OPTIMIZATION OF AIRPLANE TAKEOFF PERFORMANCE

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Abstract. *The runway length of airports is one of the main parameters that restrict airplane operation. Takeoff, despite of being one of the most important flight phases, sometimes it is neglected because it is a short phase. This work aims to demonstrate the optimal airplane trajectory described during the takeoff run, with the objective to obtain a minimum runway length necessary for the takeoff procedure. This optimization uses the runway length as the objective function subject to penalizations with different weights on required rate of climb (ROC) after the takeoff. This problem is limited to twin-engine flight subject to any altitude and temperature condition. The control variable used in this problem is the elevator deflection, parameterized in function of velocity. The velocities mentioned at the certification regulations and the optimal solution for the problem are determined simultaneously. The proposed model to simulate the aircraft trajectory during takeoff is described by the equations of motion in the X and Z cartesian coordinates, by the moment equation in the Y direction, and by cinematic and geometric expressions. This kind of problem is described by a collection of nonlinear ordinary differential equations, which are complex and the solution hard to be found. Because of this, it's necessary to use robust computational tools to obtain the results. The solution is sub-optimal because of parametric control laws limitations, and because there is no guarantee that the objective function minimum is global. The results permit to obtain the sub-optimal elevator deflection control during the takeoff process that minimizes the distance. The result of this work has as objective improve the takeoff performance, using as criterion the minimum possible runway length without injure the airplane rate of climb (ROC). The airplane used as example in this work was developed and designed during an engineering specialization program that was supported by Empresa Brasileira de Aeronáutica (EMBRAER) and by Instituto Tecnológico de Aeronáutica (ITA), both situated in São José dos Campos, Brazil.*

Keywords: *Optimization, Parametric Control, Optimal Control, Optimal Performance, Takeoff Performance*

1. Introduction

The world aeronautic market claims of their airplanes exceptional performances, which can be expressed as security, quickness, comfort, flight quality, noise levels and operational costs. Any airplane must accomplish severe requirements if desire to maintain or increase its participation in the market share. This perspective must be performed by the uncountable airplane companies around the world or by the aeronautic industries as Airbus, Boeing and Embraer. The new airplanes to be introduced by the aeronautic industries must be adapted to the principles emphasized in this paragraph, because if not, the first consequence will be the commercial ruin of these airplanes in the market.

Takeoff, climb, cruise, descent and landing must be always executed trying to get the maximum efficiency available by the airplane. Takeoff in airports with short runways, landing in runways that requires a strong rate of descent, high gradients to transpose elevated obstacles near from the airports and low noise level are example of indispensable performance items for the operation of any aircraft in the contemporaneous world. The airplane flexibility to operate of any airport under any atmospheric condition, attempting to the noise level admitted, is the warranty of a wide gap and great life cycle in the aeronautic market.

The terrorist attack of September 11 brought a shadowy perspective in the aeronautic business. People began to feel afraid of fly because of the risk of new terrorist attacks. But, with new safety schemes implanted in the airports, and with the increase of the number of people that day by day need to use airplanes to travel, the aeronautic market is taking a new breath.

The technologies advances in the aeronautic area help strongly in the new generation of airplanes projects. Aeronautic companies and industries invest hard in new technologies aiming the best satisfaction of their clients and airplane quality. Equipments installed on board make easy the pilot work and improve the airplane performance in all flight phases. The cruise, because of being the longer flight phase, is the phase preferred by the aeronautic operators to extract the maximum airplane performance. In this phase the aeronautic operators seek to obtain the lower possible operation cost, through a relation between the cruise velocity (or Mach number) and the available time to fly the distance between the airports. The target is to reach a low combustible consumption without increase so much the airplane flight time. The others flight phases, though each one present his importance, do not allow a big minimization in the operation costs. Although, this phases define important factors that have relation with the airplane performance and that guarantee the accomplishment of the principles cited in the first paragraph. The takeoff, as an example, is considered as a critical flight phase because of the risks that this phase offers during its operation and after during the airplane climb. But, on the contrary, this phase is always neglected in terms of performance, because this is a short phase and it is not possible to reach a big cost reduction.

As discussed in the above paragraph, the aeronautic market demands that the airplanes must be capable of operate (takeoff and landing) in the biggest number of airports, serving the most different places. Restrictions about the operations in airports of extreme commercial importance could compromise enormously the airplane image in a so difficult market. By other side, if the airplane demonstrate that it is capable to reach great performances in airports of short runways, as it is the example of London City airport (England), this will open for any aeronautic industry a expressive market share which is placed by a short number of airplanes. Table 1 shows the principals characteristics of London city airport.

Lodon City Airport		Altitude (*ft*)	17
Runway	TORA (*m*)	TODA (*m*)	ASDA (*m*)
10	1199	1274	1319
28	1199	1385	1319

Table 1. London City Airport data (England)

As described in the above paragraph, the optimization of takeoff and landing performance for all kinds of commercial airplanes should be a question to be observed carefully by aeronautic companies and industries. The performance optimization in these cases aim to decrease the distance necessary to land or take off, or still, allow that the airplane take off with a higher weight than that it is normally used. In the first query, the gain will allow the operation in airports where this airplane would not be capable to operate. In the second query, the possibility of increase the payload will allow that the airplane carry a bigger number of passengers, what will consequently increase the profit of the company. In others words, how bigger is the number of airports that the airplane is capable to operate, bigger will be his flexibility and agility in the clients treatment and satisfaction. This certainly establishes a schedule above the competitors, and it is more one way to reduce costs and increase profits.

Figure 1. London City Airport (England)

This work proposes to optimize specifically the takeoff of commercial airplanes. For that it was created computational routines that tries to modeling the airplane behavior during the takeoff process. The airplane is described by an aerodynamic model that defines the principals force and moments components that actuate in the airplane, and the solution are obtained over the solution of differentials equations that describe the airplane dynamic. The model presents restrictions that are imposed by the aeronautic certification agencies, or by structural and physical airplane limitations. The objective function of the problem (function to be maximized or minimized) tries to obtain the lesser takeoff distance submitted to a penalization in the airplane rate of climb with different weights, and without injure the airplane restrictions.

The takeoff simulations were made with data of Corporate Jet 1 (CJ1). This airplane is the objective of a project developed by a group of engineers from an engineering specialization program (PEE) as a result of the partnership between the Empresa Brasileira de Aeronáutica (EMBRAER) and the Instituto Tecnológico de Aeronáutica (ITA). The Figure 2 shows drawing of the CJ1 airplane and its symbol.

Figure 2. CJ1 Airplane CATIA drawing

2. Methodology

In performance studies is usual to modeling the airplane as a punctual mass. In punctual mass model, the control used is the angle of attack. Without any additional hypothesis, this model admits instantaneously variation of angle of attack and nominal load factor.

Another way of modeling the airplane in performance calculus consists in consider it as a rigid body. The rigid body model is more complete and representative of the takeoff dynamic than the punctual mass model. The rigid body model takes in consideration the airplane dynamic pitch, and the control is the elevator deflection.

The equations that rule the dynamic of any body comes from the Newton's second law. All forces and moments are translated for the body axes and the variations refer to the inertial system.

$$F = \frac{d\left(mV\right)}{dt} \quad \text{and} \quad M = \frac{dH_a}{dt} \qquad \text{Equation (1)}$$

The model adopted for this study considers the airplane as a rigid body symmetric along the XZ plan. This way, the airplane is controlled by aerodynamic surfaces, that in this case is only the elevator. Airplane forces and moments are from aerodynamic, propulsive, gravitational and earth reactions (normal forces) sources. It's assumed that the thrust acts along the X direction. The gravitational field is assumed uniform, and as consequence the gravity and mass centers are coincident, and there are not gravity acceleration variations with the altitude.

No lateral control is used because the only objective is to simulate the longitudinal movement. The pitch angle control, obtained through the elevator deflection, is used to rotate the airplane before the lift off and to maintain the speed during the climb procedure.

Some restrictions are imposed directly or indirectly in the elevator deflection, over and above the maximum and minimum deflections limits. These restrictions are from aerodynamic, structural and operational nature, whose objective priors do not exceed the load factors and the maximum positive and negative pitch speeds.

The takeoff procedure is divided in this study in three distinct phases. The first phase consists in the ground run until the airplane reaches the rotation speed (*Vr*). At this point the airplane begin to rotate around the principals landing gear until that a specific angle of attack is obtained. The second phase finishes exactly when the airplane reaches the speed that the principal landing gears lift off from ground. At this point the airplane assumes an angle of trajectory that will determinate the rate of climb (*ROC*). The third phase finishes in the moment that the lower airplane point reaches a height of 35 *ft*, complying with the certification requisites imposed by FAR-25.

Aerodynamic moments and forces acting in the airplane are characterized in terms of no dimensional coefficients. These coefficients are calculated through expressions that involve the airplane geometry, including the control surfaces deflections like flaps and elevator. The no dimensional coefficients are used in the state equations that describe the airplane dynamic.

The air density and the sound speed are calculated using the international standard atmosphere model (ISA).

The airplane movement equations are first order nonlinear ordinary differential equations. How there is no way to obtain an exact solution, it is necessary to use numerical integration methods. For this it was used Matlab algorithms that are based in second and fourth order Runge-Kutta methods.

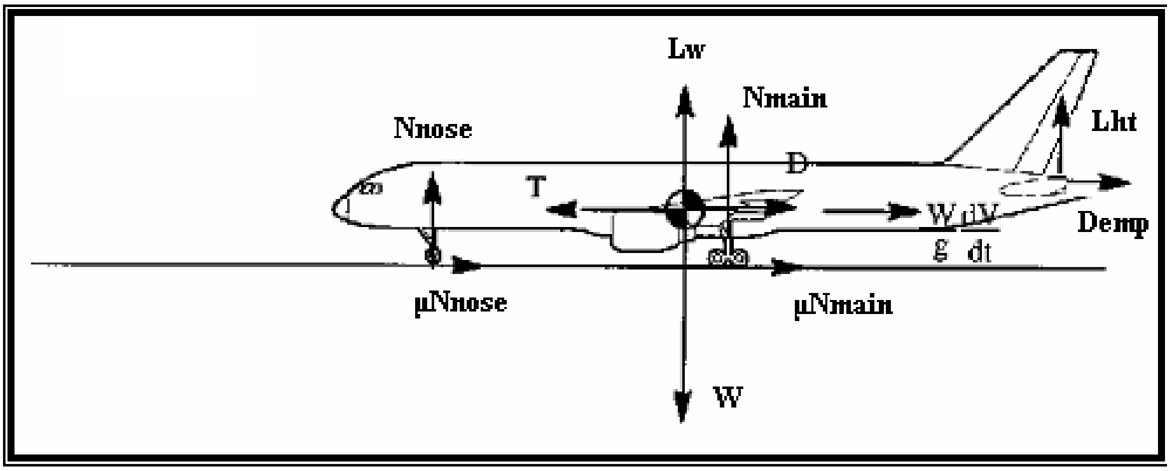

Figure 3. Forces that acts in the airplane during the takeoff process

The airplane trajectory during the takeoff is simulated by the translation movement equations in Cartesian coordinates X and Z, by the movement equation around the axes Y, and by cinematic and geometrical relations. The same group of equations describes all takeoff phases.

The translation and rotation movement equations that compose the airplane dynamic are displayed in Eq. (2), Eq. (3) and Eq. (4).

$$n_{eng} T\cos(\alpha) - D - W sen(\gamma) - \mu N_{main} - \mu N_{nose} = m \frac{dV}{dt}$$ Equation (2)

$$n_{eng} T sen(\alpha) + L + N_{main} + N_{nose} - W\cos(\gamma) = mV \frac{d\gamma}{dt}$$ Equation (3)

$$\begin{aligned} &M_0 + M_{0ht} - L_w x_1 - L_{ht} x_2 + n_{eng} T\cos(\alpha) x_3 \\ &+ n_{eng} T sen(\alpha) x_4 + D_{emp} x_5 - D_{np} x_6 - D_{Gear} x_7 \\ &- N_{main} x_5 + N_{nose} x_6 - \mu N_{main} x_7 - \mu N_{nose} x_8 = I_y \frac{dq}{dt} \end{aligned}$$ Equation (4)

The equations system is completed with the geometrical and cinematic relations displayed in Eq. (5), Eq. (6) and Eq. (7).

$$q = \frac{d\alpha}{dt} + \frac{d\gamma}{dt}$$ Equation (5)

$$V\cos(\gamma) = \frac{dX}{dt}$$ Equation (6)

$$V sen(\gamma) = \frac{dH}{dt}$$ Equation (7)

The algorithm used in the search of the optimal solution for the problem is based in the sequential quadratic programming (SQP). In this method, quadratic sub problems (QP) are created, whose solution is used to indicate the search direction of the optimal point. This method presents algorithms that approximate the gradients and the Hessian matrix values of the objective and the restriction functions based in Quasi-Newton method.

Optimization problems typically involve performance index and restriction functions. The objective is to find maximum and minimum values for the performance index without injure the restrictions and limitations imposed by the physic of the problem.

The optimization method used here is of parametric type, where the elevator control is approximated by a known function, although with unknown coefficients. One of the problems of the parametric optimization is the tendency to find the local optimal solution instead the global optimal solution. It can be added the effect of the restrictions that sometimes make difficult the determination of the optimal solution.

In this problem, the performance index (objective function) is composed by the takeoff distance necessary to the airplane reach a height of 35 *ft*, above the ground, and a penalization term over the rate of climb at a height of 35 *ft*. The penalization technique is applied in the objective function as displayed in the Eq. (8).

$$f_{objetivo} = S_{TO} + K\left|\left(ROC - ROC_{ótimo}\right)\right| \qquad \text{Equation (8)}$$

The airplane elevator works between a deflection of -25º (pitch up) and 15º (pitch down). The elevator control is modeled (parameterized) as a polynomial division of degree 2, which the only parameter presented is the airplane speed. The elevator control function is displayed at the Eq. (9).

$$\delta_p = \frac{a_0 + a_1 V + a_2 V^2}{1 + a_3 V + a_4 V^2} \qquad \text{Equation (9)}$$

The V_R, V_{LOF} and V_{35} speeds, and so the coefficients of the equation above can vary between maximum and minimum values, fixed in a convenience manner, so that the optimization process finds the optimal value of each variable that will make the elevator control to be optimal. The optimization process consists of successive minimizations until that the process finds the solution that attends the objective functions requisites, but that obeys the control limitations, the airplane restrictions and the certification authorities requisites.

The restrictions group imposed to the takeoff optimization problem is presented in the Tab. 2.

Table 2. State and control restrictions

Variable	Minimum Value	Maximum Value
δ_p	-25°	15°
H	35 *ft*	-
α	0°	9.5°
Θ	0°	15°
V_R	1.07 V_S	-
V_{LOF}	V_R	V_{35}
V_{35}	1.25 V_S	-
Φ_{35}	5.6%	-

In this work, the optimization calculus is considered only for the all engines operating takeoff case submitted to any temperature, altitude, weight and flap deflection condition. Its geometry, aerodynamic coefficients and thrust tables define the airplane.

3. Results

The programming language used to run the numerical simulations of this work is the MATLAB version 6.5 release 13. This study uses the routine "fmincon.m" to obtain the optimal elevator control during the takeoff process.

The penalization constant (*K*), presented in the objective function (Eq. (7)), will determine the importance that the rate of climb (*ROC*) has in the problem. Low values of *K* will give importance just for the minimization of the takeoff distance. Although, high values of *K* will make the program calculate a solution that should attend the objective of make the rate of climb near the desired value, and also trying to minimize the takeoff distance.

A group of simulations was defined to evaluate the algorithm "takeoff.m". These simulations involve variations of the objective function (different values of *K*), flap angle, airplane mass, atmospheric conditions and airport altitude. Figure 4 presents graphically the results obtained with this program for three of these simulations executed (flap angle 20°, sea level, ISA, airplane mass 56515 *kg* and *K* equal to 0.0, 0.5 and 1.0).

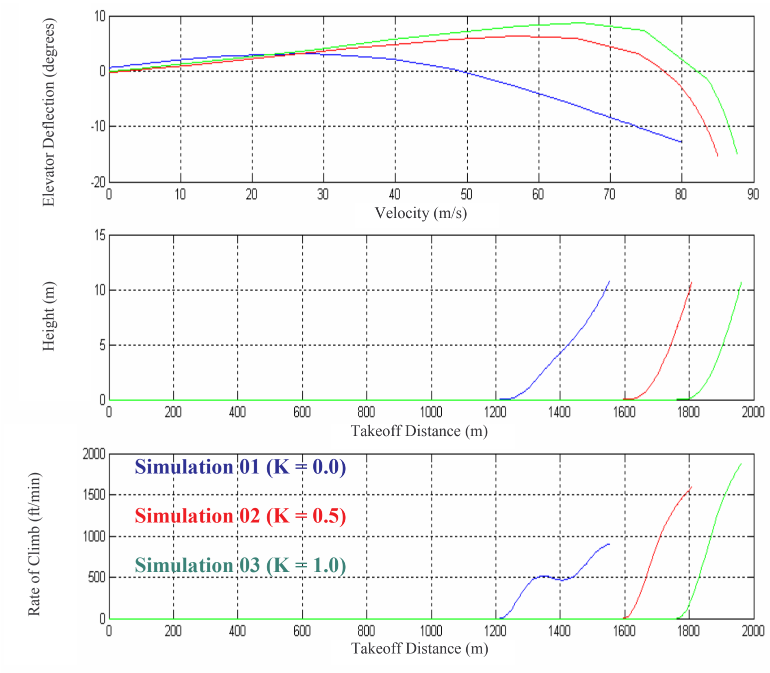

Figure 4. Optimization results for simulation 01, 02 and 03

4. Results discussion

The Figure 4 displays three of approximately 30 simulations executed. Even that these others results are not showed in this paper, its results will be discussed in the next paragraphs.

Standard takeoff procedure consists of select an adequate rotate speed, and deflects the elevator to a determined angle just when this speed is reached. For a group of rotate speeds and elevator deflections, it was selected the combination that presents the best performance accomplishing with all restrictions of the problem. Comparing the optimal control for this case with the standard takeoff procedure, the rotate speed was reduced in 17.53%, what causes a minimization of the takeoff distance at about 22.37%. This comparison tries to demonstrate the effective gain in the takeoff performance with the optimization algorithm.

The objective function is composed by two terms, the first is the takeoff distance, and the second is the rate of climb, which serve as penalization factor. The penalization factor can have a low or high effect in the objective function according to the value given for the penalization constant. The results shows that for a higher penalization constant, the rotate speed is elevated, and the elevator deflection is more abrupt. Comparing the simulations with K = 0.0 and K = 1.0, the last case has the rotate speed is increased in 20.35%, the takeoff distance is increased in 26.27%, but the rate of climb was increased too in 108.58%. This comparison tries to demonstrate the effect that the pilot can make in the rotate speed with the objective of get better rates of climb.

Higher flaps deflections decrease the takeoff distance, but there is a decrease too in the rate of climb due the increase of the drag force. Generally, higher flaps deflections are applied only in airports that have takeoff runways whose length is short. Comparing the takeoff performance with a flap deflection of 20° and 7°, the last case has the takeoff distance is increased in 28.08%. In the other side, the rate of climb is increased in 11.59%. This comparison has the objective of verify the influence of the flap deflection in the compromise between the takeoff distance and the airplane rate of climb.

If the airplane mass is increased, the lift force necessary to lift off the airplane will be increased too. How the aerodynamic forces are proportional to the square of velocity, the airplane only will be capable to lift off with a superior velocity than one used for the lesser weight. Comparing data between a takeoff with mass of 56515 *kg* and 41110 *kg*, it can be visualize that the elevator deflection is more abrupt for the case for less mass. This will result in a rotate speed decreased of 13.30%, and sure, in a takeoff distance decreased of 27.25%. This comparison tries to discuss the influence of the increase of weight and its respective impact in the takeoff performance.

The temperature increase has as consequence an expansion of the gases that compose the atmosphere, and because of this, the atmospheric density has its value decreased. How the lift force is proportional to the atmospheric density, the speed necessary for the airplane lift off the ground, for the same takeoff weight, will be higher for the case of superior temperature. The temperature increase also decreases the aeronautic motor efficiency, what means that the thrust available is lesser for the case of higher temperature. Comparing a takeoff in *ISA+0°C* and *ISA+30°C*, it can be analyzed that there is a performance reduction with the increase of the temperature. The takeoff distance is increased in

26.81%, while the rotate speed is increased in 7.84%. This comparison has as objective presents how many is penalized the takeoff performance for the cases with higher temperature.

The altitude has a similar effect as the temperature. In higher altitudes, the air is more rarefied, and as consequence the atmospheric density is lesser. The aeronautic motor efficiency is decreased with higher altitudes, what means a decrease in the thrust available. Confronting the optimizations executed at sea level and an altitude of 2000 *m*, the takeoff distance for the last case is penalized in about 44.72%, while the rotate speed is increased in 15.01%. this comparison tries to demonstrate the expressive effect caused by the altitude in the takeoff performance.

The elevator deflection, used as parameter to optimize, responses for all variations of objective functions, and for combinations between flaps deflection, takeoff weights, temperatures and airports altitudes. The comparison with standard takeoff procedure shows a takeoff performance gain in order of 20% (takeoff distance). The elevator deflection, in a great number of cases, assumes a value approximately constant and near of zero during the first instants of takeoff, assuming a higher deflection in speeds near the rotate speed. This fact is acceptable, since the elevator is only effectively in the moments before the airplane turn around the principal landing gear (drag and lift are function of the square velocity).

Finishing, this optimization allows to the user obtain the elevator deflection optimal control for different airplane configurations and environment situations, given to him the option to choose the takeoff performance level desired through the formulation of the objective function.

5. Conclusions

The takeoff performance optimization algorithm was tested exhaustively. Many optimizations were tested involving changes in the principals software input data. These simulations permit to verify the software robustness and the results coherence, over and above proportionate the elimination of some mistakes that were presented during the development phase. The simulations also have as objective to prove the optimization efficiency. Results of standard takeoffs demonstrated that the optimized takeoff reduces in approximately 20% the takeoff distance.

The simulations of objective functions modified through the penalization factor demonstrate the versatility of the software to obtain optimal results to various objectives. Optimizations between objective functions of opposed purposes demonstrate a difference in the rate of climb of about 100%.

The optimization control vector is parameterized by a polynomial division of degree two. Even not being the best parameterization form, the results obtained were in accordance with what was expected. Future studies will try to test new parameterized forms (trigonometrically, logarithmical and exponential), since then using the results obtained now it can be better estimated the elevator behavior.

Future works can complete the model with the landing gear dynamic. The landing gear can be analyzed as a mass-spring damper system, what permits displacements in Z axe while the flats still touch the ground.

The simulations made in this study just work with takeoff cases of all engines operating. Future works can improve the algorithm with a model and restrictions that permits the optimization for the cases of one engine inoperative.

The results are considered sub optimal, because it was not used any global strategy by the optimization algorithm (gradients and Hessians based methods). The solution is also said sub optimal because the parametric function used do not represent exactly the elevator behavior.

The CJ1 airplane takeoff distance optimization permits increase the number of airports in which the airplane is capable to operate. This means more flexibility and agility to serve its clients. In a market so difficult and disputed like in nowadays, this constitute a fantastic differential to the market.

6. Acknowledgements

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8. Simbology

a_n - Control Coefficients
ASDA - Accelerate Stop Distance Available
D - Drag Force
D_{emp} - Tail Drag Force
D_{Gear} - Landing Gear Drag Force
D_{np} - Nacelle an Pylon Drag Force
$f_{objetivo}$ - Objective Function
F - Force
g - Gravity Acceleration
ISA - International Standard Atmosphere
H - Height
K - Penalization Factor
k_n - Parametric Coefficient
L - Lift force
L_{ht} - Horizontal Tail Lift Force
L_w - Wing Lift Force
m - Mass
M - Pitch Moment
M_0 - Wing Pitch Moment
M_{0ht} - Horizontal Tail Pitch Moment
n_{eng} - Number of Engines
N_{main} - Main Landing Gear Normal Force
N_{nose} - Nose Landing Gear Normal Force
q - Pitch Velocity
ROC - Rate of Climb
$ROC_{ótimo}$ - Optimal Rate of Climb
S_{TO} - Takeoff Distance
t - Time
T - Thrust
TOD - Takeoff Distance
TODA - Takeoff Distance Available
TORA - Takeoff Run Distance Available
V - Velocity
V_{LOF} - Lift-off Speed
V_R - Rotation Speed
W - Weight
x_n - Distance between the Force and the Airplane CG
α - Angle of Attack
δ_p - Elevator Deflection
γ - Flight Path Angle
μ - Ground Braking Friction Coefficient
φ - Climb Gradient
φ_{35} - Climb Gradient at 35 ft Height
θ - Pitch Attitude Angle

9. Responsibility notice

The authors are the only responsible for the printed material included in this paper.