CORRELATION OF K_t MAGNITUDE AND LEVEL OF CONSERVATISM OF LOCAL STRAIN ELASTOPLASTIC MODELS

Eduardo César Gavazza Menin, ***M.Sc.***
University of Brasília – Mechanical Engineering Departament – Stress Analysis Laboratory – GaMMa Research Group
Campus Universitário Darcy Ribeiro, Asa Norte, Brasília, CEP: 70940-910, Tel (61) 3073706 Ramal 213
edumenin@unb.br

Jorge Luiz de Almeida Ferreira, ***Dr.***
University of Brasília – Mechanical Engineering Departament – Stress Analysis Laboratory – GaMMa Research Group
Campus Universitário Darcy Ribeiro, Asa Norte, Brasília, CEP: 70940-910, Tel (61) 3073706 Ramal 213
jorge@unb.br

Abstract. *This research presents simulations for the behavior of notched members under elastoplastic conditions by means of the most frequently used local strain approach models. The situations evaluated involved steel plates of 17Mn4 alloy and AISI 1141 AF associated to two notch geometries, axial and bending loads, distinct magnitudes of the theoretical stress concentration factor and loading ratios of 0 and -1. Estimated local strain and fatigue life were compared with experimental data obtained from the appropriate literature and mainly with a finite element tool in order to evaluate the local strain models under different situations. By means of the analysis, it was made evident that fatigue life is not only related to strain amplitude, but also, with the stress field configuration presented near the geometrical discontinuity. Therefore, incoherence with respect to local strain models evaluations of degradation at notch root could be observed because of their local approach, resulting in a higher degree of conservatism of such models for higher* K_t *magnitudes.*

Keywords*: notches, elastoplastic models, stress concentration, fatigue.*

1. Introduction

Material mechanics basic theories permits that designers determine with good precision stresses and strains when dealing with projects of relatively simple shape components where stress gradients are essentially uniform. However, many practical situations make it necessary the use of components that present one or more stress concentrators, such as, holes, section variations, grooves or notches (Peterson, 1997; Schijve, 1980). When intensely loaded, such components can yield locally at the root of such singularities, even when subjected to loads below the material yield limit. Under cyclic conditions, the presence of local plasticity can induce stress redistribution, material properties degradation and the nucleation of fatigue cracks that could culminate in accidents, as well as financial losses (Visvanatha, 2000).

In such cases, fatigue life could be correctly characterized by the Strain-life method, ε-N. However, the usage of this approach demands knowledge of local stress and strain levels, fatigue characteristics of the material and detailed analysis of local yielding. Therefore, for a correct fatigue life analysis in cases involving local plasticity, it is necessary to perform an elastoplastic evaluation near the geometrical discontinuity.

One possible method to evaluate the local levels of solicitation consist the use of finite element models, FEM, considering geometric non-linearities and elastoplastic behavior of the material. However, tri-dimensional analysis can demand time and computational effort, especially when involving long and arbitrary histories of solicitations. An alternative to this problem could be the usage of bi-dimensional approximations that could compromise the reliability of the results. Also, several potential sources of errors associated to the selection of the physical model, mechanical and geometrical characteristics, boundary conditions, finite element and mesh refinement level should be correctly evaluated in order to ensure the validity of the results (Oden *et al.*, 2003).

Because of the reduced computational effort, local strain approximated methods have been intensely used in engineering designs. The ones most used are the mathematical models proposed by Neuber (1961), its modification proposed by Seeger *et al.* (1980), the equivalent strain energy density method, ESED, proposed by Glinka (1985) and, its recent modification proposed by Ye *et al.* (2003). Despite the fact that most of these models were deduced for specific geometries and loading conditions, they are sometimes unconditionally used to estimate local stress and strain levels in geometrical discontinuities under elastoplastic conditions. Moreover, another inconvenient of the use of such models represents their local approach, not making it possible to evaluate stress redistribution due to local yielding and geometrical variations at the notch root. This fact could compromise the analysis and generate incoherencies between experimental observations, subjected to the stress field configuration near the notch, and local strain estimations evaluated by the elastoplastic models.

Therefore, the main objective of this study consists of an evaluation of the effects, in strain and fatigue life estimations, due mainly to the magnitude of the theoretical stress concentration factor. By this mean, study cases were analyzed involving plane components presenting two notch geometries and distinct magnitudes of K_t. The materials selected were steel alloys 17Mn4 and AISI 1141 AF evaluated under axial and bending loads with loading ratios of 0 and 1. Histories of loads were simulated at the notches in order to estimate fatigue life by means of ε-N techniques and to evaluate local strain estimated by the most used elastoplastic models and FEM. The results evaluated were compared to experimental data obtained from the literature in order to identify the difficulties associated to the use of the elastoplastic models and its relation to the increasing magnitude of K_t.

2. Local Approach Elastoplastic Models

In order to identify the possibility of material properties degradation and nucleation of fatigue cracks it is of fundamental importance to perform an analysis of the cyclic deformation occurred at the root of the stress concentrators. Due to the reduced computational effort, such analysis are commonly performed by means of local strain approaches, that intend to estimate stress and strain levels at notch roots as a function of the theoretical stress concentration factor and a constitutive equation for the material behavior. The most commonly used models are presented in the next topics.

2.1. Neuber Rule and its Generalization

Despite the fact that the model proposed by Neuber (1961) was formulated for specific geometry and loading condition, it consists one of the most used to predict stress and strains at geometrical discontinuities under elastoplastic conditions. This method is based on the fact that the factor used to correlate local stresses and strains to their nominal values can no longer be linear expressed after local yielding has been observed. This local response is primarily due to residual stresses present as results of local plasticity at the geometrical discontinuity root (Neuber 1961) and can be expressed by Eq. (1) where K_σ e K_ε represent, respectively, the stress and strain concentration factors, and $\Delta\sigma$, ΔS, $\Delta\varepsilon$, Δe, the amplitudes of local stress, nominal stress, local strain and nominal strain, respectively.

$$K_t{}^2 = K_\sigma K_\varepsilon = \frac{\Delta\sigma}{\Delta S} \cdot \frac{\Delta\varepsilon}{\Delta e} \tag{1}$$

Assuming that nominal conditions are within the linear limits and that stress-strain relation under elastoplastic conditions can be described by a constitutive law such as a Ramberg-Osgood equation, the relation between nominal stress and local stress is expressed as Eq. (2). This last expresses Neuber rule in its most used form, where n' e K' describe the cyclic behavior of the material.

$$\frac{(K_t \Delta S)^2}{4 \cdot E} = \frac{\Delta\sigma}{2}\left[\frac{\Delta\sigma}{2 \cdot E} + \left(\frac{\Delta\sigma}{2 \cdot K'}\right)^{\frac{1}{n'}}\right] \tag{2}$$

Seeger *et al.* (1977) and Amstutz (1978) came to conclusions that the model proposed by Neuber gives conservative estimates of notch root strain. Other authors, such as Seeger and Heuler rediscovered Neuber's original rule, evaluating the significant errors of the simplification. In cases where there is necessity to evaluate generalized yielding at the net section of the discontinuity, Neuber rule could be used in the form presented in Eq. (3)

$$\frac{K_t{}^2 \cdot \Delta S}{2}\left[\frac{\Delta S}{2 \cdot E} + \left(\frac{\Delta S}{2 \cdot K}\right)^{\frac{1}{n}}\right] = \frac{\Delta\sigma}{2}\left[\frac{\Delta\sigma}{2 \cdot E} + \left(\frac{\Delta\sigma}{2 \cdot K}\right)^{\frac{1}{n}}\right] \tag{3}$$

Several authors have been recently studied this generalization of Neuber rule and have studied its validity when applied to cases where local stress is even bellow the material yield limit. Terrel (1989) reported that Seeger and Heuler rule can be used especially when large magnitudes of strain are present. Glinka (1985) reported its validity in cases that high levels of local solicitation associated to low stress gradients are present and Zheng *et al.* (2001) reported that, if nominal stress is around 0.8 S_y, nominal material behavior is inelastic and models that take into account non-linear stress and strain relations should be used, as the one proposed by Seeger et al. (1980).

2.2. Equivalent Strain Energy Density – ESED and its Modification

It was adequately proved that, in cases of non-generalized yielding, the energy density distribution of the plastic zone is approximately equal to the one observed for linear elastic materials. Such fact means that the linear elastic behavior of the material around the local yield at notch root controls strain relations at the plastic zone (Walker, 1974).

For plane stress conditions and for the linear elastic regime, stress at the notch root can be calculated based on the magnitude of nominal stress and Kt values. Therefore, Eq. (4) can be expressed, meaning that elastic strain energy density at notch root, WS, is equal to the product of energy density due to nominal stress, W_{sn}, and K_t squared.

$$\frac{(K_t \cdot \Delta S)^2}{E} = \frac{\Delta\sigma^2}{E} \quad ou \quad K_t^2 \cdot W_{sn} = W_\sigma = W_S \tag{4}$$

Glinka (1985) intensely researched such energy density approach intending to estimate local levels of inelastic stress and strain at notch root of components. Based on its results it was possible to conclude that, in the presence of local yielding, the energy density should be calculated with respect to Eq. (4) associated to a material constitutive law. When using a Ramberg-Osgood equation ESED model can be expressed as presented in Eq. (5).

$$\frac{(K_t \cdot \Delta S)^2}{8E} = \frac{\Delta\sigma^2}{8E} + \frac{\Delta\sigma}{2(n'+1)}\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \tag{5}$$

This last equation is valid for local yielding at notch root. It has been proved that ESED can be associated to plane stress and/or strain approximations, and is valid for stress levels near the generalized yielding of the net section (Glinka, 1985). If there is necessity to evaluate cases involving generalized yielding, Eq. (6) should be used.

$$K_t^2 \cdot \left[\frac{\Delta S^2}{8E} + \frac{\Delta S}{2(n'+1)} \cdot \left(\frac{\Delta S}{2K'}\right)^{\frac{1}{n'}}\right] = \frac{\Delta\sigma^2}{8E} + \frac{\Delta\sigma}{2(n'+1)}\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \tag{6}$$

One interesting modification of the ESED was proposed by Ye *et al.* (2004) and is based on the fact that during a plastic deformation cycle, most of the hysteresis loop energy is converted into heat and the rest is stored in the material and associated to residual stresses. Therefore, only part of the dissipative term of ESED models contribute to stress and strain levels at the geometrical discontinuity. Based on this fact, Ye *et al.* (2004) have proposed a modification of the ESED, valid for nominally elastic stresses and expressed in Eq. (7).

$$\frac{(K_t \cdot \Delta S)^2}{4E} = \frac{\Delta\sigma^2}{4E} + \frac{(2-n')\Delta\sigma}{2(n'+1)}\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \tag{7}$$

2.3. Singularities of the Models and ε-N Fenomenological Laws

It is important to notice that if the theoretical stress concentration factor and nominal stress magnitudes are known, it is possible to assess stress and strain level at notch root with the elastoplastic models (Bannantine, 1990; Dowling, 1999). It is interesting to observe that the term $(K_t.\Delta S)^2$ is always present in local approach models. This fact is indicative of the local nature of this kind of approach, where only the magnitude of the product between nominal stress and K_t is important and indicates lack of concern with respect to stress field configuration near the notch root.

In order to estimate life when dealing with materials subjected to cyclic loads and local plasticity it is common practice to use a series of phenomenological laws. In 1910, Basquin observed that *S-N* data could be linearly presented in a bi-log graph. Coffin (1954) e Manson (1953), working independently also found that data from the plastic strain versus life graphs were correctly approximated by a linear relation in a bi-log diagram. Analogous to the methodology utilized by Basquin, they used a potential relation to express the influence of plastic deformation in fatigue life. Eq. (8) represents the base of ε-N approach and is known as Coffin-Manson relation or also Strain-life and evaluates total strain as the sum of its elastic and plastic parts. The obtainance of the number of cycles to failure, *N*, in this case represented by the nucleation of fatigue cracks demands a numerical solution of Eq. (8) or the use of a graphical method.

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_{el}}{2} + \frac{\Delta\varepsilon_{pl}}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{8}$$

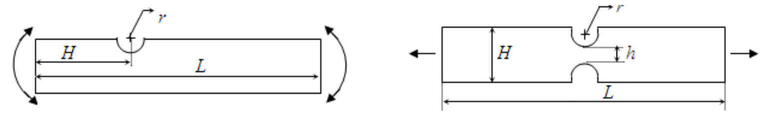
In order to incorporate the mean stress effects in the ε-N approach, several empirical equations could be used. The approach suggested by Morrow (1968) is one of the most used and can be expressed by the inclusion of σ_m in the ε-N relation, shown in Eq. (9).

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \qquad (9)$$

Other researches such as Smith-Watson and Topper also proposed equations that can be numerically solved and represent alternatives to compute mean stress effects but there is no agreement about which approach should be used (Dowling, 1999).

3. Materials and Methods

In order to evaluate the influence of the theoretical stress concentration factor magnitude in the estimates of fatigue life and notch root strain given by the most used elastoplastic models, two notch geometries presented in Fig. (1) combined to two materials were selected and studied. Tab. (1) presents the geometrical relations of the analyzed geometries, as well as the load conditions imposed in the simulations of the six evaluated study cases. The choice of such specific cases was based, not only on the availability of experimental data concerning fatigue life and notch root strain in the appropriate literature, but also trying to evaluate different materials, axial and bending load conditions, load ratios L_R of 0 and 1, and specially, similar cases involving distinct magnitudes of the K_t.



(a) (b)

Figure 1. Analyzed geometries.

Table 1. Cases of study selected.

Material	*Notch*	L_R	*r;L;H [mm]*	r/h	h/H	t[mm]	K_t	*Reference*
17Mn4	(a)	-1	3.5;125;47	---	---	16	2.14	Costa (1993)
17Mn4	(a)	-1	1.5;125;47	---	---	16	1.69	Costa (1993)
17Mn4	(a)	0	3.5;125;47	---	---	16	2.14	Costa (1993)
17Mn4	(a)	0	1.5;125;47	---	---	16	1.69	Costa (1993)
AISI 1141 AF	(b)	0	9.1; 60.3;41.1	0.399	0.55	2.5	1.77	Fatemi (2003)
AISI 1141 AF	(b)	0	2.8; 60.3;35.6	0.122	0.64	2.5	2.72	Fatemi (2003)

Figure (2) represents stress-strain cyclic curves of the evaluated materials. On Tables (2) and (3) are presented, respectively their mechanical and fatigue properties.

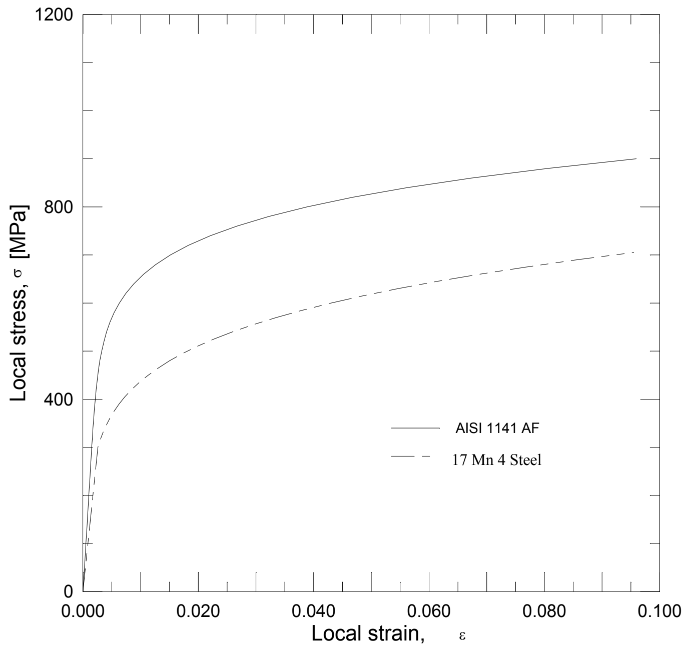


Figure 2. Cyclic stress-strain curve.

Table 2. Mechanical properties of the analyzed materials.

Material	*Reference*	S_{rt}	n'	K' [MPa]	S'_y [MPa]	E [GPa]
17Mn4	Costa (1993)	596	0,194	1120,0	412	207
AISI 1141 AF	Fatemi (2003)	875	0,122	1205,0	564	200

Table 3. Fatigue properties of the analyzed materials.

Material	*Reference*	σ_f' [MPa]	ε_f'	b	c
17Mn4	Costa (1993)	618	0,282	-0,050	-0,501
AISI 1141 AF	Fatemi (2003)	1296	1,027	-0,089	-0,687

The models proposed by Neuber (1961), Seeger *et al.* (1980), Glinka (1985) and Ye *et al.* (2003) were used to assess notch root strain when simulating the six presented study cases. The method adopted is similar to all cases of study and follows the schematic diagram showed in Fig. (3).

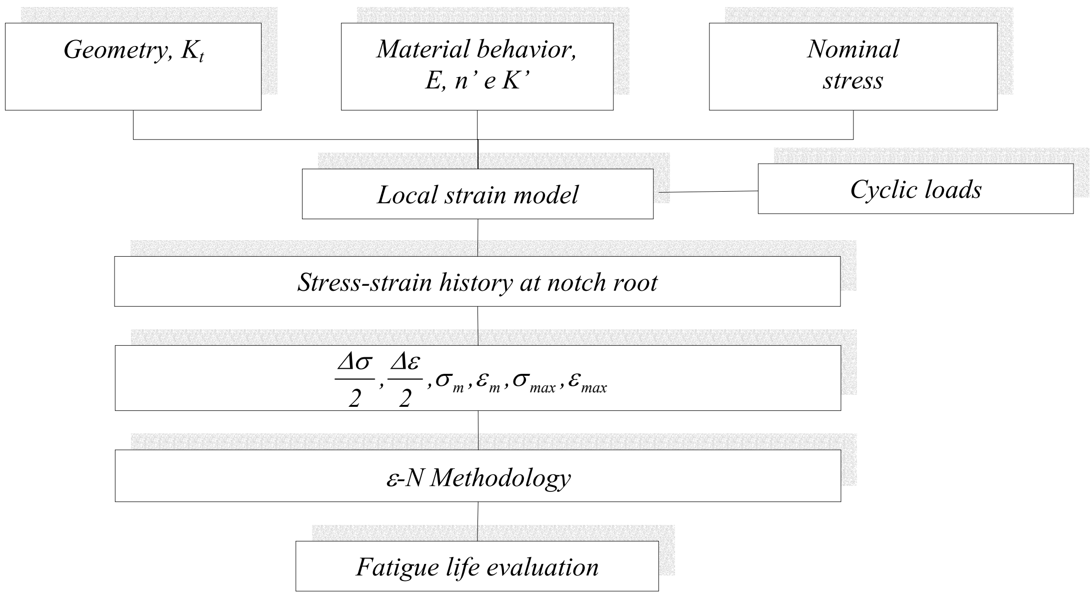


Figure 3. Schematic diagram for the method.

The magnitude of K_t used in the elastoplastic expressions was previously determined by finite element analysis for each analyzed case and compared to bi-dimensional values found in the literature (Peterson, 1997). The parameters that describe the cyclic behavior of the material, such as, young Modulus, E, strain cyclic exponent, n', strain cyclic coefficient, K', were obtained from the specialized literature, accounting for corrections if a plane strain hypothesis was intended.

Stress levels inducing linear elastic notch root response were evaluated, as well as levels that leaded to elastoplastic notch root behavior. These nominal solicitations, associated to bi-dimensional values of K_t were associated to the local strain models solved by numerical methods and iteration techniques.

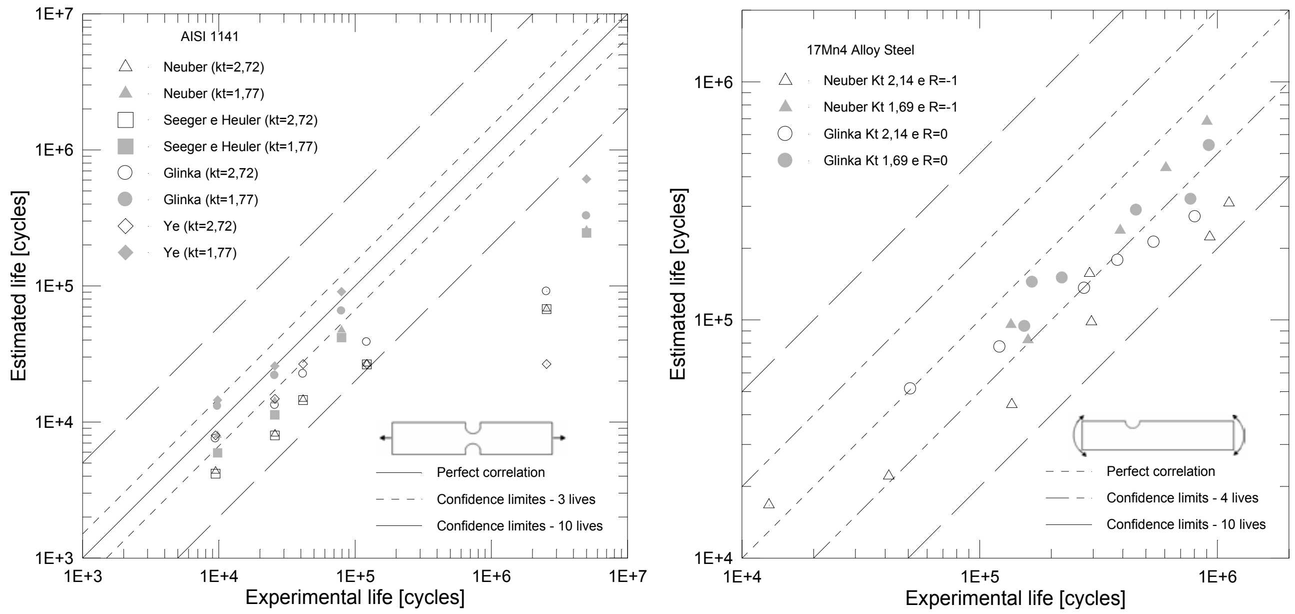
In order to assess total strain amplitudes, its elastic and plastic parts, a Ramberg-Osgood type equation was used. With the values estimated it was made possible to estimate fatigue life for the evaluated components by means of Coffin-Manson equation and its corrections to account mean stress effects, if needed.

ANSYS 5.4 software was used as a numerical comparison tool. Finite element models of the six analyzed cases were idealized and studied. The levels of mesh refinement used were selected from mesh convergence analysis based on the magnitude of K_t (Peterson, 1997). Bi-dimensional approximations were used associated to boundary conditions simulating the symmetry planes of the components. For the material strain hardening behavior, multilinear kinematic constitutive curves were used. All of the analyses were performed accounting for the stabilization of the load cycles in order to evaluate notch root stress-strain data.

Values of fatigue life and notch root strain estimated by means of local strain models were compared with data obtained from the literature and the values estimated with FEM.

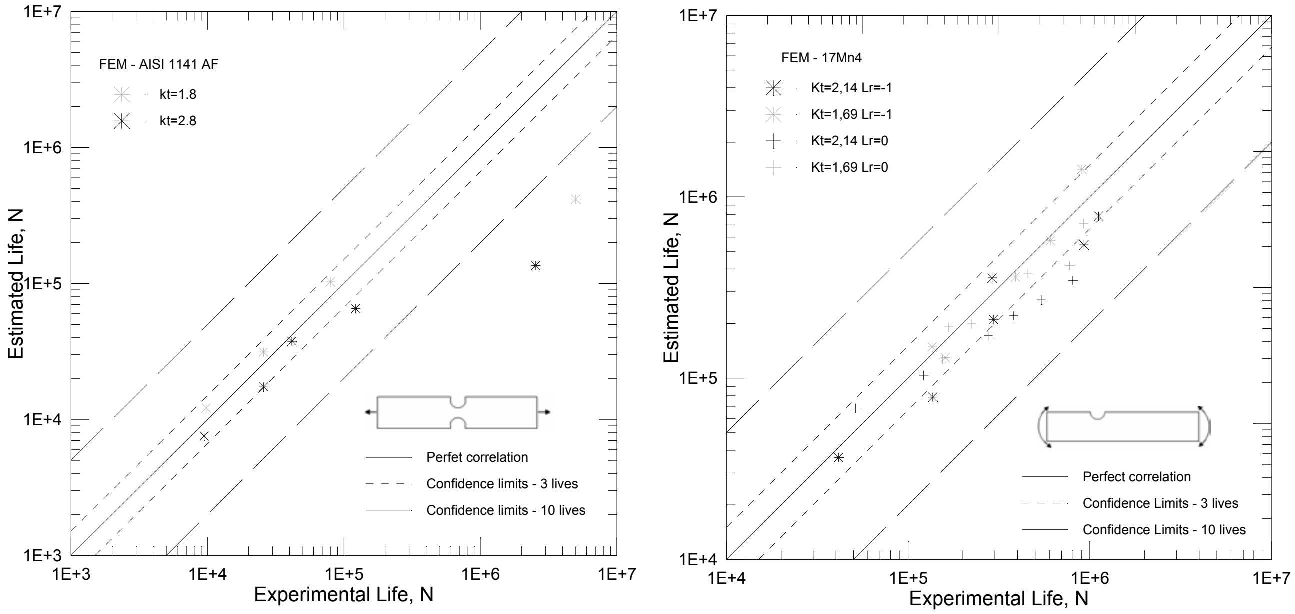
4. Results and Discussions

In order to evaluate qualitatively the correlation existing between estimated lives obtained with the use of the local strain models presented in this study and experimental obtained ones, the graphs shown on Figures 4 (a) and (b) respectively related to the cases of study involving alloy steels 17Mn4 e AISI 1141 were plotted. In these graphs, besides the correlation between estimated and experimental data, are presented the perfect correlation line and the confidence limits for 3 and 10 lives. Figure (5) exemplifies the same approach using FEM comparative tool.



(a) (b)

Figure 4. Correlation between experimental and estimated life.



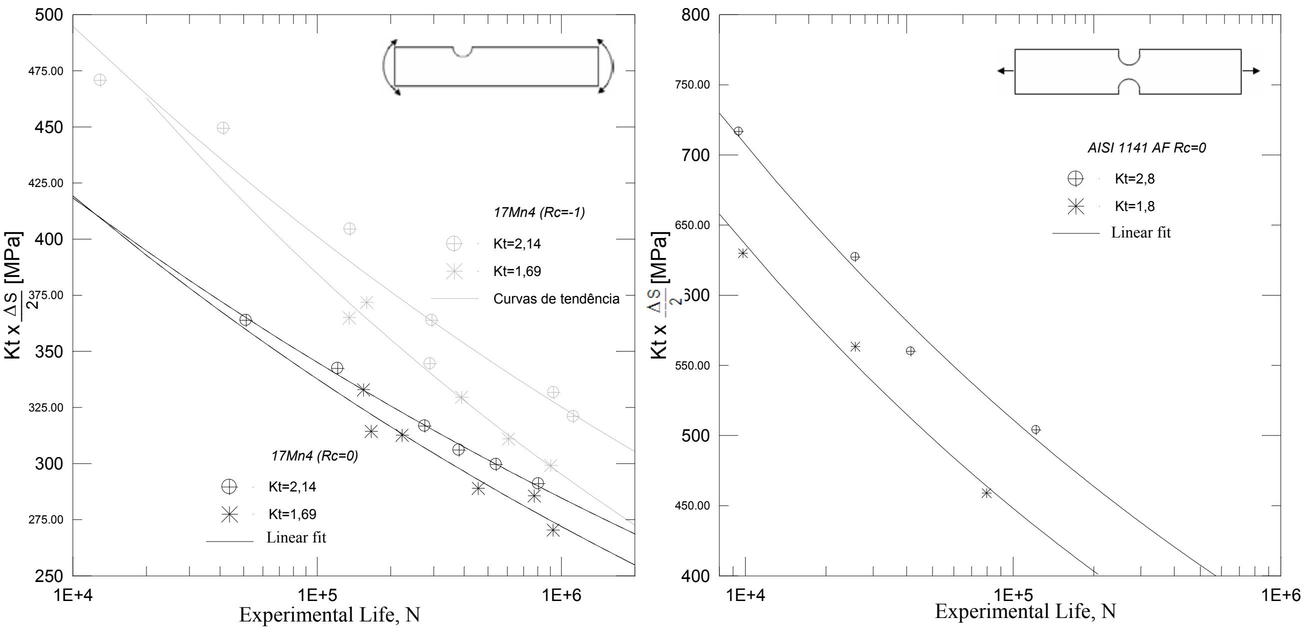
(a) (b)

Figure 5. Correlation between experimental and FEM estimated life.

By the graphs presented on Figure (4) it is possible to observe that all analytical models leaded to conservative results for most of analyzed lives. The level of conservativismo showed to be more intense in cases involving higher magnitudes of the theoretical stress concentration factor and for relatively long lives. The graphs presented on Figure (5) exemplifies that using FEM analysis, the estimations result in a significant reduction of the observed deviations between the estimations and the experimental data when compared to the results estimated by the local strain approaches. However, it is still evident that the degree of conservativismo was more intense in cases involving higher magnitudes of K_t and for relatively longer lives

As an attempt to explain such results, graphs of experimental data reported by Costa *et al.* (1993) and Fatemi *et al.* (2003) versus the product between nominal stresses and stress concentration factors, $K_t.\Delta S/2$ were plotted on Figure (6) for the study cases. By analyzing these graphs it is possible to verify that experimental data depends upon the product between nominal stresses and K_t. However, based on the same graphs it is also possible to observe that the magnitude of this product is not sufficient to completely explain fatigue life behavior because data concerning longer experimental lives were reported for higher magnitudes of K_t.

The explained facts suggest that fatigue life is, not only related to strain amplitudes of the notch root locations, but also to the stress field configuration acting at the regions near the geometrical discontinuity. In that sense, incoherence could be observed with respect to the estimates assessed with the analytical models that present a local approach to fatigue problems and do not take into account the presence of stress gradients and critical volumes theories. Such facts explain the higher degree of conservatism presented by the local strain approach models with the increase of K_t magnitudes and longer lives when correlated to the experimental data.



(a) (b)

Figure 6. Experimental data of notch effect on fatigue life.

5. Conclusions and Final Comments

The obtained results made it possible to evaluate the difficulties associated to the use of local strain approach models involving cases of study presenting different materials, notch geometries, magnitudes of the theoretical stress concentration factor and types of loads. When observing the estimated fatigue life assessed by the models for similar study cases differing only on the magnitude of K_t it was possible to verify an increasing level of conservatism of the results with the increase of K_t magnitude. This fact is related to the local characteristic of such approaches that do not take into account the stress field configuration near the geometrical discontinuity. The importance of such analysis was made evident when evaluated experimental data presented by two authors for the cases of study in question. In these results it was possible to observe higher fatigue lives for higher magnitudes of K_t where stress gradients are more expressive, corroborating the possibility of existence of incoherence when compared to the analytical model results. Also, the numerical and experimental results made evident the reduction of the notch effects for short lives probably due to the presence of plastic deformation and reduction of the local stress amplitudes.

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