EVALUATION OF LOCAL STRAIN MODELS IN THE SIMULATION OF ELASTOPLASTIC CONDITION AT THE NOTCH ROOT OF THICK PLATES

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Abstract. *This research presents the difficulties associated with the use of local strain models in cases involving components with significant thickness magnitude. In that sense, finite element modeling was used to simulate 3 notch geometries with different dimensionless thickness and notch deepness ratios. The analysis is divided in two parts: (i) linear-elastic, studying the variations of* K_t *and notch root constraint level; (ii) elasto-plastic, observing the validity of the use of plane approximations and bi-dimensional values of* K_t *when associated to local strain models, as well as, the correlation between estimated fatigue life and experimental data from the literature. The results made it possible to evaluate the dispersion of* K_t *plane values and the ones simulated for the surface and mid-thickness of the notch root. It was also observed differences between plane approximations and the real state for such locations, and quantified the validity of the use of local strain models associated to bidimensional values of* K_t *and plane state approximations in cases involving components with significant dimensinless thickness values.*

Keywords*: stress concentration, notches, thick plates, local strain models, elastoplastic conditions*

1. Introduction

Heavily loaded structural components can yield locally at stress concentrators, such as, notches and geometrical discontinuities (Peterson, 1997). In cases involving cyclic loads, the presence of local plasticity could lead to the nucleation and propagation of fatigue cracks and complete fracture of the components (Fillippini, 2000; Visvanatha, 2000). In such cases, the strain life approach should be used to evaluate fatigue damage, making it necessary to perform an elastoplastic analysis of stresses and strains at the proximities of these stress concentrators. Due to reduced computational effort requested, approximated models are widely used with that purpose. The most used are the ones proposed by Neuber (1961), Seeger ***et al.*** (1980), Glinka (1985) and Ye ***et al.*** (2003).

One factor that complicates the use of these approaches consists the evaluation of stress concentration problems, by most part of the specialized literature, as plane approximation problems that do not take into account the variations in the stress-strain field configuration along the thickness of the components, as these become larger in magnitude. To corroborate the importance of such 3D analysis, several numerical and experimental studies have been published indicating that the stress field near the geometrical discontinuities varies along the notch root, as components become thicker (Zhenhuan, 2000; Soares, 2002). In such cases, 2D values of K_t and stress/strain plane approximations should be carefully evaluated when associated to local strain models in order to correctly estimate local levels of solicitations at the notch root of thick components.

In that sense, this study presents examples of the difficulties associated to the use of local strain approximated models when evaluating cases involving notched components presenting expressive values for the ratio between thickness and notch root radius, called also dimensionless thickness, *t/r*. A preliminary linear elastic behavior analysis for the variation of K_t and the displacement constraint level of points along the notch root was performed using 3D FEM. After that, elastoplastic analyses were performed by means of the most used local strain models, as well as, finite element 3D models and 2D approximation. These simulated the local behavior at the root of 2 notch geometries and two different types of steel. The results of local strain and fatigue life estimated were compared to experimental data from the literature in order to exemplify the difficulties associated to the use of local approaches when evaluating the solicitations at the notch root of thick members.

2. Local Strain Approach

Local strain analysis are commonly performed by means of mathematical models that intend to estimate stress and strain levels at notch roots as a function of the theoretical 2D stress concentration factor and a constitutive equation for the material behavior. One of the most used models is the one proposed by Neuber (1961) and is based on the fact that the factor used to correlate local stresses and strains to their nominal values can no longer be linear expressed after local yielding has been observed, resulting on Eq. (1) where $\Delta\sigma$ and ΔS, represent, respectively, local and nominal stresses.

$$\frac{(K_t \Delta S)^2}{4 \cdot E} = \frac{\Delta\sigma}{2}\left[\frac{\Delta\sigma}{2 \cdot E} + \left(\frac{\Delta\sigma}{2 \cdot K'}\right)^{\frac{1}{n'}}\right] \quad (1)$$

This last expresses Neuber rule in its most used form, where *n'* e *K'* describe the cyclic behavior of the material. However, Eq. (1) is valid until the beginning of local yield. Therefore, if there is necessity to evaluate local strain beyond this point, as well as generalized yielding at the net section of the discontinuity, Seeger *et al.* (1980) have proposed a generalized Neuber Rule, presented in Eq. (2). Several authors (Terrel, 1989; Glinka, 1985; Zheng, 2001) have recently studied this generalization of Neuber rule and suggested its validity when applied to cases where local stress is even below the material yield limit.

$$\frac{K_t^{2} \cdot \Delta S}{2}\left[\frac{\Delta S}{2 \cdot E}+\left(\frac{\Delta S}{2 \cdot K}\right)^{\frac{1}{n}}\right]=\frac{\Delta \sigma}{2}\left[\frac{\Delta \sigma}{2 \cdot E}+\left(\frac{\Delta \sigma}{2 \cdot K}\right)^{\frac{1}{n}}\right] \tag{2}$$

Glinka (1985) intensely researched an energy density approach intending to estimate local levels of inelastic stress and strain at notch root of components. Based on its results it was possible to conclude that, in the presence of local yielding, the energy density should be calculated with respect to Walkers (1974) observations and associated to a material constitutive law, such as a Ramberg-Osgood equation. In that sense ESED model can be expressed as presented in Eq. (3).

$$\frac{\left(K_t \cdot \Delta S\right)^2}{8E}=\frac{\Delta \sigma^2}{8E}+\frac{\Delta \sigma}{2\left(n'+1\right)}\left(\frac{\Delta \sigma}{2K'}\right)^{\frac{1}{n'}} \tag{3}$$

It has been reported (Glinka,1985) that local strain energy density method, and the former proposed ones, can be associated to plane stress and/or strain approximations. In this last, the material cyclic properties *n'* e *K'* should be corrected by using an appropriate σ^*-ε^*curve, valid for plane strain approximations (Glinka,1985).

An interesting modification of ESED method was proposed by Ye *et all.* (2004) and is based on the fact that during a plastic deformation cycle, most of the hysteresis loop energy is converted into heat and the rest is stored in the material and associated to residual stresses. Therefore, only part of the dissipative term of ESED models contribute to increase the stress and strain levels at the geometrical discontinuity. Based on this fact, Ye *et all.* (2004) have proposed a modification of the ESED, valid for nominally elastic stresses and expressed in Eq. (4).

$$\frac{\left(K_t \cdot \Delta S\right)^2}{4E}=\frac{\Delta \sigma^2}{4E}+\frac{(2-n')\Delta \sigma}{2\left(n'+1\right)}\left(\frac{\Delta \sigma}{2K'}\right)^{\frac{1}{n'}} \tag{4}$$

It is important to notice that all of these equations represent crucial tools in the evaluation of local stress levels when preliminarily knowing the values for the theoretical stress concentration factor, K_t, nominal stress amplitude, S, and the cyclic properties of the material. In order to determine local strain levels, the results of such equations should be associated to equation such as, Ramberg-Osgood type expressed for cyclic solicitations. Fatigue life could be computed by a Coffin-Manson equation (Bannantine, 1990; Dowling, 1999). In that sense, it is evident that the reliability on the estimates of local strain levels and fatigue life obtained with the use of local approach are intensely subjected to the use of correct values of K_t and to the evaluation of the stress state that represents correctly the points at the notch root.

3. Materials and methods

The method used can be divided into 2 main parts: (i) elastic and (ii) elastoplastic analysis of two cases of study.

3.1 Elastic Domain Analysis

The elastic domain analysis was performed in order to evaluate the variations of K_t associated to the changes in different models thickness, and the validity of the use of bi-dimensional values of K_t for components with expressive magnitudes of the dimensionless thickness, *t/r*. In that sense, finite element models were idealized and simulating axial static loads, linear elastic behavior for the material and plates presenting three types of geometrical discontinuities: center slot (*CS*), double U notch (*DU*) and central hole (*CH*), selected because of their classical use in engineering problems and the presence of experimental data concerning fatigue life in the literature to compose the two cases of study evaluated in the elastoplastic analysis (Bannantine *et al.*, 1990). The dimensionless ratios evaluated for each notch geometry are described in Tables 1 to 3.

Table 1. Center Slot notched models.

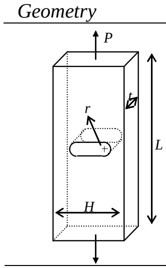
Geometry	*Model*	*H (mm)*	*H/h*	*r/h*	*t/r*	*L/2 (mm)*
	*CS1*				0.1	
	CS2				0.5	
	CS3				0.75	
	CS4				1.5	
	CS5	50.8	1.43	0.054	3	25.4
	CS6				6	
	CS7				12	
	CS8				18	
	CS9				20	

Table 2. Central Hole notched models.

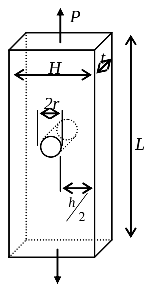
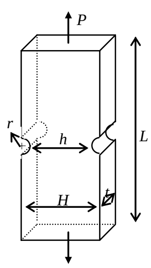
Geometry	*Model*	*H (mm)*	*H/h*	*r/h*	*t/r*	*L/2 (mm)*
	*CH1*				0,1	
	CH2				0,5	
	CH3				1	
	CH4	35	1,52	0,26	1,5	30
	CH5				2,5	
	CH6				5	
	CH7				10	
	CH8				20	
	CH9	50	5	2	0,1	45
	CH10				1,5	
	CH11				0,1	
	CH12				1	
	CH13				1,5	
	CH14	20	1,05	0,03	2,5	20
	CH15				5	
	CH16				10	
	CH17				20	

Table 3. Double U notched models.

Geometry	*Model*	*H (mm)*	*H/h*	*r/h*	*t/r*	*L/2 (mm)*
	*DU1*				0.1	
	DU2				1	
	DU3	30	1.5	0.25	1.5	20
	DU4				5	
	DU5				10	
	DU6				20	
	DU7				0.1	
	DU8				1	
	DU9	30	1.2	0.1	1.5	20
	DU10				5	
	DU11				10	
	DU12				20	

In order to evaluate the influence of the dimensionless thickness *t/r* in the assessment of K_t, magnitudes of this ratio varying between 0.1 e 20 were selected for the same notch geometry. Moreover, different notch deepness ratios, *r/h*, were simulated, resulting in different magnitudes of the theoretical 2D stress concentration factor (Peterson, 1997). The modification in the dimensionless ratios of the components with the same notch geometry were performed in a way to assure the control of the finite element sizes in the regions of interest, i.e. near the notch root.

A schematic representation for 1/8 of the FEM geometry is presented in Figure 1. The points of interest for the elastic analysis are located at the notch root in the free surface of the model, (1) and in its mid-thickness, (2). Models presenting different *t/r* relations were simulated in order to evaluate the stress distribution near the notch root concerning equivalent stress, and stresses in the orthogonal directions trough lines 1-3 and 2-4, as presented on Figure 2. For the notch root along the thickness of the components, line 1-2, observations were performed in order to evaluate different stress states, as well as, the variations of K_t, for models presenting specific *t/r* e *r/h* relations.

The values of K_t simulated trough FEM analyses were compared to the ones obtained from bi-dimensional theories (Peterson, 1997). The level of constraint of the notch root points were evaluated by means of the deformation ratio, $\varepsilon_z/\varepsilon_y$. The values observed for this relation were compared with the opposite of *Poisson* coefficient ($\nu \approx 0.3$), representative of low constraint level and plane stresses; and with values close to zero, representative of high level of restrictions to displacements and plane strain approximations (Sharpe et al., 1992).

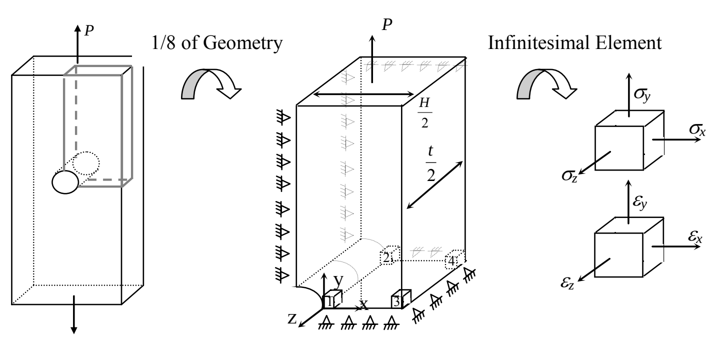


Figure 1. Schematic representation for 1/8 of the geometry.

3.1 Elastoplastic Domain Analysis

The elastoplastic domain analysis was performed through estimating the local stress-strain levels acting at the notch root by means of the most used local strain models proposed by Neuber (1961), Seeger *et al.* (1980), Glinka (1985) e Ye *et al.* (2003), solved numerically with iteration techniques. The main purpose of this analysis was to observe the estimates given by the mathematical models when evaluating study cases in thick plates notched members.

In that sense, the 2 cases of study presented on Table 4 were selected because treats of components with expressive values of *t/r* and because of the availability of experimental data in the literature (Bannantinne *et. al.*, 1999). These cases involve low and medium strength steels whose mechanical and fatigue properties are presented on Table 5.

Table 4. Cases of study used in the elastoplastic analysis.

Case of Study	*Material*	*Notch*	*r;H [mm]*	r/h	h/H	*t*[mm]	K_t
A	Medium resistance steel	Double U	2,54; 25,4	0,125	0,80	6,35	2,58
B	Low resistance steel	Center slot	1,91; 50,8	0,044	0,85	5,72	3,13

Table 5. Mechanical and fatigue properties of the materials evaluated.

Case of Study	*Srt*	*n'*	*K' [MPa*	*Sy' [MPa]*	*E [GPa]*	*σf' [MPa*	*εf'*	*b*	*c*
A	785	0,123	1061,0	648	207	1164	1,142	-0,081	-0,670
B	537	0,226	1337,6	351	207	1117	0,338	-0,110	-0,480

K_t bi-dimensional values obtained from graphics and charts (Peterson, 1997) were associated to the analytical models. The constitutive curve parameters (σ-ε), such as *E*, *n'* and *K'*, were obtained from the literature and submitted to suitable corrections, when the analysis leaded to plane strain hypothesis (σ^*-ε^*), as showed in Figure 1. The combination of bi-dimensional values of K_t and constitutive curve constants *n'* e *K'* assuming plane stress and/or strain hypothesis were associated to the local strain models in order to evaluate the difficulties of the impossibility of preliminary FEM analysis to estimate this parameters. Stress levels varying from linear elastic local response to elastoplastic behavior were simulated in the local strain models. In order to quantify total strain, as well as, its elastic and plastic parcels, a Ramberg-Osgood equation was used. By means of the estimated values of strain it was possible to estimate fatigue life through Coffin-Manson equation.

For the analysis concerning elastoplastic behavior of the notch root, FEM was used as a comparison tool. Plane and 3D models were idealized and simulated respectively, 1/4 and 1/8 of the geometry. 2D FEM models were associated to plane stress hypothesis when representing the free surface of the component, and plane strain hypothesis to simulate the mid-thickness plane. Mesh convergence analysis were performed in order to calibrate the models in respect to K_t bi-dimensional values and to conciliate computational effort and precision. In the 3D evaluation, meshes were obtained from extrusion of previously evaluated 2D meshes, with control on the size of the finite elements at the proximities of the notches. Constitutive multi linear curves were used to evaluate the elastoplastic behavior of the material.

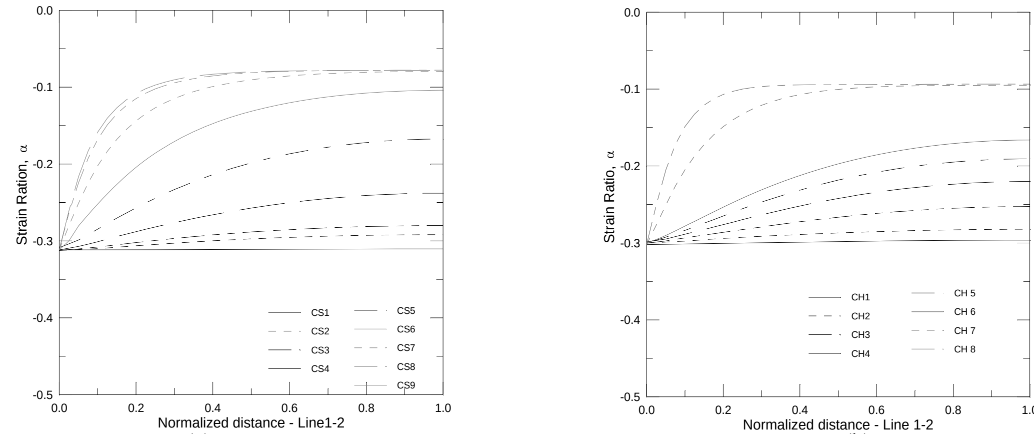
Idealized the models, they were subjected to nominal stresses similar to the ones used in the local strain models. All the analyses were performed in order to guarantee the stabilization of the hysteresis loops. These results simulated were associated to Coffin-Manson equation intending to estimate fatigue life for the cases of study. Values evaluated through this method were compared to data from the literature and the ones estimated by the analytical local strain models.

4. Results and Discussions

In order to evaluate the displacement constraint level for the finite elements along the notch root, the graphs of Figure 4 were assembled. These graphs show the evolution of the deformation ratio $\alpha = \varepsilon_z/\varepsilon_y$ for the elements along line 1-2 of center slot and central hole notch geometries, respectively on Figure 2(a) and 4(b). It was possible to make evident that α evaluated for points of the notch root at the free surface of the component, point 1, tends to the negative of the *Poison* coefficient, -0.3, indicating a low degree of displacement constraint and similarities to a plane stress state. Also, this behavior showed to be independent of the thickness of the models.

By these graphs it was also possible to verify that for components with low magnitudes of *t/r*, α showed to be approximately constant. The variations of such ratio became more intense, as the ratio became more expressive. For components presenting *t/r*=1.5 variations between the free surface and the mid-thickness of the models were already about 10%. For points located at the mid-thickness of models with expressive values of the dimensionless thickness, magnitudes of α close to zero were observed, indicating high degree of displacement constraint for this location. However, values distinct from zero were evaluated suggesting differences between the tri-axial stress state observed for the mid-thickness of thick components and the approximation of plane strain condition.

These results suggest that when dealing with components presenting unexpressive values of dimensionless thickness, plane stress approximations are coherent to describe the stress state for both the free surface and the mid-thickness of the models, as suggested by Dowling (1999). On the other hand, α variations for components with expressive values of *t/r* made evident that points at the free surface can still be correctly approximated by a plane stress condition but points located at the mid-thickness of the notch root, besides its high degree of displacement constraint, presents divergences to plane strain approximations.

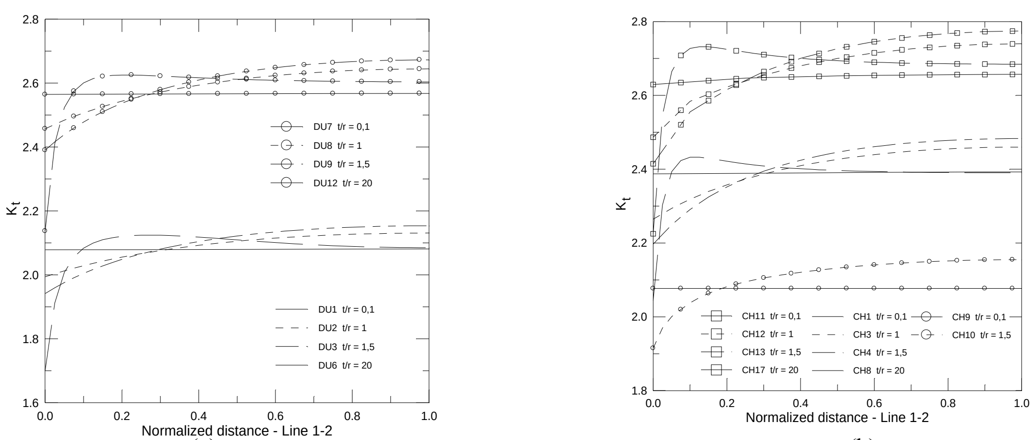


(a) (b)

Figure 2. α= εz/εy for the notch root along line 1-2: (a) Center slot; (b) Central hole.

Figures 3 (a) e (b) presents the behavior of K_t for line 1-2 of components with central hole and double U notches, respectively. This analysis showed to be interesting in order to evaluate the use of bi-dimensional values of K_t (Peterson, 1997) for components presenting expressive magnitudes of dimensionless thickness.

Based on these results it was possible to verify that K_t magnitude for components presenting $t/r \approx 0$ is practically constant and equal to bi-dimensional values of plane approximations. As the ratio *t/r* rises in magnitude, higher variations of this factor are observed. Such results are in agreement with Soares *et al*. (2002) and Zhenhuan ***et al.*** (2000). It is interesting to notice that bi-dimensional values of K_t are within the limits of values observed for this factor at the surface and mid-thickness of thick components. This factor evaluated for the mid-thickness is higher in magnitude when compared to bi-dimensional values. On the other hand, K_t estimated for the free surface presented magnitude inferior to the same factor 2D values. In the same figure it is also possible to notice similar behavior when evaluating distinct 2D magnitudes of K_t for the same notch geometry.



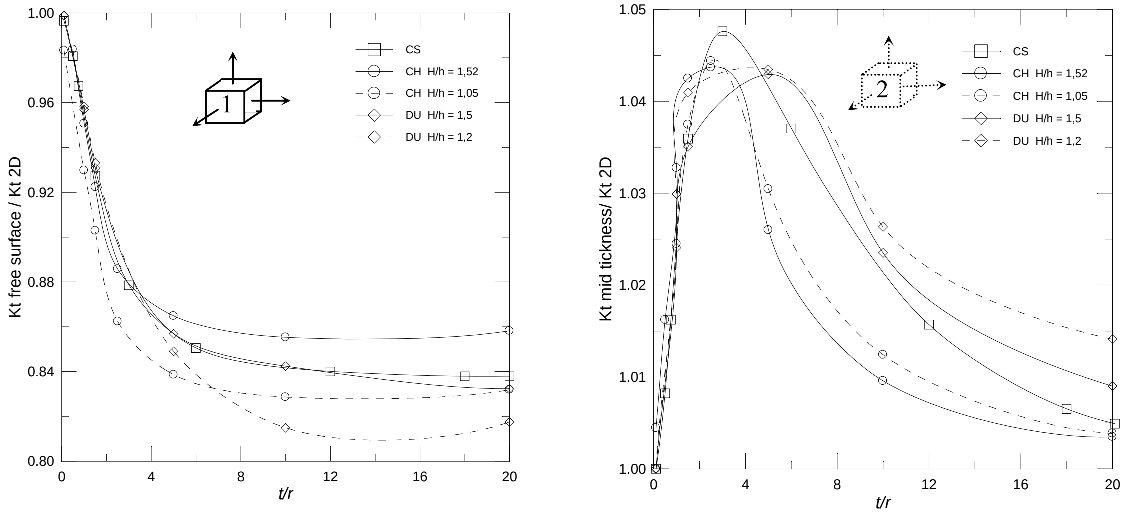
(a) (b)

Figure 3. K_t variations for the notch root along line 1-2: (a) Double U; (b) Central hole.

Results related to the behavior of K_t in thick plates can be more appropriately discussed when evaluated separately two distinct points of the notch root: (1) free surface and (2) mid-thickness. Figure 4 (a) details K_t behavior evaluated in point 1, normalized in respect to its bi-dimensional value, as the relation *t/r* of the component varies. The several curves represent distinct notch deepness ratios and different notch geometries. Figure 4 (b) expresses similar results for point 2 (Menin, 2004).

The graph of Figure 4 (a) makes it evident that models with non-expressive values of *t/r* present K_t evaluated for the free surface similar to bi-dimensional values. However, expressive magnitudes of *t/r* lead to higher deviations of K_t when compared to theoretical results. In such cases, values of K_t around 15% lower than the bi-dimensional values were observed. Several authors (Peterson , 1997) have developed experimental tests concerning distinct notch geometries and came to deviations up to 23%. It is also possible to notice that for the same notch geometry, high values of de *H/h* leaded to higher K_t deviations when compared to components with relations of *H/h* less intense.

Evaluating the behavior of K_t for the mid-thickness at Figure 4 (b), it is possible to notice that the deviation of this factor is not a monotonic function of the dimensionless thickness. Such deviations rise from their respective bi-dimensional values in $t/r \approx 0$, to approximately $t/r \approx 2$ a 6, where critical values of about 5% higher than 2D values can be observed, than they tend to decrease gradually to a plane stress/strain equivalent value as t/r becomes sufficiently high. It was also possible to notice that the curves are independent of the notch configuration for relations of $t/r \approx 0$ as suggested by Zhenhuan ***et al.*** (2000).



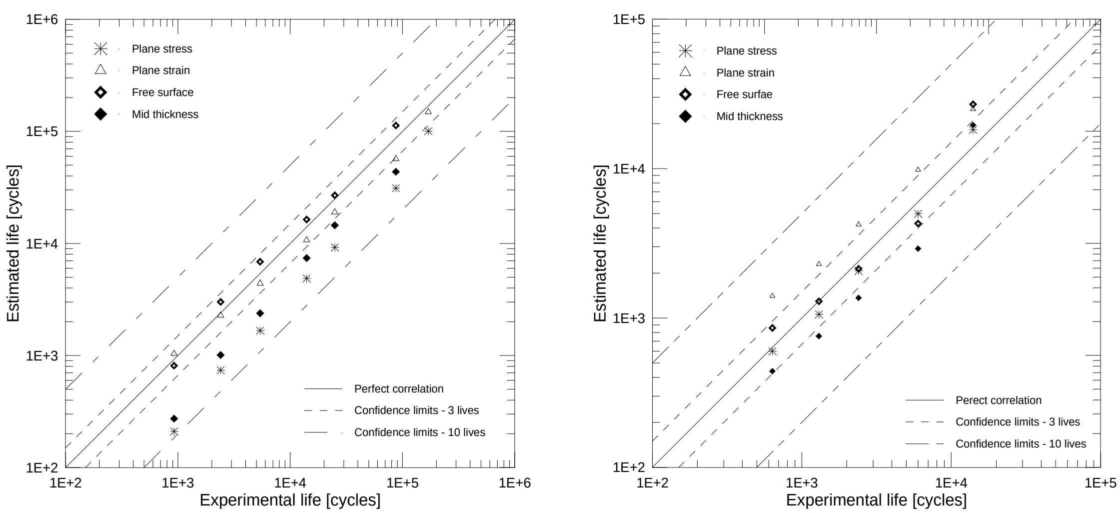
(a) (b)

Figure 4. K_t behavior along notch root for different t/r and H/h: (a) surface, point 1; (b) mid-thickness, point 2.

In order to evaluate the difficulties associated to the use of local strain elastoplastic models when associated to plane stress/strain hypothesis and 2D K_t magnitudes, the two cases of study presented on Table 4 were studied.

Preliminary analysis for the variations of K_t and α along the thickness of 3D finite element models evaluated in the linear elastic regime for cases of study A and B were performed and made evident interesting results. For K_t behavior along line 1-2, it was possible to notice significant variations of that factor with respect to 2D values for both cases of study. The values of K_t evaluated for the free surface were found about 6% and 15% lower than 2D values for, respectively, cases A and B. When evaluating the values obtained for the mid-thickness of the models, values of 8% e 5% higher than 2D values were assessed. Therefore, it was made evident that the unconditional use of bi-dimensional values of K_t could lead to imprecise estimates of stress and consequently, deviations in the fatigue life previsions for both cases of study. When analyzing α , significant variations of this factor where also noticed when compared to models with t/r = 0.1. Besides that, this ratio when evaluated for the free surface of the components, showed to be close to -0.3, indicating low degree of displacement constraint and plane stress state approximated. For the mid-thickness, this ratio tends to values close to zero, indicating increase in the level of constraint. However, values inferior to the ones observed for models with extremely high t/r were observed indicating differences between the stress states evaluated for the mid-thickness and the approximations of plane strain.

FEM estimates for fatigue life are presented for cases A and B, respectively, in Figures 5 (a) and (b) taking into account the evaluation at: (i) free surface of 3D model, (ii) mid-thickness, (iii) plane stress approximation and (iv) plane strain approximation (Menin,2004). This graphs also show perfect correlation line and confidence limits for 3/10 lives.



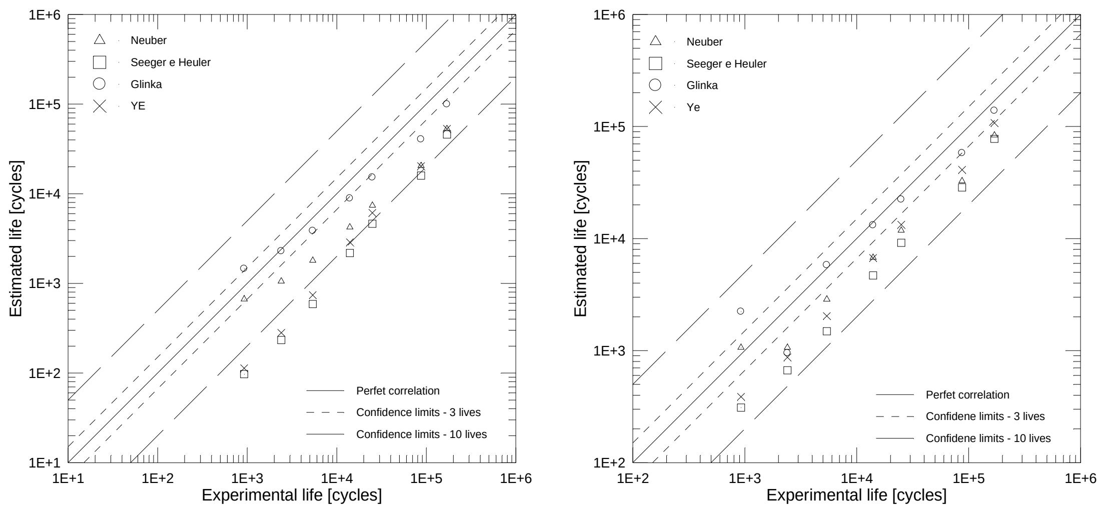
(a) (b)

Figure 5. FEM fatigue life results: (a) case A; (b) case B, (Table 4).

Analyzing the graph for study cases B it is possible to notice good agreement between lives estimated by deformation assessed at the free surface of 3D model and the experimental data. This fact suggests the validity of the hypothesis of greater probability of occurrence of material intrinsically imperfections although presenting lower levels of stress when compared to the mid-thickness. Estimates obtained from plane stress approximations showed to be conservative due to the fact that they were based in 2D values of K_t about 15% higher than the ones evaluated for the free surface. On the other hand, the plane strain approximation, despite the fact that leaded to relatively good results, were based on assumptions of a K_t value that do not represent in an adequate way, the value observed in the 3D model for the mid-thickness.

By means of Figure 5 (a) it is possible to analyze FEM results for case of study A. It is once again observable a high level of concordance between estimated lives for the free surface and the experimental obtained ones, as well as, that the estimates based on the results for the mid thickness are more conservative. For this specific case of study, the plane stress approximations showed to be coherent in respect to the ones obtained for the free surface of the 3D model because the deviations of K_t values evaluated for this two occasions are lower than 6%. On the other hand approximations of plane strain leaded to non conservative results because were based in a K_t value approximately 8% lower than the one observed for the mid-thickness of the model.

In order to exemplify the use of elastoplastic models in cases where it is not possible to use a FEM tool for the correct evaluation of K_t and α and when associated to components presenting expressive values of *t/r*, 2D values of K_t associated to plane stress/strain constitutive curves were used into the elastoplastic models to estimate local strain and fatigue life for the selected study cases. These results correlated to experimental data are exemplified for cases of study B associated to plane stress and plane strain hypothesis, respectively, in Figure 8 (a) and (b) (Bastos, 2004).



(a) (b)

Figure 8. Local strain models fatigue life results: (a) case B – plane stress; (b) case B – plane strain.

It is possible to observe that the estimates of the models taking into account plane stress approximation showed to be more conservative than the ones assuming plane strain approximations. However, it is important to remember that the free surface of the 3D model presented K_t values lower than the ones evaluated by Peterson (1997). This fact explains these conservative estimates despite the fact that the degree of displacement constraint evaluated for the free surface is similar to a plane stress hypothesis. Moreover, it is important to inform that, although the plane strain state hypothesis leaded to estimates close to the experimental, these estimates were obtained with values of K_t lower than the ones observed for the mid-thickness of 3D models and that the degree of displacement constraint evaluated for this location presents divergences to the plane strain hypothesis.

4. Conclusions and Final Comments

The scope of this research was to evaluate the difficulties related to the use of local strain models when associated to the cases of components presenting expressive values of dimensionless thickness. In that sense the analysis was divided in two distinct parts: (i) elastic domain evaluation of notches and (ii) elastoplastic domain evaluation of two study cases.

The results presented for the linear elastic domain analysis made it possible to evaluate that components with $t/r \approx 0$ presents low stress gradients with respect to its thickness. In that sense, the stress state and values of K_t associated to the free surface do not vary along its thickness and the use of plane stress approximations and bi-dimensional values of K_t are coherent to describe the stress concentration phenomena, as suggested by Dowling (1993).

The differences evaluated for the theoretical stress concentration factor at the free surface and the mid-thickness of models with t/r =1 e t/r = 1,5 was respectively, about 5% e 10% of 2D values, indicating the deviation expected when dealing with such approximations.

When evaluating models with expressive magnitudes of *t/r*, and with hypothesis of non existence of inherent imperfections, the nucleation of fatigue cracks is more probably to occur at the mid-thickness of the components due to the high levels of stress there present. By means of the analysis of the models thickness variations it was possible to conclude that the values observed for the K_t at the mid-thickness of the components are only up to 5% higher than 2D values. In that sense, it is possible to understand the motivation to use plane values of K_t for all thicknesses. However it is important to pay attention to two important facts:

(i) Although sufficiently thick models present K_t values for its mid-thickness tending to its 2D values, these could not be the most solicited points. Notch root points located somewhere between the mid-thickness and the free surface could be in a higher solicitation status, fact made evident by the K_t variations evaluated for models with $t/r \approx 20$.

(ii) the difficulty to obtain material completely disproved of inherent imperfections, mainly that ones due to fabrication processes. In cases where such imperfections are present, fatigue cracks would be more likely to occur at the free surface of the components. In these cases, despite the fact that 2D approximations of Kt are always on the conservative side, the use of such practice could lead to overestimates up to 20% with respect to values evaluates for the free surface. Therefore, the unconditional use of 2D values of K_t in cases involving models presenting expressive values of t/r could lead to imprecision and errors, as suggested by Zhenhuan et al (2000).

The results presented for the elastoplastic analysis of the two cases of study made evident the difficulties associated to the use of local strain approach in cases involving components with expressive values of the dimensionless thickness. Among these are the ones associated to the determination of correct values of K_t and the constitutive curve constants to be used along with the local strain models, the determination of the points subjected to the most severe conditions, as well as, the type of bi-dimensional approximation to be used with FEM.

The analysis suggested the necessity of 3D FEM evaluation to determine in a precise way the local strain and fatigue life of components with expressive values of t/r. Moreover, the results made evident that, in the impossibility to use such practice, one should be careful when using plane stress and strain approximations and 2D values of K_t because these could present divergences with respect to the real conditions evaluated for the models. However, in these cases, the use of 2D values of K_t associated to plane stress hypothesis showed to be conservative in both cases of study analysed.

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6. Responsibility notice

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