PARAMETRIC OPTIMIZATION OF INTEGRALLY MACHINED PANELS REGARDING FOUR STABILITY CRITERIA

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Abstract. *The aim of this article is to establish the optimal geometry of integrally machined panels regarding four stability criteria. Initially the panel is idealized as a structural component subjected only to compressive loads. Its geometry is defined as a function of five design variables: panel length, web width, flange height, web thickness and flange thickness. The parametric optimization is performed achieving the most effective geometry in attendance to four stability criteria that concur simultaneously to the physical phenomenon under study: section crippling, web buckling, flange buckling and column collapse. Results of parametric optimization for different panels lengths demonstrate that the panel geometry is scalable regarding the compressive allowable stress and the structure still presents an elastic behavior. FEM buckling analysis validates this theoretical result. Despite the assumed hypothesis, the resultant geometry can be very helpful as a baseline on the initial dimensioning of aeronautical structures – especially for wing upper skin design.*

Keywords*: structures, stability, optimization*

1. Introduction

It is known that if a panel is critical for crippling and not for buckling, it could be substituted for an arrangement of panels with reduced cross section area, minimizing the global volume and increasing the available payload. On the other hand, if a panel is critical for buckling and not for crippling, it could have its length reduced in such a way to increase its resistance, and since the applied load is the same, the cross section area could be reduced, minimizing the global volume and increasing the available payload again. I.e., the optimal structure is the one that attends the equilibrium between concurrent design criteria.

The aim of this article is to establish the optimal geometry of integrally machined panels regarding four stability criteria. Initially the panel is idealized as a structural component subjected only to compressive loads. Its geometry is defined as a function of five design variables: panel length, web width, flange height, web thickness and flange thickness. The parametric optimization is performed achieving the most effective geometry in attendance to four stability criteria that concur simultaneously to the physical phenomenon under study: section crippling, web buckling, flange buckling and column collapse.

Despite the assumed hypothesis, the resultant geometry can be very helpful as a baseline on the initial dimensioning of aeronautical structures – especially for wing upper skin design.

2. Development

Geometry and Hypothesis: The panel is idealized as a structural component subjected only to axial compressive loads – this is the only assumed hypothesis. The panel geometry is defined as a function of five design variables: panel length L, web width bs, flange height bw, web thickness ts and flange thickness tw. Figure 1 presents a sketch of the panel section as well as its reference coordinate system.

Material: The AL7050-7451 Plate 2.75” is assumed for the purposes of this study. Its material properties are: elasticity modulus E = 71020 MPa, Poisson coefficient ν = 0.33 and yield compressive stress F_{cy} = 421 MPa.

Section Crippling: The crippling allowable stress is calculated using Gerard Method. Gerard (1958) presents the results of a comprehensive study on the subject of crippling stresses. Equation (1) describes the crippling allowable stress F_{cc} of an arbitrary section, where: F_{cy} is the yield compressive stress, E is the elasticity modulus, A is the section area and t is the section thickness.

$$\frac{F_{cc}}{F_{cy}} = \beta \left[\left(\frac{gt^2}{A} \right) \sqrt{\frac{E}{F_{cy}}} \right]^m \tag{1}$$

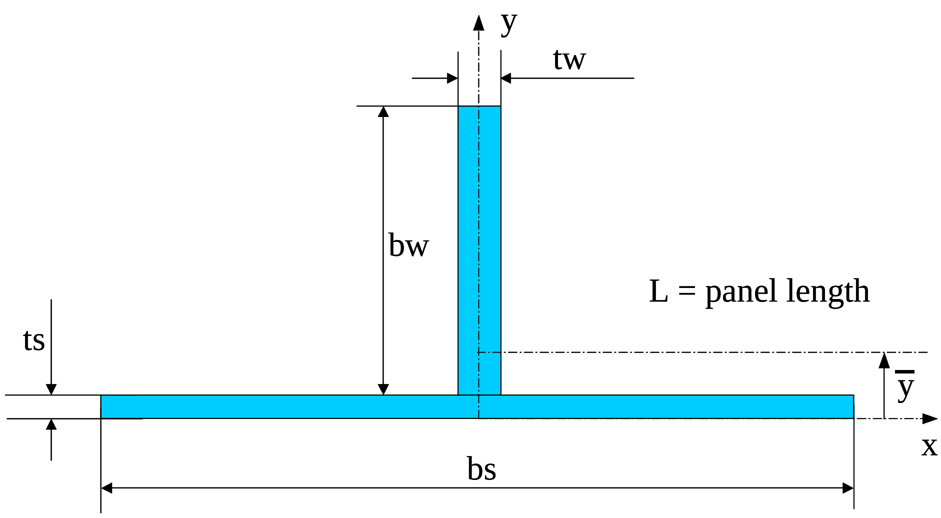

Figure 1 – Panel Geometry

The remaining variables of Eq. (1) are dependent on specific section shapes and considering the panel geometry – a T-section with straight unloaded edges – they result in g = 3, β = 0.67 and m = 0.40. Since the panel geometry presents independent web thickness ts and flange thickness tw, the mean section thickness t is calculated as presented in Eq. (2). Additionally Gerard recommends the cut-off of crippling allowable stress at 0.8 F_{cy}.

$$t = \frac{t_w b_w + t_s b_s}{b_w + b_s} \tag{2}$$

Web buckling and flange buckling: The web and the flange are analyzed as simple supported plates under axial loads in accordance with plate theory, Timoshenko and Gere (1961). Figure 2 presents the geometry of a generic plate and its loads, where b is the element width and a is the element length.

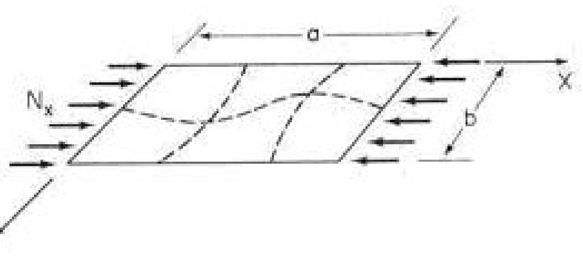

Figure 2 – Simple Supported Plate under Axial Loads

Equation (3) presents the calculation of buckling failure stress F_{cb}, where E is the elasticity modulus, ν is the Poisson coefficient, t is the thickness and K_c is the buckling coefficient – dependent on the plate boundary conditions.

$$F_{cb} = \frac{K_c \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \tag{3}$$

Equation (4) presents the calculation of the buckling coefficient K_c for a simple supported plate – or the panel web – where m represents the sequence of integers from 1 up to infinity. So, Eq. (4) defines a series of $K_c(m)$ values for each ratio a/b of the panel. The effective buckling coefficient K_c is the lowest value of this series; and K_c = 4.0 can be assumed for practical purposes if a/b > 4.

$$K_c = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 \tag{4}$$

Equation (5) presents the calculation of the buckling coefficient K_c for a simple supported flange, due to Lundquist & Stowell.

$$K_c = \frac{6}{\pi^2}(1-\nu) + \left(\frac{b}{a}\right)^2 \quad (5)$$

Equation (6) results from equalizing the buckling allowable stress of web F_{cb_web} to the buckling allowable stress of flange F_{cb_flange}, where L is the panel length, bs is the web width, bw is the flange height, ts is the web thickness and tw is the flange thickness – as defined in Fig. 1.

$$4\left(\frac{t_s}{b_s}\right)^2 = \frac{6}{\pi^2}(1-\nu)\left(\frac{t_w}{b_w}\right)^2 + \left(\frac{t_w}{L}\right)^2 \quad (6)$$

It must be observed that Eq. (6) leads to a dependency of the flange height bw on the remaining variables of panel geometry (L,bs,ts,tw) – described in Eq. (7) – as a result of the concurrence of web and flange buckling design criteria.

$$b_w = \frac{t_w}{\pi}\sqrt{6(1-\nu)}\left[4\left(\frac{t_s}{b_s}\right)^2 - \left(\frac{t_w}{L}\right)^2\right]^{-1/2} \quad (7)$$

Column collapse: The panel is also verified as simple supported column under axial loads in accordance with Euler theory, Bruhn (1973), Chapter A18 *'Instability of Columns'*. The Euler formula for the column allowable stress F_{cr} is presented in Eq. (8) where (L'/ρ) is the slenderness ratio.

$$F_{cr} = \frac{\pi^2 E}{\left(L'/\rho\right)^2} \quad (8)$$

In the case of a simple supported panel, the slenderness ratio is a pure geometrical property whose calculation is fully described in Eq. (9) up to Eq. (12) – Bruhn (1973), Chapter A3 *'Properties of Sections'* – and referred to Fig. 1.

$$\left(\frac{L'}{\rho}\right)^2 = \frac{L^2 A}{I_x} \quad (9)$$

$$A = t_w b_w + t_s b_s \quad (10)$$

$$\bar{y} = \frac{1}{A}\left[\frac{t_s^2}{2}(b_s - t_w) + \frac{t_w}{2}(b_w + t_s)^2\right] \quad (11)$$

$$I_x = \frac{t_s^3}{3}(b_s - t_w) + \frac{t_w}{3}(b_w + t_s)^3 - A\bar{y}^2 \quad (12)$$

Optimization: The problem consists in establishing the optimal geometry of integrally machined panels regarding four stability criteria.

The panel was idealized as a structural component subjected only to compressive loads. Its geometry is defined as a function of five *Design Variables*: panel length L, web width bs, flange height bw, web thickness ts and flange thickness tw. The parametric optimization is performed in order to achieve the most effective geometry in attendance to four stability criteria that concur simultaneously to the physical phenomenon under study: section crippling, web buckling, flange buckling and column collapse.

The *Objective Function* is to maximize the panel effectiveness Eff as defined in Eq. (13). It must be observed that the buckling allowable of the flange F_{cb_flange} doesn't appear explicitly in the *Objective Function* since it is implicitly taking into account by Eq. (7) which describes the dependency of the flange height bw on the remaining variables of the panel geometry (L,bs,ts,tw).

$$\begin{cases} \max\left[Eff(L,b_s,b_w,t_s,t_w)\right] \\ Eff(L,b_s,b_w,t_s,t_w) = \dfrac{1}{\left|F_{cb\_web} - F_{cc}\right| + \left|F_{cr} - F_{cc}\right|} \end{cases} \tag{13}$$

The assumed *Design Space* is presented in Eq. (14).

$$\begin{cases} L = 1,10,100,1000mm \\ 0.1L \le b_s \le 0.5L \\ 0.005b_s \le t_s \le 0.2b_s \\ 0.5t_s \le t_w \le 2t_s \end{cases} \tag{14}$$

3. Results

Optimal geometry of panels: The results of the optimization problem stated in the previous item are reported in Tab. 1.

Table 1 – Establishing the Optimal Geometry for Different Panel Lengths

Integrally Machined Panel Optimization:						AL7050-T7451 Plate 2.75"			
Eff	**Fcb_web**	**Fcb_flange**	**Fcc**	**Fcr**	**L**	**bs/L**	**bw/bs**	**ts/bs**	**tw/ts**
388.36	278.0	278.0	278.0	278.0	1.0	0.1832	0.3684	0.03256	1.1480
388.36	278.0	278.0	278.0	278.0	10.0	0.1832	0.3684	0.03256	1.1480
388.36	278.0	278.0	278.0	278.0	100.0	0.1832	0.3684	0.03256	1.1480
388.36	278.0	278.0	278.0	278.0	1000.0	0.1832	0.3684	0.03256	1.1480

It is important to notice that the geometric ratios, as presented in Eq. (15), obtained by the optimization procedure are independent on the panel length. So, the result is scalable regarding the compressive allowable stress F_c.

$$\begin{cases} b_s/L = 0.1832 \\ b_w/b_s = 0.3684 \\ t_s/b_s = 0.03256 \\ t_w/t_s = 1.1480 \end{cases} \tag{15}$$

Table 2 – Additional Results of Optimization

Results:					
Fcy =	421.0	**MPa**	**bs =**	0.1832	**L**
Fc =	278.0	**MPa**	**bw =**	0.06749	**L**
Fc =	66.0%	**Fcy**	**ts =**	0.005965	**L**
Pc =	0.43224	**L^2 N**	**tw =**	0.006848	**L**
Pc/bs =	2.35936	**L N/mm**	**A =**	0.001555	**L^2**

Optimal Geometry of Panels:				L = 100.0mm			
L	**bs**	**ts**	**bw**	**tw**	**A**	**y**	**Ix**
100.0	18.32	0.5965	6.749	0.6848	15.55	1.390	61.67

Some additional results of this optimization problem are presented in Tab. 2. The compressive allowable stress – i.e. the minimum of (F_{cc}, F_{cb_flange}, F_{cb_web}, F_{cr}) – is F_c = 278.0 MPa for the assumed material, AL7050-7451 Plate 2.75". It must be observed that this compressible allowable stress F_c corresponds to 66.0% of the yield compressive stress F_{cy}.

So, the structure still presents an elastic behavior and is not necessary modeling the tangent modulus of elasticity E_t in order to take plasticity effects into account. A final observation is that the optimal geometry of the panels is dependent on material properties (E, ν, F_{cy}).

Numerical validation: A finite element model of integrally machined panel considering its optimal geometry is constructed in order to validate this theoretical result, as presented in Fig. 3.

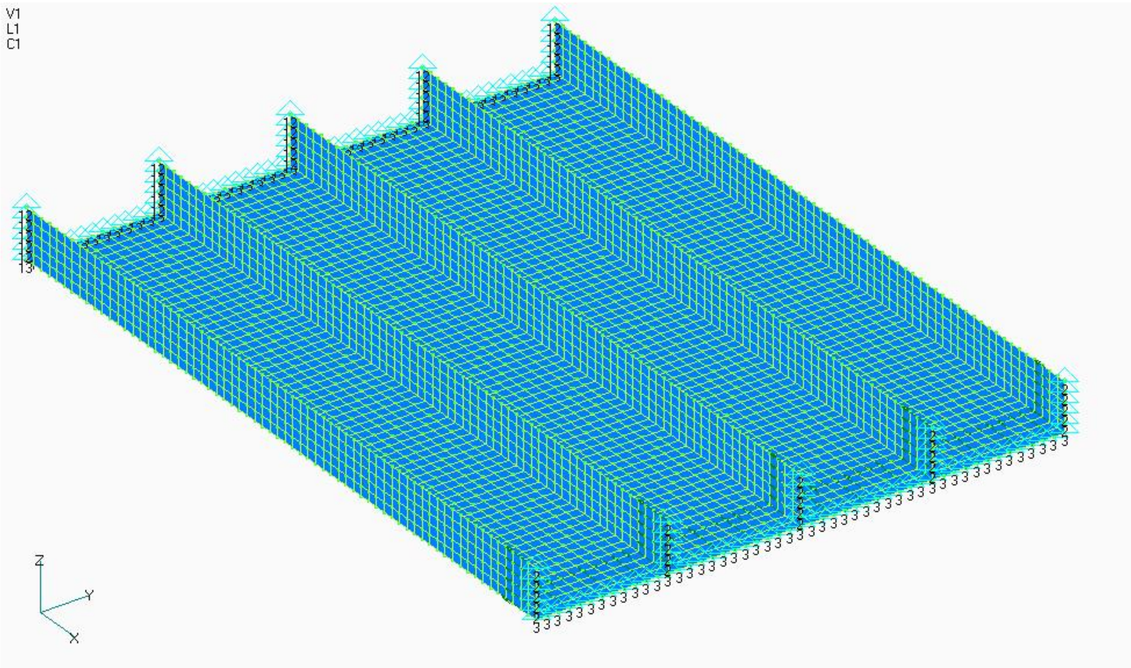

Figure 3 – FEM Model of Integrally Machined Panel

This computational model was specially prepared for buckling analysis, i.e. Nastran solution 105, and only shell elements CQUAD4 (isoparametric quadrilateral FE) were used, The MacNeal-Schwendler Corp. (1994). The geometry, material data, applied loads and boundary conditions are described in Tab. 3, as well as the analysis results.

Table 3 – FEM Analysis Data

A) Material:	AL7050-T7451 Plate 2.75"	
E =	**71020**	**MPa**
ν =	**0.33**	**[-]**

B) Optimal Geometry of Panels:				**[mm]**
L	**bs**	**ts**	**bw**	**tw**
200.000	**36.6400**	**1.19300**	**13.4972**	**1.36956**

C) Compressive Load:		
Fc =	**278.0**	**MPa**
	174.8	mm^2 = web section area
	49	grids along web width
	991.9	**N per web grid**
	92.4	mm^2 = stringer section area
	30	grids along stringer height
	856.4	**N per stringer grid**
A =	267.3	mm^2
Pc =	**74296**	**N**

D) Loads & Boundary Conditions:
Web UX and UZ constrained in X = 0.0 mm
Flange UX and UY constrained in X = 0.0 mm
Negative FX applied in X = 200.0 mm
Web UZ constrained in X = 200.0 mm
Flange UY constrained in X = 200.0 mm

E) Results:	
First Eigenvalue =	**1.037988**
Second Eigenvalue =	**1.059703**
Third Eigenvalue =	**1.079122**
Fourth Eigenvalue =	**1.116983**
Fifth Eigenvalue =	**1.130286**

F) Numerical x Theoretical Results Comparison:		
Pc_num =	**77119**	**N = Pc x First Eigenvalue**
Pc_theo =	**74296**	**N**
Delta =	**3.80%**	**= (Pc_num - Pc_theo)/Pc_theo**

The theoretical critical load of the panel is P_{c_theo} = 74296 N, while the obtained numerical result is P_{c_num} = 77119 N, i.e. an error of only 3.8% is observed. Figure 4 up to Fig. 6 present the three first failure modes of the structure. The simultaneous collapse of the panel regarding different stability criteria, as predicted by theory, can be verified in these figures.

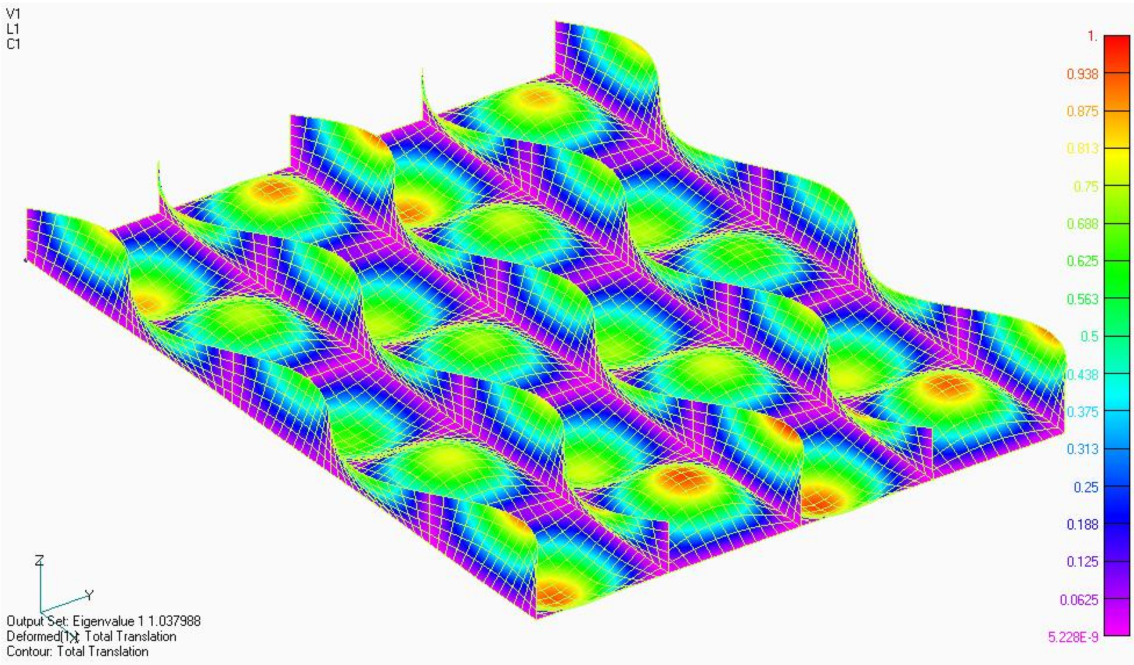

Figure 4 – Buckling Analysis, First Failure Mode

4. Conclusions

This paper established the most effective geometry of integrally machined panels under compressive loads in attendance to four stability criteria that concur simultaneously to this physical phenomenon: section crippling, web buckling, flange buckling and column collapse.

The most important results of parametric optimization are: (a) The panel geometry is scalable regarding the compressive allowable stress F_c. (b) The compressive allowable stress is F_c = 278.0 MPa for the assumed material, AL7050-7451 Plate 2.75". (c) The structure still presents an elastic behavior, F_c = 66.0% F_{cy}.

FEM buckling analysis validated this theoretical result. An error of only 3.8% was found for the critical load of the panel. The simultaneous collapse of the panel regarding different stability criteria, as predicted by theory, could be verified.

Despite the assumed hypothesis, i.e. the panel is subjected only to axial compressive loads, the results presented herein can be very helpful as a baseline on the initial dimensioning of aeronautical structures – especially for wing upper skin design.

In fact, this hypothesis doesn't properly represent a limitation since the wing panels are subjected to three main load components: the direct compression induced by bending of the entire section, the shear flow caused by wing box torsion, and local bending effects induced by aerodynamic pressure; being the compressive load the most significant one. And still some authors recommend optimizing the cover panel only for compression loading by maintaining a 10-15% allowance for taking into account these secondary effects, Niu (1993), Chapter 8 *'Wing Box Structure'*.

Another important observation is that the development presented herein doesn't take into account the lateral constraint of the panel set by the wing spars, i.e. the approach is exact for a infinite set of panels, being conservative otherwise.

The state-of-art in pre-dimensioning of integrally machined panels, e.g. Abdo, Piperni and Kafyeke, (2003), proposes the simultaneous optimization of the complete set of panels regarding experimental stability data. This procedure conduces to a large amount of design variables, since there are five geometric parameters per panel. The proposed approach presents three main advantages in this sense: (a) Most of the background is theoretical (except Gerard Method that is a semi-analytical/experimental result). (b) Simpler formulas. (c) The number of design variables is approximately reduced by a factor of five.

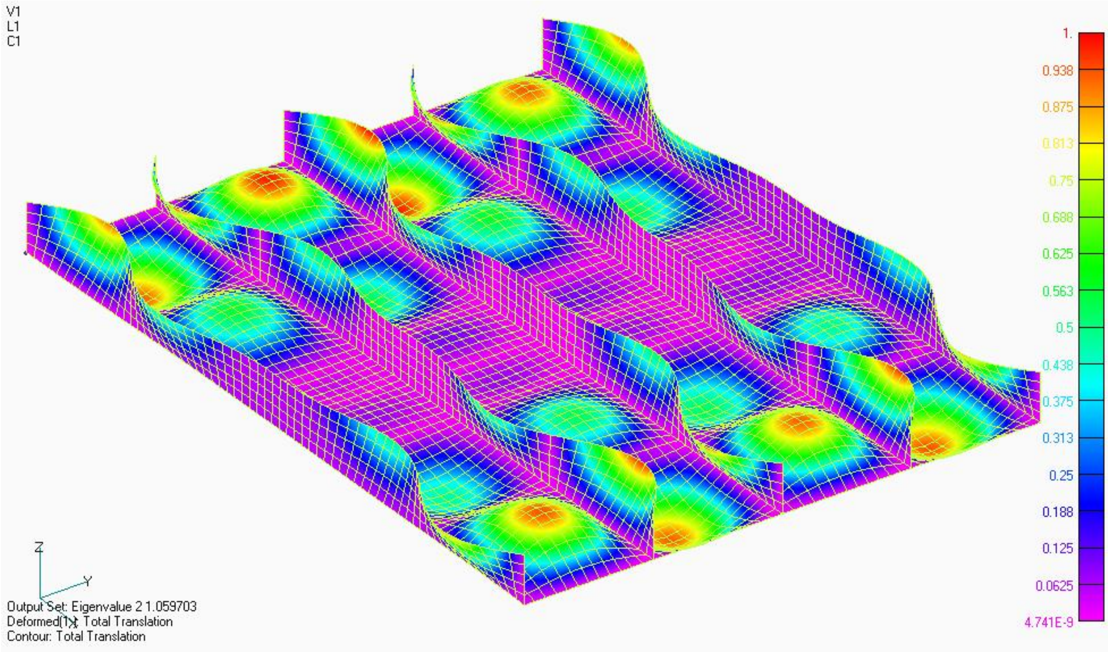

Figure 5 – Buckling Analysis, Second Failure Mode

5. Acknowledgements

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7. Responsibility notice

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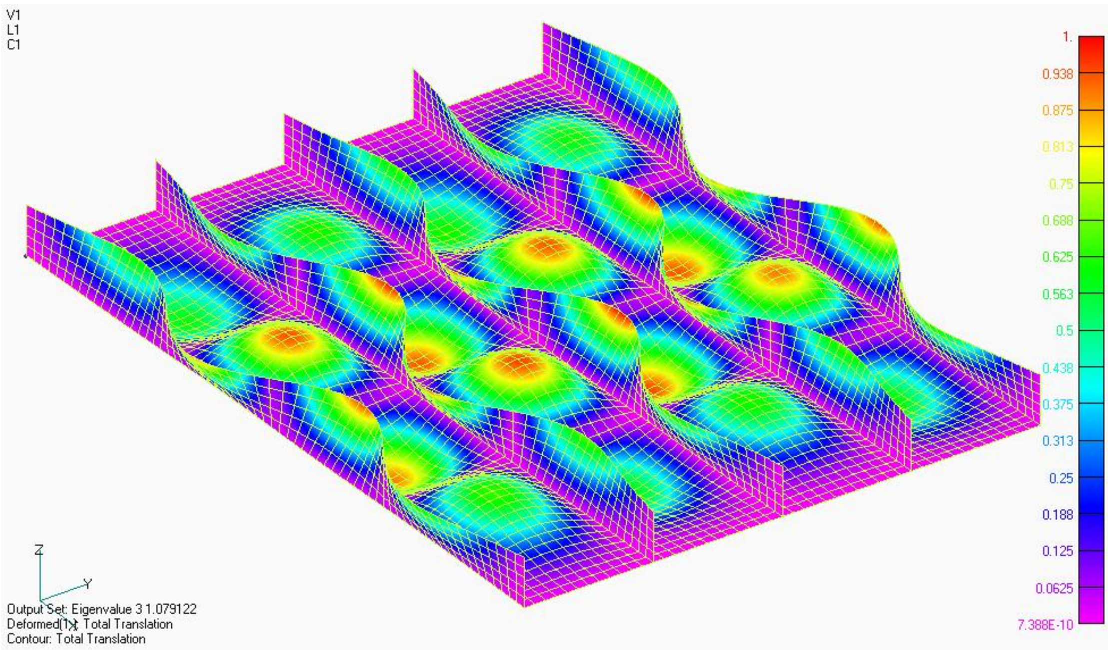

Figure 6 – Buckling Analysis, Third Failure Mode