SIMULATION OF HORIZONTAL TWO-PHASE SLUG FLOWS USING THE TWO-FLUID MODEL WITH A CONSERVATIVE AND NON-CONSERVATIVE FORMULATION

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Abstract. *The two-fluid model has been widely used on the simulation of two-phase flows in pipelines. The model consists of two sets of conservation equations – mass and linear momentum – for the liquid and gas phases on its transient, one-dimensional form. The "slug capturing methodology" involves the numerical solution of the equations using a finite volume formulation, which is capable of naturally predicting the onset of slugging from stratified flow regime, as well as the growth and collapse of the slugs. When a slug is formed the gas volume fraction is zero, and as a result the gas momentum equation becomes singular. A non-conservative and a conservative approach were employed to handle the problem. Although the non-conservative approach is more stable, it is also too dissipative.*

Key words: *slug-flows, one-dimensional, horizontal, conservative and non-conservative formulation.*

1. Introduction

Two-phase flow in the slug pattern can be found in several engineering applications, such as flow of hydrocarbons through pipelines, liquid-vapor flow in power-plants, etc (Dukler e Fabre, 1992). The slug is a flow pattern which is highly intermittent, it is formed by sequences of large gas bubbles followed by packs of liquid (slug) flowing in random fluctuating frequencies (Fabre e Liné, 1992; Woods et al., 2000).

The slug pattern can be formed in horizontal and inclined pipelines from a stratified pattern by basically two mechanisms: the natural growth of hydrodynamic instabilities and by the accumulation of liquid due to irregularities on the terrain where the pipes are installed. In the first case, small perturbations in the form of small waves naturally emerge. These waves can continue to grow, capturing the liquid that flows in front of them until the cross section becomes saturated with liquid, thus forming the *slugs*. (Ansari, 1998). The growth mechanism is the classic Kelvin-Helmholtz instability (KH) (Lin e Hanratty, 1986; Fan et al., 1993a,b). At inclined pipelines, the slug can be formed due to the delay and sub-sequent accumulation of liquid at the down points of the pipeline, leading to a cross section completely filled with liquid. This type of slug pattern induced by the terrain is called "*severe slugging*" and it can formed when a slightly inclined pipeline joins a riser, that is, a vertical pipeline (Jansen et al., 1996). The flow in the slug pattern can also be formed by a combination of the mechanisms described above. Small undulations of the terrain can lead to slug formation in addition to the ones formed by the inherent instabilities of the flow. In theses cases, the slug formed by one mechanism interacts with those formed by the other, leading to a complex slug pattern.

The intermittence of the flow in the slug pattern causes large instabilities, which propagates through out the pipeline and any other equipment connected to it. This often increases the design problems and it usually leads to a reduction of the efficiency and/or size of a processing plant. Thus, it is important to be able to predict the beginning and subsequent development of the slug pattern, as well, as the prediction of its characteristics such as size and frequency.

There are several methods to predict the slug formation, among them, the "*slug capturing*" methodology can be mentioned, which is based on the one-dimensional two fluid model (Ishii,1975), to predict two-phase flows. In this model, each phase is described by a set of mass, momentum and energy conservation equations, which are obtained by an average process for each phase. The "*slug capturing*" methodology was employed by Issa e Kempf (2003), to predict the transition of a stratified flow to a slug pattern, and by Oliveira e Issa (2003) who investigated numerical aspects of the solution of the two-fluid model, presenting methodologies to limit the volume fraction within physical limits. In 2003, Bonnizi e Issa presented two papers employing the *"slug capturing"* model. At the first one, the slug pattern for a three phase fluid was obtained with the multi-fluid model, while the second one concerns the entrainment of small bubbles of gas in the liquid.

One of the problems in predicting the slug formation is related to the fact that, when a slug is formed the gas volume fraction becomes zero, and as a result the gas momentum equation becomes singular (Issa and Kempf, 2003; Olivera and Issa, 2003). Therefore, the objective of the present work consists on investigating the slug formation by two approaches: a non-conservative and a conservative one, and identify which one is more suitable to handle the problem.

2. Mathematical Modeling

The mathematical model selected is based on the "*slug capturing*" technique, in which the slug formation is predicted as a result of a natural and automatic growth of the hydrodynamic instabilities (Issa e Kempf, 2003). Both stratified and slug pattern are modeled by the same set of conservation equations based on the two-fluid model. Additionally, closure relations are also included. The first hypothesis to be mentioned, consists on assuming a uniform pressure P along the cross section, being the same, for the liquid, gas and interface. The liquid is considered as incompressible, while the gas follows the ideal gas law, $\rho_g=P/(RT)$, where R is the gas constant and T is its temperature, which was considered here as constant. The governing mass and momentum equations in the conservative form can be written as

$$\frac{\partial(\rho_g\ \alpha_g)}{\partial t}+\frac{\partial(\rho_g\ \alpha_g\ u_g)}{\partial x}=0 \quad ; \quad \frac{\partial(\rho_\ell\ \alpha_\ell)}{\partial t}+\frac{\partial(\rho_\ell\ \alpha_\ell\ u_\ell)}{\partial x}=0\,, \tag{1}$$

$$\frac{\partial(\rho_g \alpha_g u_g)}{\partial t}+\frac{\partial(\rho_g \alpha_g u_g^2)}{\partial x}=-\alpha_g\frac{\partial P}{\partial x}-\rho_g\alpha_g g\,\mathbf{sen}(\beta)-\rho_g\alpha_g g\frac{\partial h}{\partial x}\mathbf{cos}(\beta)-F_{gw}-F_i\,, \tag{2}$$

$$\frac{\partial(\rho_\ell \alpha_\ell u_\ell)}{\partial t}+\frac{\partial(\rho_\ell \alpha_\ell u_\ell^2)}{\partial x}=-\alpha_\ell\frac{\partial P}{\partial x}-\rho_\ell\alpha_\ell g\,\mathbf{sen}(\beta)-\rho_\ell\alpha_\ell g\frac{\partial h}{\partial x}\mathbf{cos}(\beta)-F_{\ell w}+F_i\,, \tag{3}$$

where $\alpha_g+\alpha_l=1$. The subscripts g, l, and i concern the gas, liquid phases and interface, respectively. The axial coordinate is x, ρ and α are the density and volumetric fraction, u is the velocity. The pipeline inclination is β, h is the liquid level inside the pipe, and g is the gravity acceleration. The third term on the right side of Eqs. (2) and (3) are related with the hydrostatic pressure at the gas and liquid, respectively. The term $F=\tau\ S\ /\ A$ is the friction force per unit volume between each phase and the wall and between the phases (at the interface), where τ is the shear stress, S is the phase perimeter and A is the pipe cross section area.

The shear stress is $\tau=f\,\rho\,|\,u_r|\ u_r\ /\ 2$, where u_r is the relative velocity between the liquid and wall, the gas and wall, or gas and liquid. Closure relations are needed to determine the friction factor f.

The flow was considered in the laminar regime, when the Reynolds number **Re**, was smaller the 2100 ($\mathbf{Re}_g$; $\mathbf{Re}_i$ and $\mathbf{Re}_l$ for the gas, interface and liquid, respectively). The Hagen-Poiseulle formulas were employed for the gas-wall and interface friction factor and the correlation of Hand (1991) for the liquid-wall friction factor:

$$f_g=16/\mathbf{Re}_g\,, \qquad f_i=16/\mathbf{Re}_i\,, \qquad f_l=24/\mathbf{Re}_l^s\,, \tag{4}$$

while the Taitel e Dukler (1976) was adopted for the turbulent gas-wall and interface friction factor and the Spedding and Hand (1997) correlation for turbulent the liquid-wall friction factor:

$$f_g=0.046\left(\mathbf{Re}_g\right)^{-0.25}, \qquad f_i=0.046\left(\mathbf{Re}_i\right)^{-0.25}, \qquad f_l=0.0262\left(\alpha_l\ \mathbf{Re}_l^s\right)^{-0.139} \tag{5}$$

where α_l is the *hold-up* (liquid volumetric fraction). The Reynolds numbers are defined as

$$\mathbf{Re}_g=\frac{4A_g u_g \rho_g}{\left(S_g+S_i\right)\mu_g}, \quad \mathbf{Re}_i=\frac{4A_g\left|u_g-u_l\right|\rho_g}{\left(S_g+S_i\right)\mu_g}, \quad \mathbf{Re}_l=\frac{4A_l\ u_l\ \rho_l}{S_l\mu_l}, \quad \mathbf{Re}_l^s=\frac{\rho_l U s_l D}{\mu_l} \tag{6}$$

where μ is the absolute viscosity and D is the pipe diameter. The last Reynolds in Eq. (6) is based on the liquid superficial velocity $U_{sl}=Q_l\ /\ A$, i.e., the ration of the liquid volume flow rate to the total cross section area of the pipe.

When a slug is formed, the liquid volume fraction becomes one and the gas volume fraction goes to zero. As a result, the gas momentum equation, Eq. (2), becomes singular, since α_g appears in both sides of the equation. Oliveira and Issa (2003) have suggested that this singularity can be avoided by rewriting this equation is a non-conservative form as

$$\rho_g\frac{\partial u_g}{\partial t}+\rho_g u_g\frac{\partial u_g}{\partial x}=-\frac{\partial P}{\partial x}-\rho_g g\,\mathbf{sen}(\beta)-\rho_g g\frac{\partial h}{\partial x}\mathbf{cos}(\beta)-\frac{F_{gw}}{\alpha_g}-\frac{F_i}{\alpha_g} \tag{7}$$

This form does not completely solve the singularity problem, due to the presence of $F_{gw}\ /\ \alpha_g$ and $F_{,i}\ /\ \alpha_g$, but Oliveira and Issa (2003) argue that this form is less restrictive.

Figure (1) presents a sketch of the pipeline, with a cross section area A and diameter D. The areas and wetted perimeters of the gas and liquid are $A_{g,}$, S_g and A_ℓ, S_ℓ, respectively, and the interface width is S_i. All these geometric parameters depend only on liquid height h, by straightforward geometric relations. Further, the liquid height is a geometric function of the gas volumetric fraction ($\alpha_g = A_g / A$), and can be obtained by the solution of

$$\alpha_g = \frac{1}{\pi}[\cos^{-1}(\xi) - \xi\sqrt{1-\xi^2}\,] \quad ; \quad \xi = 2\,(D/h) - 1 \tag{8}$$

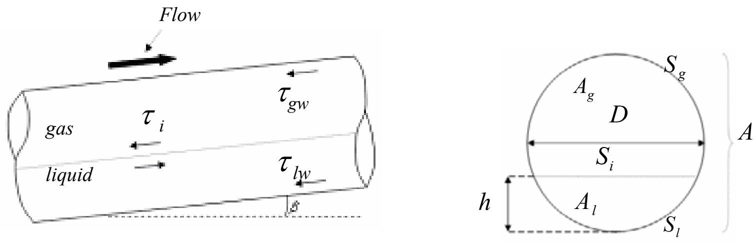

Figure 1 Sketch of the pipe, and its cross section

3. Numerical Method

The conservation equations were discretized by the Finite Volume Method (Patankar, 1980). A staggered mesh was employed, with both phases' velocities stores at the control volume faces and all other variables at the central point. Figure 2 illustrates the mesh, where the upper case symbols refer to the main node and lower case symbols to control volume faces. The interpolation scheme *Upwind* and the implicit *Euler* scheme were selected to evaluate the space and time derivates, respectively. The conservative approach is presented first, followed by the non-conservative one.

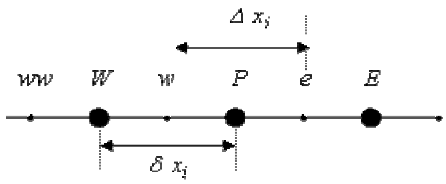

Figure 2. Control volume

3.1. Conservative Formulation

3.1.1. Volumetric fraction

The mass conservation equation for the gaseous phase, Eq. (1) is discretized for the main control volume. The volumetric fraction is obtained from its previous value, α^o, by solving the following discretized equation

$$a_P \alpha_P = a_E\, \alpha_E + a_W\, \alpha_W + b \tag{9}$$

$$a_W = \left\| \tilde{F}_w, 0 \right\| \quad ; \quad a_E = \left\| -\tilde{F}_e, 0 \right\| \quad ; \quad a_P^o = \rho_P^o\, A \frac{\Delta x_i}{\Delta t} \quad ; \quad a_P = a_W + a_E + a_P^o \quad ; \quad b = a_P^o\, \alpha_P^o \tag{10}$$

$$\tilde{F}_w = \rho_w\, u_w\, A \;\; ; \;\; \tilde{F}_e = \rho_e\, u_e\, A \;\; ; \;\; \rho_w = (\rho_W + \rho_P)/2 \;\; ; \;\; \rho_e = (\rho_P + \rho_E)/2 \tag{11}$$

where the symbol $\| a\, , b \|$ means the maximum between a and b.

3.1.2. Velocities

Both phases' velocities are obtained from the momentum equation, Eq. (2) and (3), discretized for the staggered control volume. Since pressure is unknown, it is kept explicitly in the equation. Further, due to the non-linearity characteristic of the momentum equation, an under-relaxation factor, γ, is also included.

$$a_w u_w = a_{ww} u_{ww} + a_e u_e + b - \alpha_w A (P_P - P_W) \tag{12}$$

$$a_{ww} = \| F_W, 0 \| \quad ; \quad a_e = \| -F_P, 0 \| \quad ; \quad F_P = (\tilde{\alpha}_w \tilde{F}_w + \tilde{\alpha}_e \tilde{F}_e)/2 \quad ; \quad F_W = (\tilde{\alpha}_{ww} \tilde{F}_{ww} + \tilde{\alpha}_w \tilde{F}_w)/2 \tag{13}$$

$$a_w^o = \rho_w^o \alpha_w^o A \frac{\delta x_i}{\Delta t} \quad ; \quad a_w = (a_{ww} + a_e + a_w^o + S_P\, \delta x_i)/\gamma \quad ; \quad b = a_w^o\, u_w^o + S_C\, \delta x_i + (1-\gamma)\, a_w\, u_w^* \tag{14}$$

where the volumetric fraction at the faces are obtained from their corresponding upwind values, in order to guarantee mass conservation in the staggered control volume, i.e., for the west face, $\tilde{\alpha}_w \tilde{F}_w = (\alpha_W \| \tilde{F}_w, 0 \| - \alpha_P \| -\tilde{F}_w, 0 \|)$. u_w^* corresponds to the velocity of the previous iteration. The source terms S_C e S_P for the gaseous phase are:

$$S_C = b_{grav,g} + b_{h,g} + b_{int}\, u_{l,w} \quad ; \quad S_P = b_{wall,g} + b_{int} \tag{15}$$

$$b_{grav,g} = -\rho_{g,w}\ \alpha_{g,w}\, g\, A\ \mathbf{sen}(\beta) \quad ; \quad b_{h,g} = -\rho_{g,w}\ \alpha_{g,w}\, g\ A\ \mathbf{cos}\,(\beta)\,\frac{h_P - h_W}{\delta\, x_i} \tag{16}$$

$$b_{wall,g} = \frac{1}{2} f_{g,w}\ \rho_{g,w} S_{g,w}\left|u_{g,w}\right| \quad ; \quad b_{int} = \frac{1}{2} f_{i,w}\ \rho_{g,w}\ S_{i,w}\left|u_g - u_l\right|_w \tag{17}$$

In these equations, the volumetric mass fraction is obtained at the faces by linear interpolation, $\alpha_w=(\alpha_W+\alpha_P)/2$. The geometric parameters, S_g and S_i are determined from the volumetric fraction, and the friction factors $f_{g\text{-}wall}$ and f_{int} by Eqs. (4) and (5) applied at the control volume faces. The liquid momentum equation is discretized on a similar manner.

Since the gas momentum equation becomes singular when the gas volumetric fraction becomes zero, this equation must not be solved when a slug is formed (α_g<0.02), and the gas velocity is arbitrarily set to zero, as recommended by Issa e Kempt (2003) and Bonizzi (2003).

3.1.3. Pressure

The pressure equation is derived from a combination of both phases' continuity equations, resulting in an overall continuity equation. Since the order of magnitude of the density of each phase is quite different, each equation is normalized by a reference density, resulting in

$$\left[\frac{(\rho_P\alpha_P - \rho_P^o\alpha_P^o)_g}{\rho_{g,ref}} + (\alpha_P - \alpha_P^o)_l\right]\frac{A\Delta x_i}{\Delta t} + \left[\frac{\rho_{e,g}\ \tilde{\alpha}_{e,g}}{\rho_{g,ref}} u_{e,g} - \frac{\rho_{w,g}\ \tilde{\alpha}_{w,g}}{\rho_{g,ref}} u_{w,g}\right] A + \left[\tilde{\alpha}_{e,l}\ \ u_{e,l} - \tilde{\alpha}_{w,l}\ \ u_{w,l}\right] A = 0 \tag{18}$$

The pressure is introduced in the global mass conservation equation through its relation with the velocity and density, from both momentum equations and the equation of state for the gas, respectively. The momentum equation, Eq. (12), can be rewritten in an explicit form as:

$$u_w = \hat{u}_w - \frac{\alpha_w A}{a_w}(P_P - P_W) \quad ; \quad \hat{u}_w = \frac{a_{ww}\ u_{ww} + a_e\ u_e + b}{a_w} \tag{19}$$

Substituting Eq. (19) and the ideal gas law in Eq. (18), the discretized equation for the pressure is

$$a_P P_P = a_W P_W + a_E P_E + b \tag{20}$$

$$a_W = \left(\frac{\rho_{g,w}\ \tilde{\alpha}_{g,w}}{\rho_{g,ref}}\frac{\alpha_{g,w}A}{a_{w,g}} + \tilde{\alpha}_{l,w}\frac{\alpha_{l,w}A}{a_{w,l}}\right) A \quad ; \quad a_E = \left(\frac{\rho_{g,e}\ \tilde{\alpha}_{g,e}}{\rho_{g,ref}}\frac{\alpha_{g,e}A}{a_{e,g}} + \tilde{\alpha}_{l,e}\frac{\alpha_{l,e}A}{a_{e,l}}\right) A \quad ; \quad a_P^o = \frac{\alpha_{g,P}\ A}{P_{ref}}\frac{\Delta x_i}{\Delta t} \tag{21}$$

$$a_P = a_W + a_E + a_P^o \quad ; \quad b = \left[\left(\frac{\rho_{g,w}\tilde{\alpha}_{g,w}}{\rho_{g,ref}}\hat{u}_{g,w} + \tilde{\alpha}_{l,w}\hat{u}_{l,w}\right) - \left(\frac{\rho_{g,e}\tilde{\alpha}_{g,e}}{\rho_{g,ref}}\hat{u}_{g,e} + \tilde{\alpha}_{l,e}\hat{u}_{l,e}\right)\right] A + a_P^o\ P_P^o - \frac{(\alpha_P - \alpha_P^o)_l\ A\,\Delta x_i}{\Delta t} \tag{22}$$

When a slug is formed, the contribution from the gas phase must be eliminated from the overall mass conservation equation.

3.2. Non-Conservative Formulation

3.2.1. Volumetric fraction

Besides the singularity problem of the gas momentum equation, another problem that has to be handled is to guarantee that the volumetric fraction is bounded between zero and one, duration the iterative process (Oliveira e Isaa, 2003). The use of the *Upwind* approximation leads to a scheme that limits the volumetric fraction to zero, but it does not guarantee, that it will be limited by one. In order to force these limits, both continuity equation are solved separately and the resulting volume fractions (α_g^+ e α_l^+) are corrected by a factor ξ, such as $\alpha_g = \xi\,\alpha_g^+$ and $\alpha_l = \xi\,\alpha_l^+$ and satisfy the restriction equation $\alpha_g + \alpha_l = 1$ where $\xi = 1/(\alpha_g^+ + \alpha_l^+)$.

3.2.2. Velocities

The main consideration regarding the discretization of the non-conservative gas momentum equation, Eq. (7), is to guarantee that the terms F_{gw} and F_i tend to zero faster than α_g, when $\alpha_g \to 0$. Oliveira and Issa (2003) stated that to avoid division by zero, it is sufficient to represent the face volumetric fraction as a mean value between adjacent nodes. The discretized equation, Eq. (12) is the same, but there is a small change in the coefficients, which are

$$a_{ww} = \left\| \tilde{F}_w, 0 \right\| \quad ; \quad a_e = \left\| -\tilde{F}_w, 0 \right\| \quad ; \quad a_w^o = \rho_w^o A \frac{\delta x_i}{\Delta t} \quad ; \quad a_w = (a_{ww} + a_e + a_w^o + S_P\, \delta x_i)/\gamma \tag{23}$$

$$b = a_w^o\, u_w^o + S_C\, \delta x_i + (1-\gamma)\, a_w\, u_w^{*} \quad ; \quad S_C = b_{grav,g} + b_{h,g} + b_{int}\, u_{l,w} \quad ; \quad S_P = b_{wall,g} + b_{int} \tag{24}$$

$$b_{grav,g} = -\rho_{g,w}\; g A\; \mathbf{sen}(\beta) \quad ; \quad b_{h,g} = -\rho_{g,w}\; g\; A\; \mathbf{cos}(\beta) \frac{h_P - h_W}{\delta x_i} \tag{25}$$

$$b_{wall,g} = f_{g,w}\; \rho_{g,w} S_{g,w} \left| u_{g,w} \right| / (2\, \alpha_{g,w}) \quad ; \quad b_{int} = f_{i,w}\; \rho_{g,w}\; S_{i,w} \left| u_g - u_l \right|_w / (2\, \alpha_{g,w}) \tag{26}$$

3.3. Solution Procedure

The set of resulting equations consists of two momentum equations, one pressure equation (global mass conservation), one gas volumetric fraction (gas mass conservation) and a restriction equation for the liquid phase. These equations were solved sequentially, through an iterative method, which can be described by the following steps:

i) Definition of initial conditions.
ii) Determination of each phase velocity by the solution of each momentum equation,
iii) Determination of pressure, through the solution of the overall mass conservation equation.
iv) Explicit correction of the velocities by Eq. (19), such as to guarantee overall mass balance.
v) Determination of the gas volumetric fraction by the solution of the mass conservation equation of the gas.
vi) Determination of the liquid volumetric fraction: by the restriction equation (α_l=1-α_g) for the conservative approach and by the solution of the mass conservation equation of the liquid for the non-conservative approach, followed by the volumetric fraction correction ($\alpha_g = \xi\, \alpha_g^+$ and $\alpha_l = \xi\, \alpha_l^+$)
vii) Repeat the iterative process, items (ii) through (vi), until convergence is obtained, that is until the residue of all equations smaller than 10^{-4}.
viii) Advance in time, returning to item (ii), and marching until the desired final time step is reached.

4. Results

Figure (3) illustrates the problem configuration. It is exactly the same as the one investigated by Issa & Kempf (2003) and Bonizzi (2003). The pipeline is horizontal, with length equal to L=36 m, and internal diameter D = 0.078m. The flow was considered isothermal at a reference temperature T = 281.15 K. The outlet pressure P_{out} was kept constant equal to the atmospheric pressure. The gas phase is air (gas constant, R = 287 N m /(Kg K) and the absolute viscosity is μ = 1.796× 10^{-5} Pa-s) and the liquid phase water (density, ρ= 998.2 kg/m^3, absolute viscosity is μ= 1.139 × 10^{-3} Pa-s). At the inlet the superficial liquid and gas superficial velocities, U_{sl} and U_{sg} , were specified as constant, as well the liquid *hold-up* α_l.

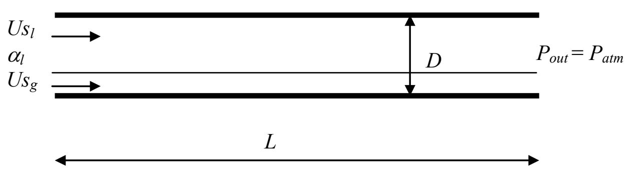

Figure 3. Problem test configuration.

The initial condition was defined as a stratified steady state flow, that is, constant liquid height along the pipeline, with constant liquid and gas velocities. The pressure distribution was obtained by integrating the following equation, resulting from a combination of the momentum equation of the liquid and gas phase, for the equilibrium stratified flow

$$\partial P / \partial x = -(\tau_{lw} S_l + \tau_{gw} S_g) / A \tag{27}$$

To be able to predict the slug pattern, it is necessary a well posed system of equations. Additionally, the transition from the stratified to the slug pattern can only happen if the boundary conditions induce an unstable flow. Therefore, a Kelvin-Helmoltz stability analysis was performed (Taitel & Dukler, 1976) aiming the selection of the boundary conditions (U_{sl} ,U_{sg} , α_l) suitable for the present problem.

A map of flow pattern (Taitel & Dukler, 1976) can be seen in Fig. (4), for the pipeline configuration adopted at the present work. This map shows the range of gas and liquid superficial velocities which leads to the different flow patterns (bubbles, slug, annular, stratified). Based on the analysis presented by Bonizzi (2003), at the same map, an additional curve was plotted, separating the regions where the system of equations is well and ill posed. Further, the test cases presented here are also indicated at the map. For the first case the boundary condition employed by Issa & Kempf (2003) were selected: Us_g=2.0 m/s; Us_l=1.0 m/s, α_l=0.4. For the second case, the following superficial velocities were defined Us_g=6.0 m/s, Us_l=0.4 m/s and liquid volume fraction corresponding to the stratified equilibrium condition, α_l=0.566 as in Bonizzi (2003). In both cases, the data selected is above the line that defines the regions of well and ill posed solutions for the problem. These data were selected here to allow comparison with the available literature. It such be mentioned that Ortega Malca (2004) investigated several situations located above and below the stability line, i.e., both ill and well posed situations.

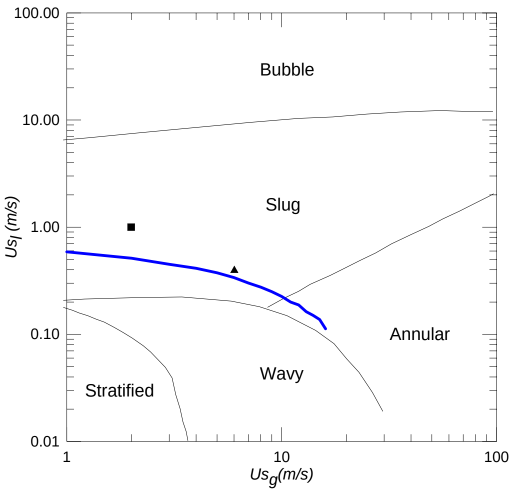

Figure 4. Flow pattern map of Taitel & Dukler (1976). Location of test cases on the map.

A uniform mesh was specified in the domain, with 1250 nodes (Δx/D=0.369) as employed by Bonizzi (2003). The time step was specified to guarantee a Courant number equal to 0.5 (Issa e Kempf, 2003), therefore, the time step was obtained from $\Delta t = 0.5\, \Delta x_i / | u_{\max} |$, where u_{max} is the maximum velocity in the domain.

Figure 5 illustrates the liquid *hold-up* evolution along the pipeline, for different time instants (each curve corresponds to a different time). Fig.(5a) corresponds to the results of Issa and Kempf (2003) and Fig. (5b) to the present results. It can be seen in both results the formation of the slug, its growth, as well as its displacement along the pipeline. The position where the first slug appears is similar in both cases, however, it took a smaller time period to appear with the present formulation. Further, the slugs displaces along the pipeline with a slightly higher velocity. Although the results are not exactly the same, it can be concluded that both formulation predicted similar results.

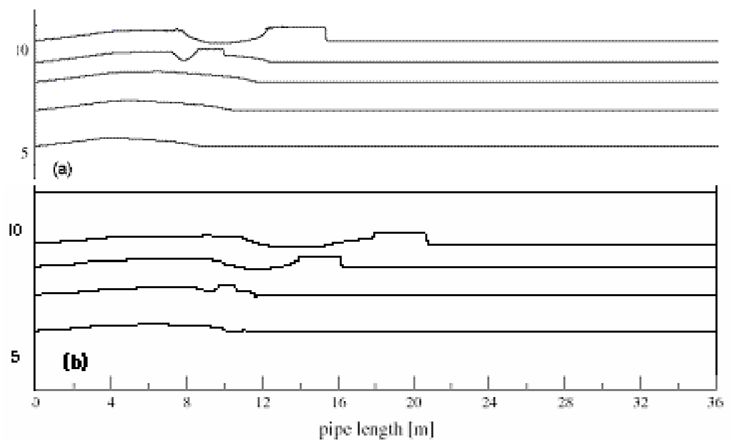

Figure 5 – Liquid *hold-up* distribution along the pipe for different time instants until the formation of the first slug. Us_g = 2,0 m/s e Us_l =1,0 m/s. (a) Issa & Kempf (2003) (b) present results.

As already mentioned, when a slug is formed the gas momentum equation becomes singular, since the gas volume fraction is zero, leading to convergence problems. As explained in the Numerical Method section, an ad-hoc procedure is employed to be able to obtain a solution. To investigate the influence of the conservative and non-conservative approach to hand the problem, the second test was solved with both formulations.

The liquid *hold-up* time evolution along the pipeline can be seen at Fig. (6) for the second test case, employing the conservative approach. Once again, it can be seen the slug formation, growth and displacement along the pipeline.

Due to the singularity at the gas momentum equation, unrealistic very large gas velocity and pressure gradient are obtained, as can be seen in Fig. (7), which corresponds to the time instant of formation of the first slug. The velocity and pressure distribution are not presented by Issa & Kempt (2003) and Bonzini (2003). However, through a private communication (Issa, 2004) informed that unrealistic high velocities were also predicted by his algorithm.

Analyzing Fig. (7), it can be seen a large increase of the pressure at the liquid phase, leading to abrupt pressure drop, resulting in a pressure oscillation (pressure under peak). The large pressure gradient induces an unrealistic gas velocity at the slug tail. Therefore, another factor that can be a cause of such unrealistic behavior is the hypothesis of a uniform pressure through the cross-section, i.e., the pressure is the same at the liquid and gas phase, as well as at the interface. (Ishii, 1975, Mishima & Ishii, 1986).

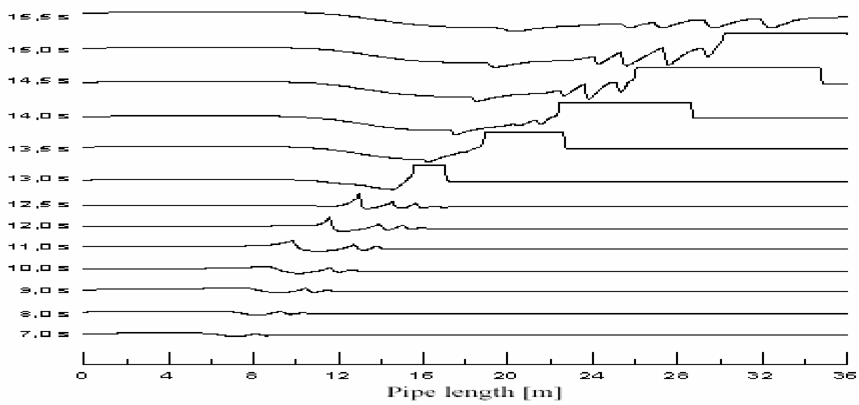

Figure 6. Liquid *hold-up* evolution in space and time, Us_g = 6.0 m/s e Us_l = 0.4 m/s, conservative approach.

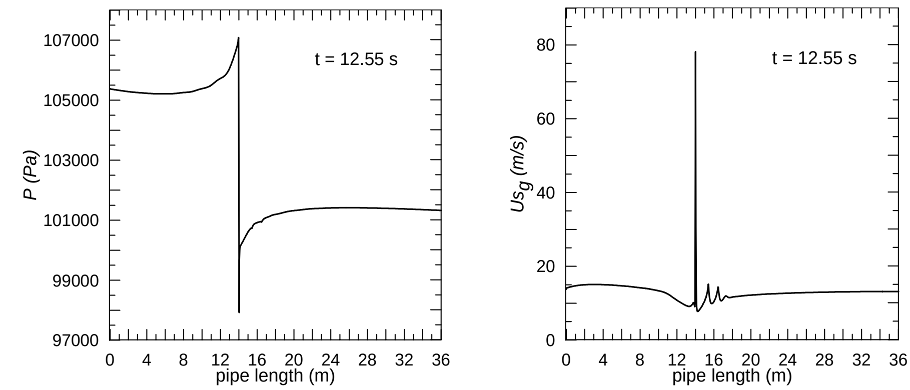

Figure 7. Pressure and gas velocity along the pipe, at t =12.55 s, when the first slug occurs.

Oliveira and Issa (2003) suggested the non-conservative approach to avoid the singularity problem. They also suggested the limiting procedure to the volumetric fraction of both phases. Therefore, the same problem was solved again, incorporating these changes. The time evolution of the liquid *hold-up* is shown in Fig. (8).

It can be seen in Fig. (8), that the non-conservative approach damped the instabilities which originates the slugs, and even after a very long period of time, only small waves traveling with approximately constant velocity were predicted.

5. Final Remarks

Depending on the superficial velocity ratio different flow patterns can be found. Further, due to physical instabilities, transition from one flow pattern to another occurs. These instabilities by themselves can introduce difficulties in predicting these types of flows, which are increased by numerical instabilities that are also encountered. Therefore, the prediction of slug flow is highly complex, due to the presence of both physical and numerical instabilities.

At the present work the one-dimensional two-fluid model as described by Issa & Kempf (2003) and Bonizzi (2003), was employed to predict the transition from the stratified flow pattern to the slug pattern. Conservative and non-conservative approaches for the momentum equations were investigated, to avoid the singularity problem that arises at the gas momentum equation when the slug is formed.

The conservative approach was able to predict the slug formation, growth and displacement along the pipeline, however unrealistic velocities were also predicted at the slug tail. On the other-hand, the non-conservative approach was too diffusive, damping the instabilities that lead to the slug formation. Therefore, the non-conservative approach is not recommended.

As a final remark, it is suggested that a pressure jump must be considered at the interface, to avoid the unrealistic velocity at the slug tail.

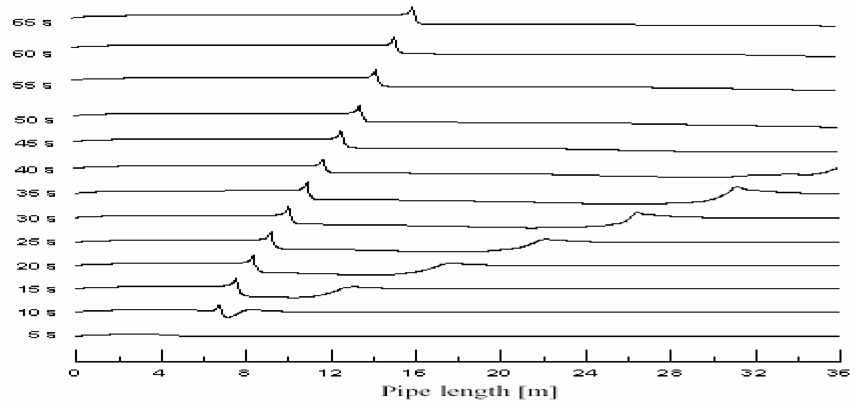

Figure 8. Liquid *hold-up* evolution in space and time, Us_g = 6.0 m/s e Us_l = 0.4 m/s, non-conservative approach

6. Acknowledgement

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