CASE STUDY: NON-LINEAR RESONANT OBJECT CONTROL

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Abstract. *This paper describes a teaching experience obtained by the author while lecturing the discipline "Mechatronics Design" at the University of Passo Fundo (UPF), Rio Grande do Sul, BRAZIL. Here, a case study relating to the control of an electronic rocking chair for children, which is discussed with the students, is presented. Learning this case study, the studens make itself familiar with the technique of modelling and simulation of complex non linear system The case study shows the importance of the mechatronics approach to the design of electro-mechanical systems, when mathematical modeling and numerical simulations permit more detailed insight into complex non-linear dynamics.*

Keywords*: non-linear dynamics, resonant object, mechatronics, digital control, simulation*

1. Introduction

Mechatronics education was introduced at the UPF in 2001 when the curriculum of the Department of Electrical Engineering was modified to offer two areas of specialization. These areas were "Electronics and Telecommunications" and "Mechatronics". The area of Mechatronics received some disciplines of the Mechanical Engineering and some new ones, including the discipline "Mechatronics Design", which offers some case studies, including the one related to electronic rocking chair for children. Originally, the project of electronic rocking chair was elaborated by the author in 2000 when one of the Brazilian firms tried to fabricate the chair. The idea was to guarantee a stable move of the chair under load variations, using the power of a small electrical motor and some simple electronic controller. The initial design, which was based on an open-loop PWM controller, failed and the firm appealed to the Academy looking for some cheap and quick solution of the problem.

The rocking chair and its mechanism are depicted in Fig.1. The mechanism includes one low power DC electrical motor, worm screw transmission, gear wheel-driving crank, connecting rod and flexible propulsion link which moves the chair suspension. The diagram of the mechanism is detailed in Fig.2.

Figure 1. Rocking chair for children (on the left) and its mechanism.

The particularity of the project included the impossibility of any serious modification of the original mechanical design and the obligation of using the originally selected DC motor.

2. Design methodology

The adopted design methodology includes model development, identification of parameters, simulations and analysis of the results of the simulation. The principal limitation of the design is the impossibility of using of neither any sensor nor any advanced microcontroller. As a possible candidate to controller an 8-bits microcontroller with embedded A/D (analog/digital) converter was considered. The main idea about how to implement a control strategy was to use the resonance phenomena forcing the chair to oscillate at the resonant frequency.

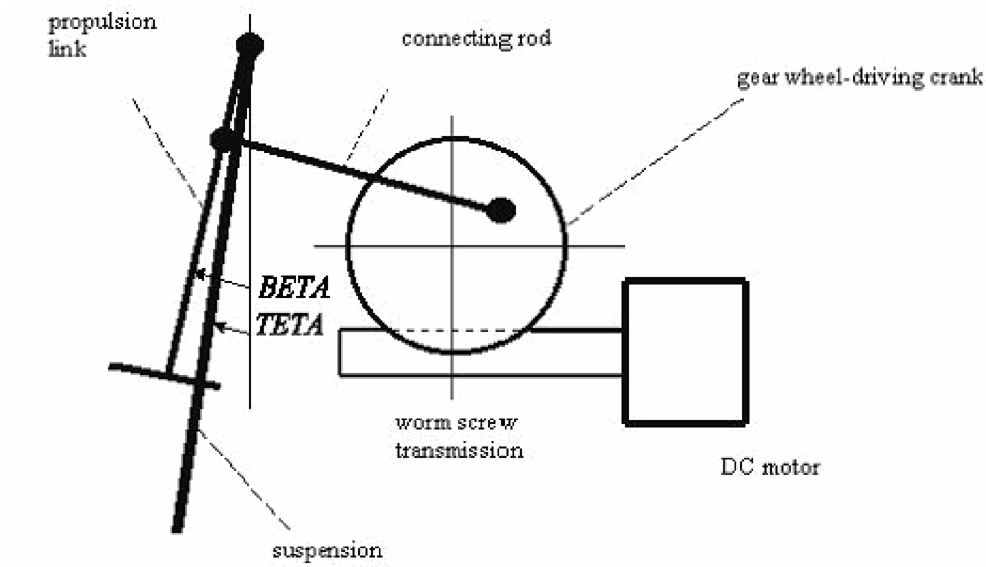

Figure 2. Diagram of the mechanism. *TETA* ≡ θ; *BETA* ≡ β.

2.1. Model development

The transmission mechanism has been fabricated of plastic and is very light so only its cinematic model was developed. The analysis of the mechanism was performed using the diagram depicted in Fig.3. Initially, using numerical calculation in MATLAB (Hanselman and Littlefield, 1997), the function $\beta(\varphi)$ was deduced as in Eq.(1).

$$\beta = 0.26 \cdot \sin(\varphi - 3.927) \tag{1}$$

The propulsion link has some back-lash which permits a movement of the suspension without interaction with this link. The deformations of the propulsion link cause a torque which is applied to the suspension of the rocking chair. The reaction of the chair, on the form of a force, is applied, through the connecting rod, to the driving crank, which is the gear wheel. This gear wheel is drove by the worm screw transmission and DC motor. The following parameters are used in the model:

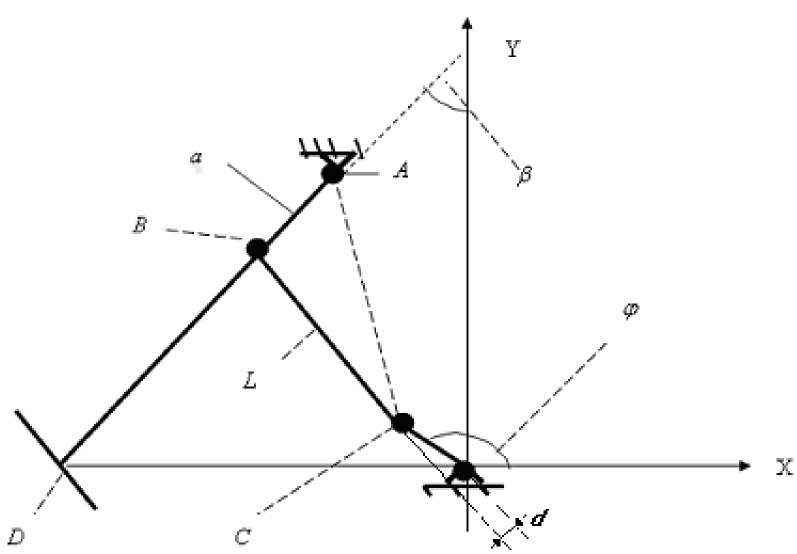

Figure 3. Diagram of cinematic analysis. $AB = a = 0{,}023$ m; $AD = 0{,}063$m.

$K = 31.1\, N / rad$ is the stiffness of the propulsion link;
$l = 0.063\, m$ is the length of the propulsion link;
$T = F_c \cdot l$ is the torque applied to the suspension, here F_c is the force applied to the chair suspension;
$R = \frac{63}{23} \cdot F_c$ is the force applied to the point *B* as in Fig. 3; 23 mm is the distance *a* between the point A and the point B as in Fig. 3; l = 0.063 = 63 mm;
$N = R \cdot d$ is the reaction force torque applied to the gear wheel, here *d* is the moment arm of *R* related to the axis of the driving crank as in Fig.3. The $d(\varphi)$ should be deduced to calculate the mechanism reaction on the motor axe.

To deduce the $d(\varphi)$ function, a MATLAB numerical calculation was performed, resulting in Eq.(2)

$$d(\varphi) = 0.006 \cdot sin(\varphi + 0.6981) \tag{2}$$

Based on above functions and parameters, the dynamic model of the rocking chair was developed as the following:

$$V(t) = V_b(t) + L_m \cdot \frac{di(t)}{dt} + i(t) \cdot r \tag{3}$$

Here, $V(t)$ is the voltage applied to the motor, $V_b(t)$ is the back-EMF, L_m is the motor inductance, r is the motor resistance, $i(t)$ is the motor current.

$$V_b(t) = K_2 \cdot \omega(t) \tag{4}$$

Here, $K_2 = 0{,}01 = const$ and $\omega(t)$ stands for motor velocity.

$$T(t) = i(t) \cdot K_1 \tag{5}$$

Here, $T(t)$ is for motor torque, $K_1 = 0{,}01 = const$. Using the 2nd law of Newton, the following equation was derived:

$$J \cdot \frac{d\omega(t)}{dt} = T(t) - K_4 \cdot R(t) \cdot d(t) - T_a(t) \tag{6}$$

Here, J is the moment of inertia of the rotor, $R(t)$ is the force of mechanism reaction, $d(t)$ is the moment arm of $R(t)$, $T_a(t)$ stands for friction torque, K_4 = 1/16 = const stands for the gain which transmits the mechanism reaction torque to the motor axis.

$$T_a(t) = 0{,}004 \cdot sign(\omega) \tag{7}$$

$$\frac{d\varphi(t)}{dt} = \frac{\omega(t)}{K_3} \tag{8}$$

Here, K_3 stands for the gear ratio, $K_3 = 48 = const$. The interaction between the suspension and the propulsion link:

$$d(t) = 0{,}006 \cdot sin(\varphi(t) + 0{,}6981) \tag{9}$$

$$\beta(t) = 0{,}26 \cdot sin(\varphi(t) - 3{,}927) \tag{10}$$

$$if \;\; \beta + \Delta \le \theta \qquad F_c(t) = K \cdot (\beta(t) + \Delta(t)) - \theta(t) \tag{11}$$

Here, Δ stands for the angle that is defined by the backlash (0,007 m) and the length of the propulsion link (0,063 m):

$$\Delta = \text{arctg}(\frac{0{,}007}{0{,}063}) = 0{,}1124\text{rad} \tag{12}$$

$$if \;\; \beta + \Delta > \theta \qquad F_c(t) = K \cdot (\beta(t) - \Delta(t)) - \theta(t) \tag{13}$$

$$Torque(t) = F_c(t) \cdot 0{,}063 \tag{14}$$

Here, *Torque* stands for the torque applied to the chair suspension

$$R(t) = F_c(t) \cdot \frac{0{,}063}{0{,}023} \tag{15}$$

Here, $R(t)$ is the reaction force applied to the connecting rod by the suspension. Finally, the following equation, which describes a simple pendulum, is a very simplified approximation of the real complex dynamic.

$$(m \cdot L_1^2 + J_1) \cdot \frac{d^2\theta(t)}{dt^2} = Torque - m \cdot g \cdot L_1 \cdot sin\theta(t) - 0{,}003 \cdot sign(\frac{d\theta(t)}{dt}) \cdot m \cdot g \cdot \cos\theta(t) \tag{16}$$

Here, m stands for the mass of the load (4 to 11 kg), $J_1 = 0.1$ kg·m^2 is the moment of inertia of the mechanism, $g = 9{,}81$ m/s^2, L_1 =0,3 m is the length of the suspension. All the numerical values used here were measured directly, deduced or extracted from data sheets. The third term on the right of Eq. (16) represents the torque of friction, including only gravitational component, in the mechanism. Thus, the dynamic model of the rocking chair represented by Eq. (1) to Eq. (16) is non-linear and is relatively complex. A complete model based on the above description has been introduced into MATLAB/Simulink (MathWorks, 1996). The model is shown in Fig.4 and includes 3 subsystems: *Chair, Mechanism* and *Motor,* which are shown in Fig.5*,* Fig.6 and Fig.7 respectively.

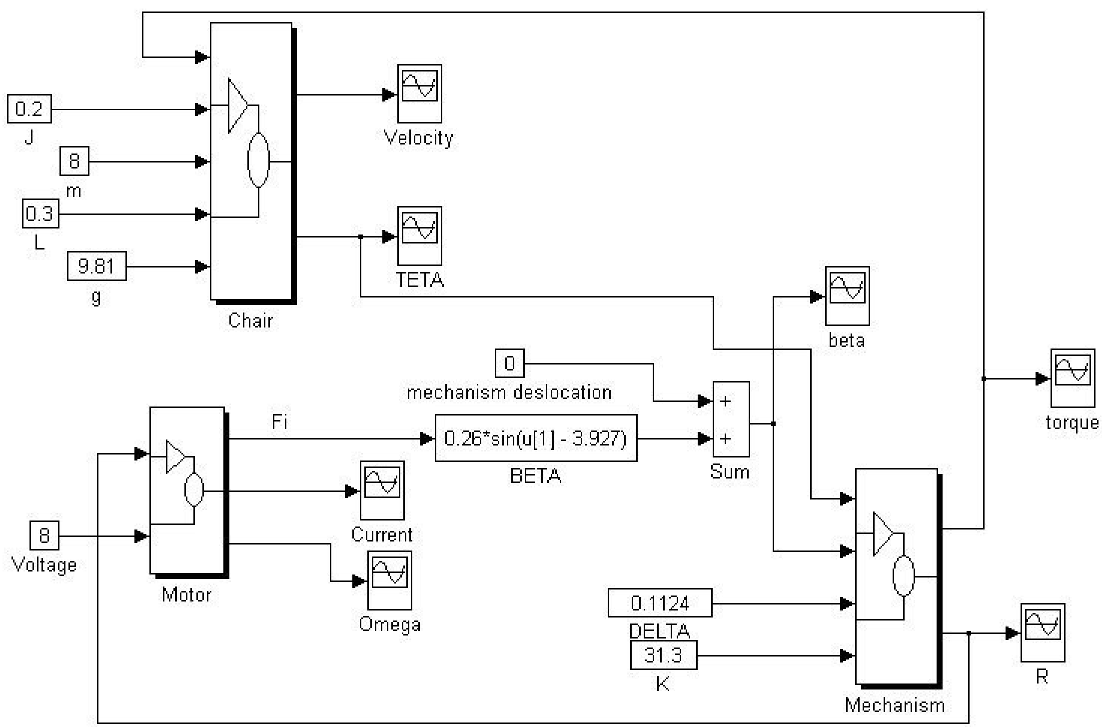

Figure 4. Complete dynamic model of the rocking chair.

Figure 4 shows the complete system model, including three subsystems and Eq.(1). The *Chair* subsystem in Fig.5 was developed using Eq.(16). The *Motor* subsystem in Fig.6 was developed using Eq.(2) to Eq.(6), while the *Mechanism* subsystem was developed using Eq.(7) to Eq.(15).

2.2. Simulations

At first, a free movement (motor is turned off) of the chair was simulated. The transition of the suspension angle θ is depicted in Fig. 8. This simulated dynamics was compared with the real one and an error of about 5% was detected. Than, a PWM control without feedback was tested. As was expected, the amplitude of oscillations was insufficient (about 0,0004 rad) and was noted that the frequency of forced oscillations was greater than the frequency of the free oscillations shown in Fig. 8. This suggested the control strategy which consists in motor velocity regulation. To control the motor velocity, it is necessary to use a velocity feedback. Because of the impossibility to install any additional sensor, it was decided to use the motor back-EMF as feedback information. A relay type controller was tried for velocity control execution. Many simulations were executed for different values of the reference signal OmegaRef, but

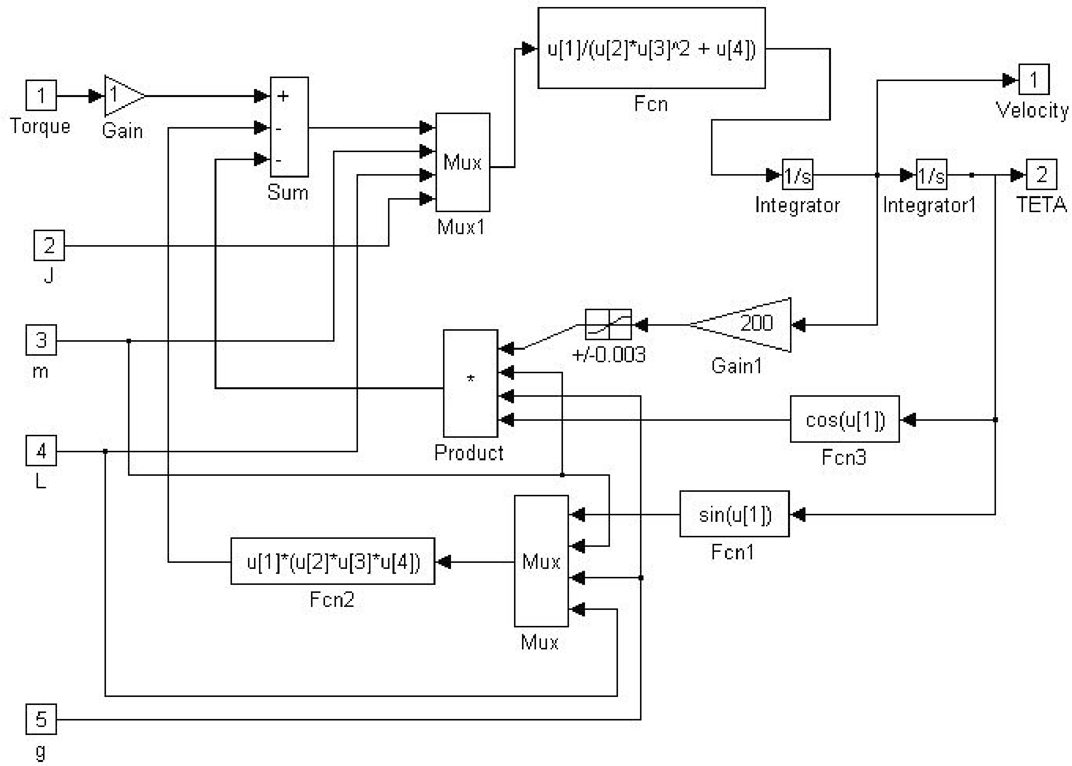

Figure 5. *Chair* subsystem.

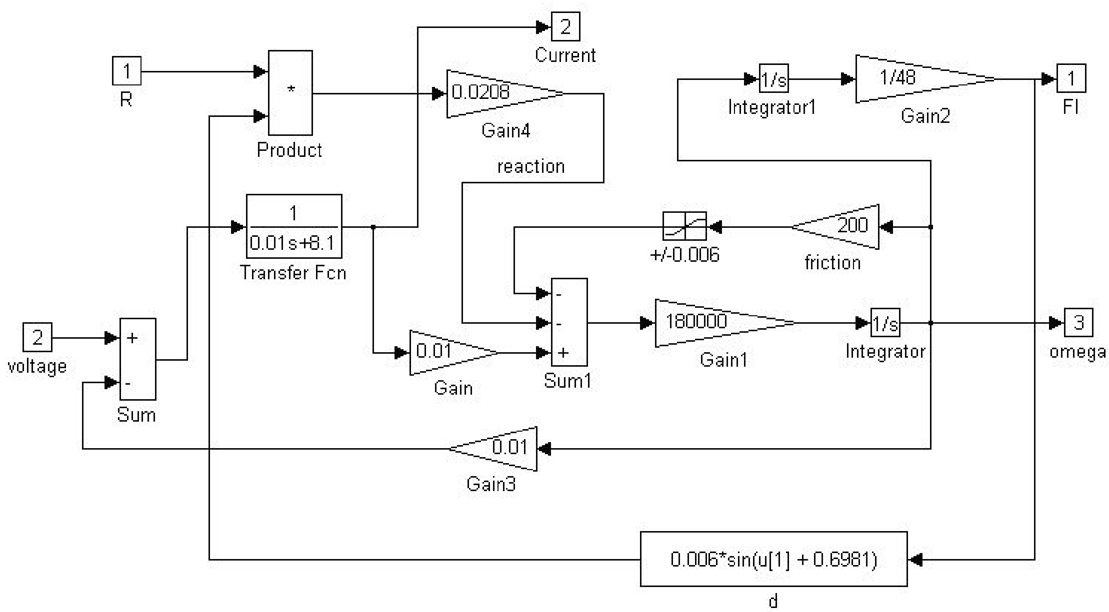

Figure 6. *Motor* subsystem.

the chair did not achieve any significant amplitude. The explication was that the value of the backlash Δ (DELTA in Fig. 4) was very high. So this one was reduced from 0,1124 *rad* to 0,03 *rad* and this resolved the problem. The simulation result with DELTA =0,03 is shown in Fig.9.

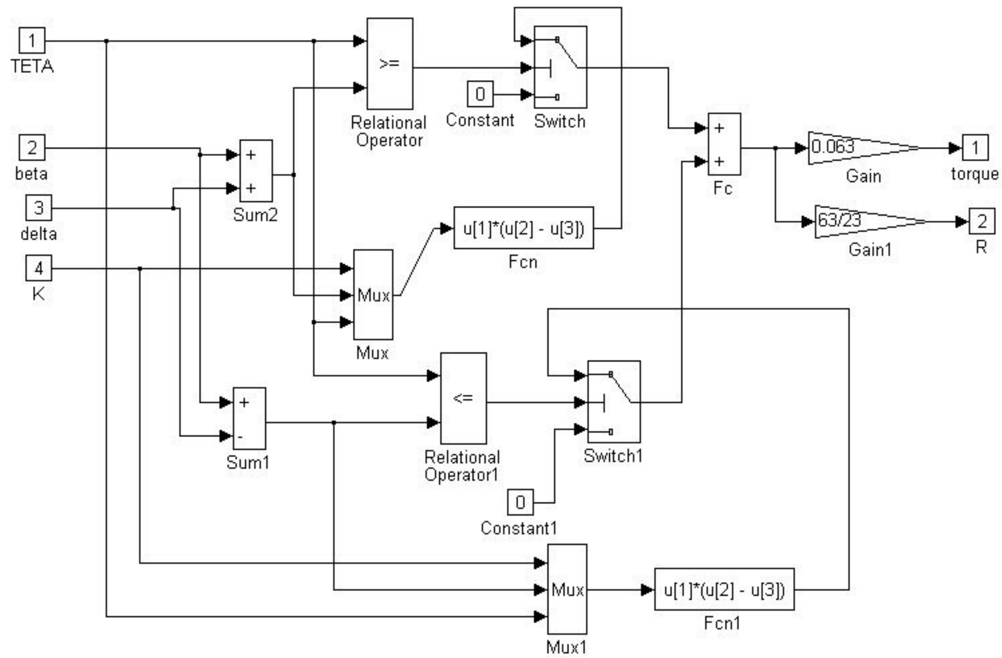

Figure 7. *Mechanism* subsystem.

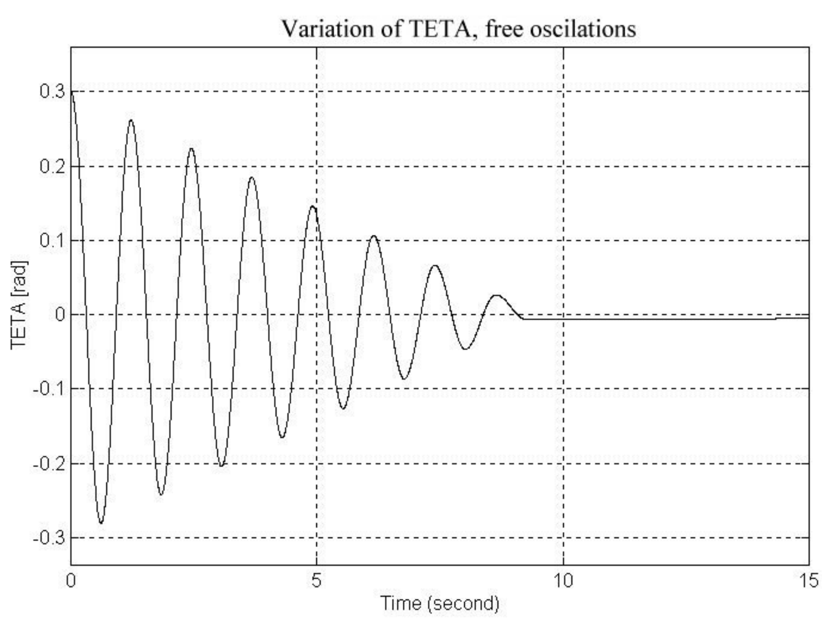

Figure 8. Free oscillations of the chair.

3. Controller prototyping and implementation

To verify the effectiveness of the developed relay controller, a prototyping is necessary. A prototype was constructed using a PC, one I/O module and a simple 2-transistors power amplifier. The I/O module has one 8-bits A/D and some logic level outputs. The PC which runs the C++ Borland DOS compiler was used as a programming tool. The idea was to program the proposed relay controller using only logical operations and delay function, because the final

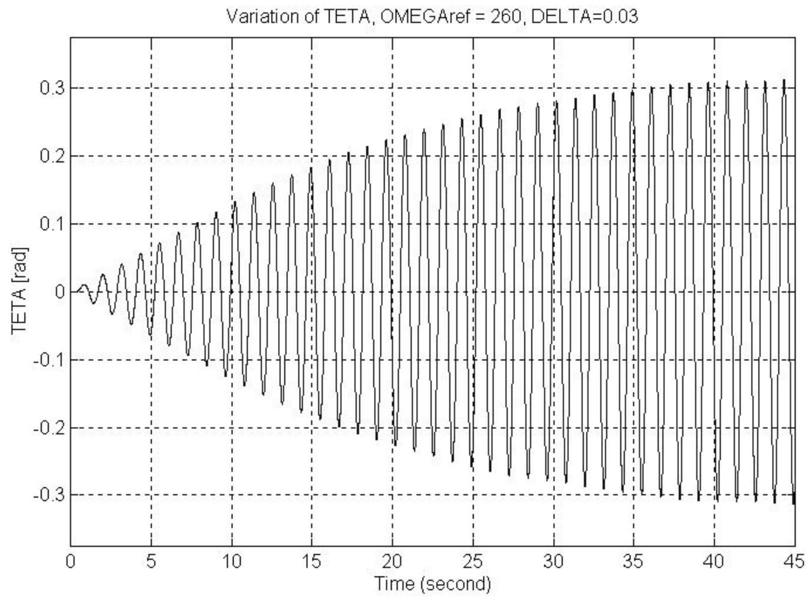

Figure 9. Forced movement of the chair with the relay closed-loop controller. DELTA = 0.03.

version of the controller should be implemented using a simple 8-bits microcontroller. During this stage of design, some details of the controller implementation were discovered. Based on the "try-and-error" approach, one delay of 70 ms and another one of 2 ms were introduced into the control program as in the following:

```
Inicialization;
while(1)
{ switch_off_motor();
delay(2);                        // delay of 2 ms
OMEGA=read_back_EMF();
if (OMEGA < OMEGAref
        {
        switch_on_motor();
        delay(70); }                    //    delay of 70 ms
else delay(2);
}
```

The controller prototype was tested with the rocking chair. With the back-lash (DELTA) equal to 0,1124 rad, the chair did not developed some considerable amplitude. So the back-lash was reduced to approximately 0,03 rad by fixing two ribbon sticks on the chair suspension and this really resolved the problem. The tests showed the possibility of varying amplitude of the chair oscillation by varying amplitude of the reference signal. The chair was tested under the load variations from 4 to 11 kg and showed a very satisfactory functioning.

4. Discussion

There are many publications reporting mechatronics education curriculums and relative laboratory activities. Here we can mention (Acar and Parking, 1996), (Wikander, Torngen and Hanson, 2001), (Lima *et al*., 2002), (Ume *et al*., 2002). The main idea is that Mechatronics is an interdisciplinary area and hands-on experience in laboratory is very important for the teaching practice.

The case study reported here shows the importance of the mechatronics approach which includes modeling and simulation since the early stages of design. Besides the model used for the chair dynamics was simplified, the model permitted to disclose a simple control strategy, which guarantees the stable movement under wide range of load variation. Using these techniques, it was possible to refine the initial mechanical design of the rocking chair and to develop one simple relay digital controller. Learning this case study, the students make itself familiar with the technique of complex system modeling and hierarchical simulation in MATLAB/Simulink.

The case study presented here, introduces such a fundamental concept of digital control as fixed-point calculation and permits to modify and test the control strategy easily through C language programming in DOS environment.

5. References

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6. Responsibility notice

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