *C12* refer to staking sequences of the laminates with 10 and 12 layers, respectively. The symbol *"s*" refers to the symmetry of the material relating the staking sequences. It is observed that the *C10* laminate is symmetric while the *C12* does not.

Preliminary tests of volumetric density and calcination were carried out to determine the fiber volume, whose were: 24.6 and 24.9% to *C10* and C12 laminates, respectively. Based on these results it was observed that both laminates have equal fiber percentage. Consequently, it could be considered that any variation in the mechanical properties between them will only be a consequence of the variation on their configuration and their number of layers.

The plates were sectioned using a diamond disk (DIFFER D252) to prevent fiber pullout or any other damage to the specimens. The specimen dimensions for uniaxial tensile tests are based on the ASTM D 3039 (1990), while the uniaxial compression and fatigue tests are based on work developed by Mandel et. al (1997). All specimens were cut in the shape of a rectangle, in a predetermined orientation of the laminate with the following dimensions: 200 x 25 mm for the uniaxial tensile and fatigue tests, and 100 x 25 mm for the uniaxial compression tests. The gage length was 127 mm for the fatigue specimens with $R$ = 0.1 and 0.7; 40 mm for the fatigue specimens with $R$ = 1.43, 10, 1.57 and -1; and 35 mm for the uniaxial compression specimens. Fatigue tests were carried out with constant tension amplitude in which for each value of maximum load selected 3 specimens were used, totaling 175 specimens for all the tests. All results of the fatigue tests showed a high cycle fatigue behavior.

For the uniaxial tensile tests a Mechanical Universal Test machine PAVITEST was used, and for the compression tests and those of fatigue a MTS-810 was used. A displacement velocity of 1.0 mm/min was used for the tensile and compression testing and a frequency of 5 Hz for the fatigue tests. All the tests were carried out at room temperature ($25^oC$) with 50% of relative humidity.

For the analysis of the S-N curves of the laminates will be applied the exponential equation and the power law generalizations for this work; presented in equations 1 and 2, respectively.

$$\sigma_{max} = a - b \cdot [\log(N)]^k \tag{1}$$

$$\log(\sigma_{max}) = A - B \cdot [\log(N)]^P \tag{2}$$

Where: *A, B, P, a, b* and *k* are constants that should be obtained during the adjustment of the equation to the data, *N* is the number of cycles to failure and $\sigma_{max}$ is the maximum stress applied to the material.

For modeling the constant life curves of the Goodman Diagram, the equation proposed by Adam et al (1989) was used, shown in equation 3.

$$\frac{\sigma_a}{\sigma_{ultT}} = f \cdot \left(1 - \frac{\sigma_{med}}{\sigma_{ultT}}\right)^u \cdot \left(\frac{\sigma_{ultC}}{\sigma_{ultT}} + \frac{\sigma_{med}}{\sigma_{ultT}}\right)^v \tag{3}$$

In the above equation $\sigma_a$ stands for the alternating stress (obtained subtracting the maximum and minimum stress and dividing the result by two), $\sigma_{med}$ represents the mean stress (obtained adding the maximum and minimum stress and dividing the result by two), $\sigma_{ultT}$ stands for the ultimate tensile strength, $\sigma_{ultC}$ stands for the ultimate compressive strength, *u, f* and *v* are the constants obtained during the curve fitting to the data.

## 3. Experimental Results

### 3.1. Static Tests

Table 1 shows the mechanical properties results obtained in the uniaxial tensile and compressive tests for both configurations. Based on these data it was observed that the results for both laminates are very similar as in the case of the ultimate tensile strength, that is approximately equal to 115 MPa for both. It is important to emphasize that the elasticity modulus was measured in the direction of the load applied.

Considering that the fiber percentages are nearly the same, the introduction of two additional layers (1 M and 1 T) did not cause important changes in static mechanical result of the laminate.

Table 1. Mechanical Properties of *C10* and *C12* laminates.

| | **C10 Laminate** | **C12 Laminate** |
|---|---|---|
| **Tensile Elastic Modulus (GPa)** | 4.81 | 4.50 |
| **Compressive Elastic Modulus (GPa)** | 4.27 | 4.79 |
| **Ultimate Tensile Strength (MPa)** | 116.7 | 115.3 |
| **Ultimate Compressive Strength (MPa)** | 171.3 | 181 |
| **Maximum Tensile Strain (%)** | 2.45 | 2.54 |
| **Maximum Compressive Strain (%)** | 4.07 | 3.92 |

### 3.2. Fatigue Tests: S-N Curves

With the objective to compare both equations for each stress ratio (using the correlation coefficient (*r*)) for the *C10* and *C12* laminates, table 2 and 3 were drafted, respectively. Based on these tables it was observed that both equations are able to model the *S-N* curves satisfactorily for all stress ratios (*R*). The values of the correlation coefficient are always higher than 0.90 for the exponential equation and higher than 0.92 for the power law in the case of the *C10* laminate, and higher than 0.96 to the *C12* laminate regardless the type of equation used.

Table 2. Constants obtained for the equations 1 (exponential) and 2 (power law), as well the correlation coefficient (*r*) for each stress ratio – *C10* Laminate.

| ***R*** | **Power Law (eq. 2)** | | | ***r* (eq. 2)** | **Exponential (eq. 1)** | | | ***r* (eq. 1)** |
|---|---|---|---|---|---|---|---|---|
| | ***A*** | ***B*** | ***P*** | | ***a*** | ***b*** | ***k*** | |
| 1.43 | 2.23 | 0.00513 | 1.64 | 0.922 | 171.4 | 2.16 | 1.55 | 0.907 |
| 10 | 2.23 | 0.00893 | 1.91 | 0.985 | 171.5 | 4.56 | 1.62 | 0.984 |
| -1.57 | 2.23 | 0.0407 | 1.35 | 0.991 | 251.3 | 30.98 | 0.941 | 0.991 |
| -1 | 2.06 | 0.0279 | 1.63 | 0.990 | 115.4 | 12.60 | 1.04 | 0.991 |
| 0.1 | 2.06 | 0.00195 | 2.84 | 0.985 | 115.7 | 0.971 | 2.31 | 0.987 |
| 0.7 | 2.21 | 0.0619 | 0.953 | 0.949 | 166.0 | 28.63 | 0.660 | 0.948 |

Table 3. Constants obtained for the equations 1 (exponential) and 2 (power law) as well as the correlation coefficient (*r*) of each stress ratio – *C12* Laminate.

| ***R*** | **Power Law (eq. 2)** | | | ***r* (eq. 2)** | **Exponential (eq. 1)** | | | ***r* (eq. 1)** |
|---|---|---|---|---|---|---|---|---|
| | ***A*** | ***B*** | ***P*** | | ***a*** | ***b*** | ***k*** | |
| 1.43 | 2.26 | 0.00458 | 2.04 | 0.972 | 181.0 | 2.37 | 1.83 | 0.973 |
| 10 | 2.26 | 0.0255 | 1.57 | 0.994 | 181.1 | 13.77 | 1.20 | 0.993 |
| -1.57 | 2.26 | 0.0615 | 1.29 | 0.994 | 181.0 | 32.68 | 0.82 | 0.994 |
| -1 | 2.05 | 0.0330 | 1.68 | 0.993 | 113.1 | 13.04 | 1.10 | 0.991 |
| 0.1 | 2.05 | 0.0163 | 1.78 | 0.991 | 113.0 | 6.05 | 1.37 | 0.988 |
| 0.7 | 2.05 | 0.00512 | 2.12 | 0.966 | 112.9 | 1.76 | 1.83 | 0.966 |

By comparing the correlation coefficients of both equations it was observed that in most cases they are nearly equal, being the power law correlation coefficients a little higher than those of the exponential equations. Only regarding $R$ = 1.43 of the *C10* laminate a little more difference was noticed, in which the power law equation has a correlation coefficient of 0.922 while the exponential equation has $r$ equal to 0.907, thus showing a percentage difference of 1.6%.

Since the power law shows little better results than those of the exponential equation, the power law equation will be used for fatigue modelling of the composite materials.

Another reason that stimulates the use of the power law in the modelling of the *S-N* curves is the smooth characteristic showed by it for high number of cycles.

The *S-N* curves obtained for the *C10* and *C12* laminates are shown in fig. 1 and 2. These curves show that the lower values of the normalized tension were $R$ = - 1 and $R$ = -1.57. These results were already expected since for these cases the highest tension amplitudes are applied to the laminate when the results of the stress ratio are compared concerning the same number of failure cycles. Similarly, for $R$ = 0.7 and $R$ = 1.43 the highest values of the normal tension are found, once the amplitudes of the applied tension for these stress ratios are very small when compared for a similar number of failures.

Regarding these graphs, it is important to observe that the symbols marked with an arrow points to fully tested specimens which did not rupture. Besides, in these graphs $\sigma_{max}$ or $\sigma_{min}$ and $\sigma_{ultT}$ or $\sigma_{ultC}$. were used according to the type of stress ratio used thus, for $R$ = 1.43 and $R$ = 10, the minimum stress ($\sigma_{min}$) and the ultimate compression strength ($\sigma_{ultC}$) were used. For $R$ = 1.57, $R$ = -1, $R$ = -0.1 and $R$ = 0.7 the maximum stress ($\sigma_{max}$) and the ultimate tensile strength ($\sigma_{ultT}$) were used. These variations are shown in these graphs with the objective to enable comparison among the results obtained for each stress ratio.

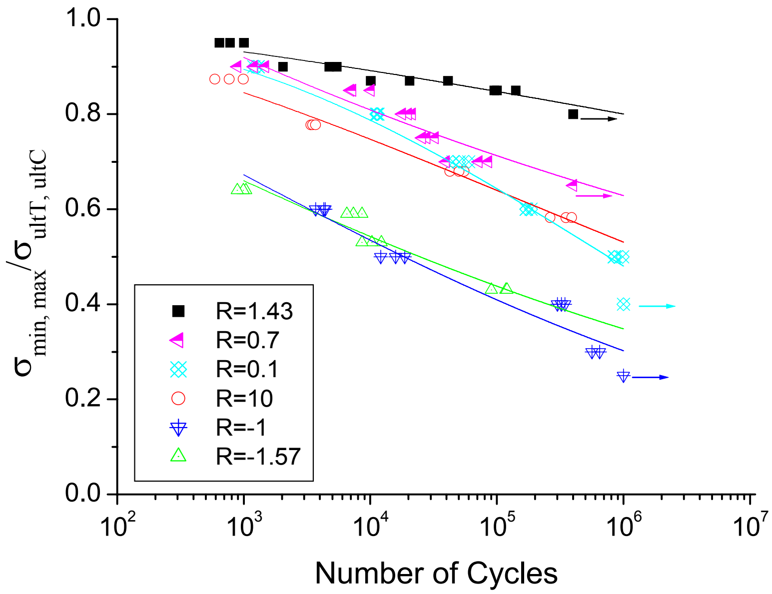


Figure 1. *S-N* curves of the *C10* material, experimental data and model obtained from the power law.

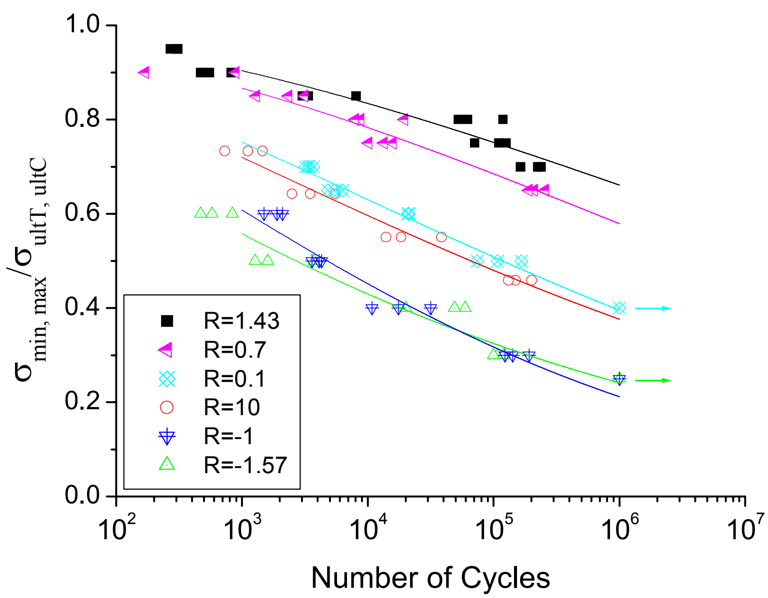


Figure 2. *S-N* curves of the *C12* material, experimental data and model obtained from the power law.

A comparative study between these two laminates showed that the *C10* laminate is more resistant to fatigue than the *C12* laminate in all stress ratios as shown in fig. 3 for $R$ = 1.57. This occurs mainly as a result of the symmetry of the *C10* laminate that enables a better distribution of the internal tensions thus decreasing damage propagation.

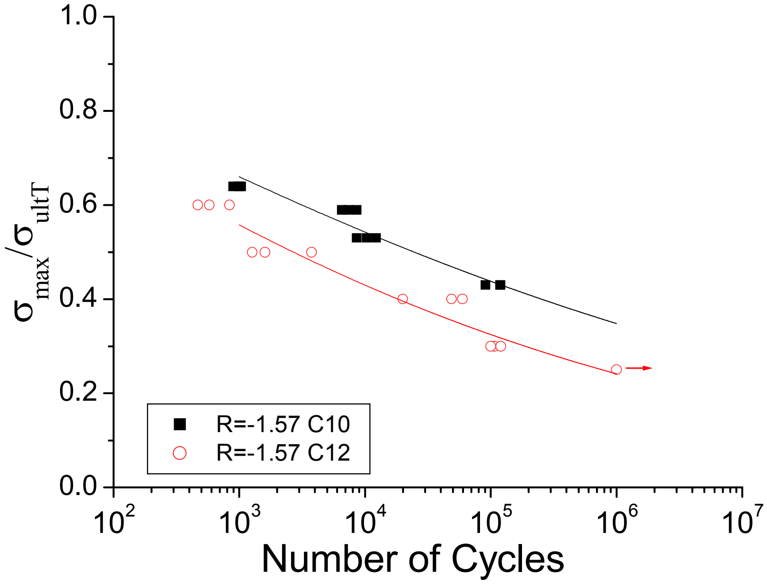


Figure 3. Comparative study of the results obtained for *C10* and *C12* laminates for $R$ = 1.57.

**3.3. Fatigue tests: Goodman Diagram Modelling**

Based on the results obtained by the *S–N* curves (using equation 2) the Goodman Diagram was built whose constant life curves were modeled according to equation 3, for the *C10* and *C12* laminates. These diagrams are shown in figures 4 and 5. By studying these graphs it is evident the importance of the $R$ = 1.57 testing since, for this stress ratio were obtained the highest tension amplitude values among all constant life curves analyzed, considering an equal number of failure cycles. Thus, it is for $R$ = 1.57 that the highest safety region concerning fatigue behavior of these materials is observed. Based on this behavior it could be proved that both laminates bear a greater capacity to endure compression-dominated fatigue loading.

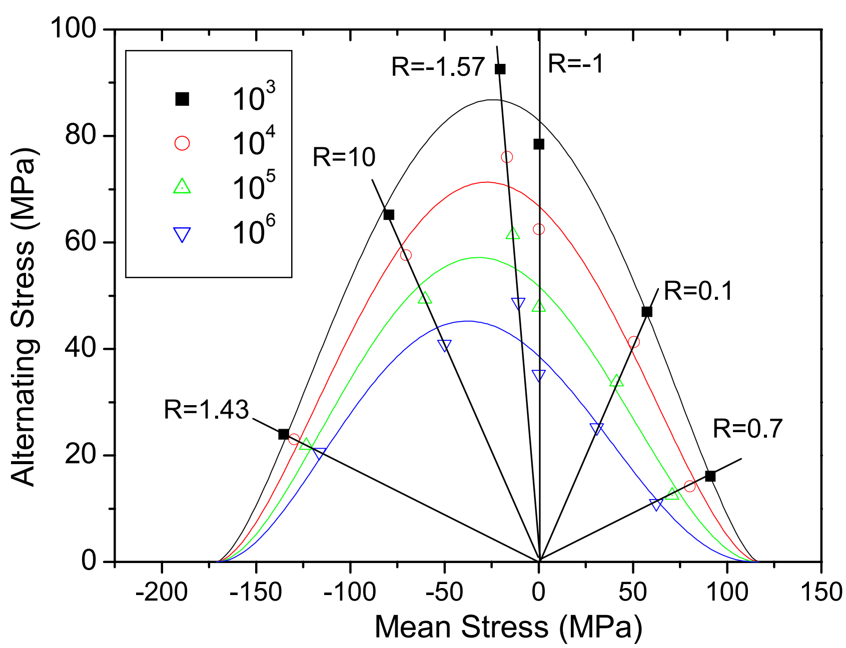


Figure 4 Goodman Diagram modeled from Adam's Equation, obtained for *C10* laminate.

Analyzing figures 4 and 5, it was observed that for $R$ = 1.43, 0.1 and 0.7, the results of the curves of equation 3 are satisfactorily similar to the results shown by $S - N$ curves (equation 2), however, for the others stress ratio there is a difference a little more pronounced.

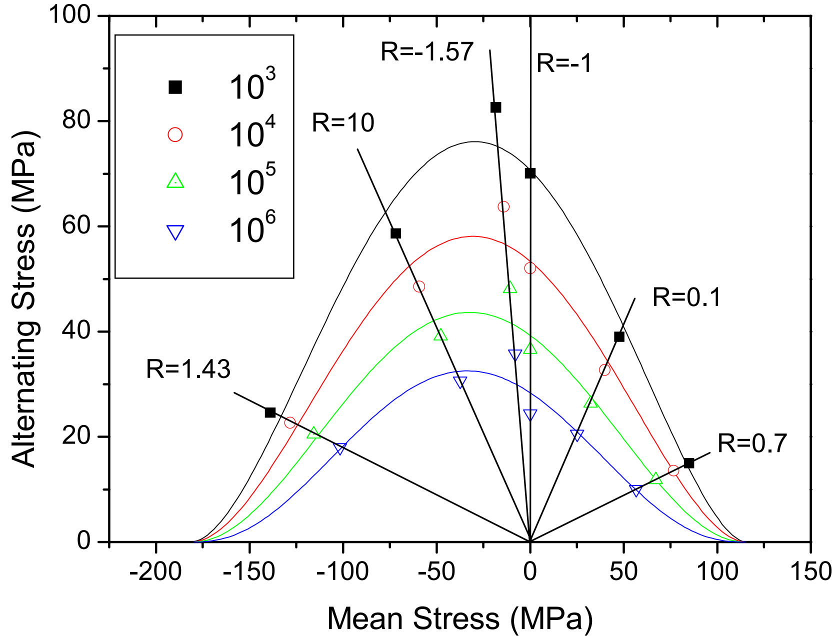


Figure 5. Goodman Diagram modeled from Adam's equation, obtained for the *C12* laminate.

A quantitative analysis of the mathematical model adjustment to the *S-N* curves can be obtained from the values of the correlation coefficients. These were obtained between all the constant life curves and the *S-N* curves. The values correspond to 0.988 and 0.985 for the *C10* and *C12* laminates, respectively. Based on these values it was concluded that for both laminates Adam's equation (equation 3) represent, satisfactorily, the fatigue behavior of these laminates.

According to some authors (Beheshty et al, 1999; Harris, 2003) equation 3 constants are values that suffer variation as a function of the number of cycles, and could be represented by $u(N)$, $v(N)$, and $f(N)$. In Figures 6 and 7 the behavior of these constants as a function of the cycle number is shown. In this case the curves which approximate the $u$, $v$ and $f$ values were obtained from equation 4.

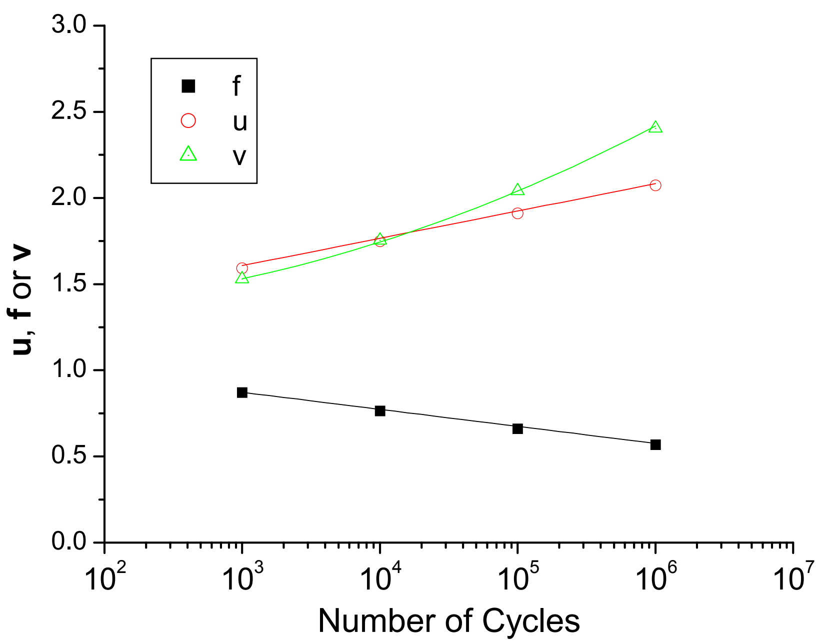


Figure 6. *f, u,* and *v* constants of equation 3 as a function of *N*, obtained for the *C10* laminate.

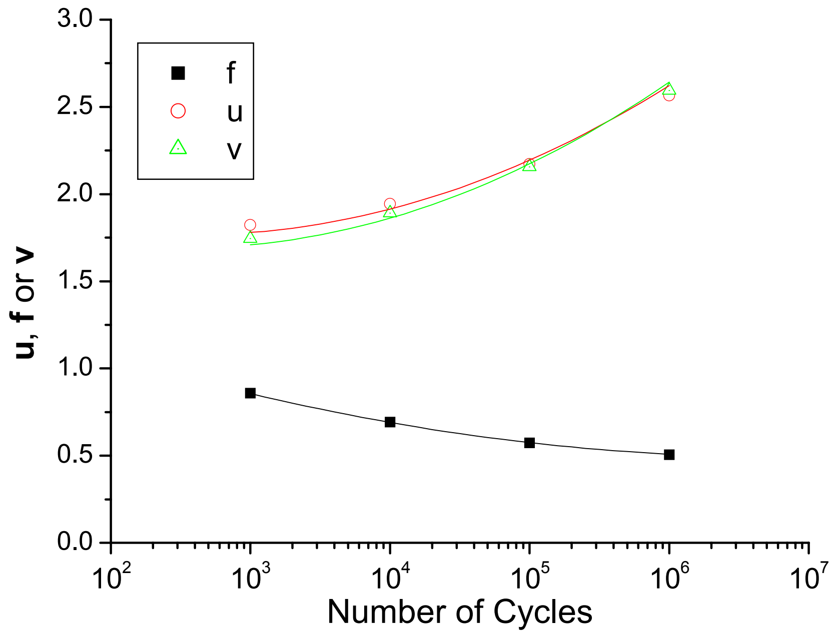


Figure 7. *f, u* and *v* constants of equation 3 as a function of *N*, obtained for the *C12* laminate.

$$[f \quad u \quad v] = c \cdot [\log(N)]^2 + d \cdot \log(N) + e \tag{4}$$

Where *c, d* and *e* are vectors of the constants for the adjustment of the polynomial curve to the data obtained for *u, f* and *v*.

It is interesting to mention that in Beheshty's et al (1999) and Harris's (2003) works the equation obtained that correlates the *u, f* and *v* constants to the number of cycles are totally different from the one obtained in the present work. This shows that, although there is always a relation between the number of cycles and the constants, this relationship could vary depending on the type of composite material analyzed.

**4. Conclusion**

Based on the results obtained from the static tests, it could be concluded that the material configuration and the number of layers used had little influence on most of the mechanical properties of the material, such as ultimate strength and elastic modulus in the direction of applied loading.

In modeling the *S-N* curves the power law is better adapted to the experimental data than the exponential equation, however, this difference was also of little significance.

The *C10* composite laminate shows a better fatigue behavior than the *C12* composite laminate in all the tests carried out thus demonstrating the importance of the material symmetry in the distribution of the internal tensions linked to the formation and propagation of the accumulative damage.

Through the results obtained the importance of the stress ratio analysis, $R = 1.57$, was observed, because the greatest tension amplitudes applied to two composite laminates were found in this analysis, by comparing the results for the same number of failure cycles, as it was shown by the Goodman Diagram.

By analyzing the Goodman Diagrams one could observe that Adam's equation satisfactory models the results obtained for both laminates, presenting correlation coefficients in the order of 0.98.

With view to the data shown in figures 6 and 7, it was concluded that there is a relation between the constants of Adam's equation and the number of cycles of the material, however this relation does not agree with equations found in the literature concerned.

**5. Acknowledgements**

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