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# TIME DOMAIN IDENTIFICATION OF LOW ORDER EQUIVALENT SYSTEM FROM FLIGHT DATA

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***Abstract.*** *The identification of low order equivalent system is a fundamental task in airplane certification, since some international guidelines must be obeyed in the process, regarding safety and flight quality. In this paper a time domain approach is applied for identifying low order equivalent systems for both longitudinal and lateral-directional dynamics. Since the time delay must also be identified, a 2 step approach is employed: in the first, SVD is used for fitting competing models with different time delays. The model producing the smallest prediction error is selected for the second phase, in which the model parameters are refined via the Nelder-Mead optimization technique, so as to better tackle non-ideal experimental data. The benefits of the proposed approach are twofold: identification is performed directly in the time domain, in which the cost is based in the prediction error, thereby having direct physical meaning, and both time delay and dynamic parameters are estimated simultaneously.*

***Keywords****: System identification, LOES, aircraft modeling, flight data*

## 1. Introduction

The true airplane dynamic behavior should be fully characterized during its flight test campaign. This task has multiple purposes but one of the most important is to check and compare airplane flying-qualities against other patterns or some specific international standard (Anon, 1987). Flying qualities can be interpreted as the way pilot "feels" the airplane. Since it is a subjective evaluation, one tries to find a way to translate this into numbers. Despite the high order dynamics, common in modern fly-by-wire open and closed loop controlled airplanes, longitudinal and lateral-directional dynamics can be quite well represented through Low Order Equivalent Systems (LOES). After that, airplane flying qualities can somehow be evaluated comparing obtained natural frequencies and damping against standard values.

In this work a time domain approach is employed for LOES identification, since in this case the cost function used is the pediction error, which has immediate physical meaning. However, in the time domain approach care must be taken in order to avoid mixing of the parameters, since the standards require the model to be written in a well defined form. To comply with that, we employ a procedure similar to that proposed in (Hsia, 1968): the continuous time model is discretized via finite difference approximation and the discrete model is then fitted to the experimental data.

The approach proposed here differs considerably from that in (Hsia, 1968) in 2 ways: SVD is used for fitting the competing models, with different time delays, and most important, there is a second stage in which the parameters of the best model is refined via the output error identification method based on Nelder-Mead optimization procedure. It is our experience that this second phase is very important to deal with non-ideal experimental data and model approximations: short data acquisition, nonstationarity and nonlinearities can spoil considerably the results obtained by least squares method, such as SVD, used here, and linear Kalman filter, used by (Hsia, 1968).

Airplane LOES can be estimated also in the frequency domain. (Morelli, 200a, 200b) has done this in a relaxation process estimating iteratively model parameters and time delay. Frequency domain has some advantages, like treating time delay as a continuous variable, in contrast to discrete time domain techniques. Nevertheless, there are some issues related to flight data. Frequency domain identification, as proposed by Morelli, requires accurate experimental frequency responses functions (FRF) determination. This can only be done through proper airplane excitation, which in general is obtained using frequency sweeps. Such maneuvers are not usual and it is not easy to obtain a large bandwidth input spectrum using pilot generated inputs. Good experimental FRF's also requires an average process to deal with correlated process noise, and this leads to long time histories or multiple test points to average the results.

The main contributions of this paper are:

1) proposition of a time domain approach for identifying LOES models, comprising 2 steps: time delay estimation via SVD and parameter refining via Nelder-Mead optimization method, which enable us to tackle non-ideal data in a more efficient way;
2) application of the proposed method to experimental flight data obtained for a typical Embraer regional airplane, for both longitudinal and lateral-directional dynamics.

This paper is organized as follows: in section 2 the procedure for LOES identification is described. In sections 3 the models for longitudinal and latero-directional dynamics are summarized, and experimental results are presented and discussed. Comparison with the frequency domain approach is presented in section 4, and the conclusions follow in section 5.

## 2. Proposed Identification Procedure

The models of interest in this work are described in continuous time by the ordinary time invariant differential equation

$$\frac{d^n}{dt^n}y(t)+a_1\frac{d^{n-1}}{dt^{n-1}}y(t)+\cdots+a_n y(t)=\frac{d^m}{dt^m}u(t-\tau)+b_1\frac{d^{m-1}}{dt^{m-1}}u(t-\tau)+\cdots b_m u(t-\tau) \tag{1}$$

where *y(t)* and *u(t)* are the system output and input, respectively, and $\tau$ is the time delay. The parameters $a_i$, $b_j$ and $\tau$ are supposed unknown and must be identified from experimental data.

If usual discretization method, such as *z* transform, were applied to (1), then there would appear a nonlinear mapping between the continuous and the discrete model parameters. This problem is further complicated here due to the unknown time delay. Therefore, bearing in mind that in the aeronautic industry usually signals exhibiting low noise levels are available, here we can apply a procedure similar to that in (Hsia, 1968), which uses Taylor series expansion for obtaining the derivatives, for instance,

$$\frac{d}{dt}y(t)\approx\frac{y(k+1)-y(k)}{T} \quad , \frac{d^2}{dt^2}y(t)\approx\frac{y(k+1)-2y(k)+y(k-1)}{T^2} \quad , \ldots \tag{2}$$

where *T* is the sampling time.

In case the signals were noisier, then a more complicated procedure, as for instance that proposed by (Puthenpura and Sinha, 1986), based on trapezoidal pulse functions, could be used.

For keeping the presentation simple, lets us consider the particular second order model

$$\frac{d^2}{dt^2}y(t)+a_1\frac{d}{dt}y(t)+a_2 y(t)=b_1\frac{d}{dt}u(t-\tau)+b_2 u(t-\tau) \tag{3}$$

with

$$\tau=d.T \quad , d\in N \tag{4}$$

Then, by using (2), from (3) we get the discrete time model

$$y(k+1)=\left(2-a_1T-a_2T^2\right).y(k)+\left(a_1T-1\right).y(k-1)+b_1T.u(k-d)+\left(b_2T^2-b_1T\right).u(k-1-d) \tag{5}$$

where it is seen that the parameters of the discrete time model are linear combinations of the original continuous time parameters. By supposing a number of readings equal to $n_r$, from (5) we obtain the matrix representation

$$\Phi.\Theta=Y \tag{6}$$

where

$$\Phi=\begin{bmatrix} y(d+2) & y(d+1) & u(2) & u(1) \\ y(d+3) & y(d+2) & u(3) & u(2) \\ \vdots & \vdots & \vdots & \vdots \\ y(n_r-1) & y(n_r-2) & u(n_r-d-1) & u(n_r-d-2) \end{bmatrix} \tag{7}$$

and

$$Y=\begin{bmatrix} y(d+3) & y(d+4) & \cdots & y(n_r) \end{bmatrix}^T \tag{8}$$

From model (6) and experimental data (7) and (8), for a given time delay *d* in (4) we can use the Least Squares method, in its batch form, in order to calculate $\hat{\Theta}$, i.e., an estimate for the parameter vector $\Theta$. It should be noted that the use of SVD for solving the LS problem is highly advisable, since it provides a measure of how adequate a given parameterization is for explaining the experimental data. For further details, see (Golub and Van Loan, 1984).

By using this particular model (3) as example, we can summarize the method proposed here for time delay and parameter estimation:

- For each $d \in D,\ D = \{0, 1, \cdots, d_{max}\}$, the SVD method is employed to obtain the parameter estimate $\hat{\Theta}$ in (6) and then the associated mean squared prediction error

$$J(d) = \frac{\sum_{k=1}^{n_r - d - 2} \left( y(k+d+2) - \hat{y}(k+d+2) \right)^2}{n_r - d - 2} \tag{9}$$

is calculated. The time delay estimate is then taken as the one which minimizes the cost function (9), i.e.,

$$\hat{d} = \underset{d \in D}{Arg\ Min}\ J(d) \tag{10}$$

- The parameter estimate associated with $\hat{d}$ obtained in (10), denoted here by $\hat{\Theta}_{SVD}$, is given as the initial condition for the output error identification method based on Nelder-Mead algorithm (Press et.al, 1990), aiming at improving it. The final parameter estimate $\hat{\Theta}$ is thereby obtained. The Nelder-Mead method is adequate in the present application, since only the cost function evaluation is required and the dimension of the parameter vector used here, see section 3, is small, at most 5.

The flowchart summarizing the proposed identification procedure, in the time domain, is shown in Fig. 1.

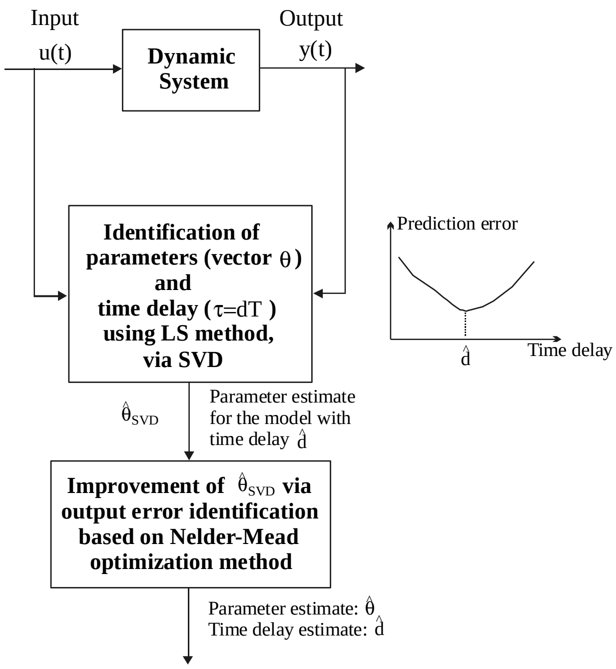


Figure 1. Two steps approach for time delay and parameter estimation

It is worth mentioning that in the ideal case (large value for $n_r$, linearity and stationarity), the estimate $\hat{\Theta}_{SVD}$ in Fig. 1 would be optimal, thence no further refining would be necessary. It should also be pointed out that the time delay estimation based on the prediction errors after improving all the parameter estimates, and not before as indicated in Fig. 1, provides only marginal improvement. Therefore, it is not worth the additional numerical burden.

## 3. Longitudinal and Lateral-Directional Models

Five dynamic modes can describe aircraft response to any equilibrium disturbance like turbulence or control inputs. Although being a simplified assumption, it is accurate enough for some modeling and simulation purposes. These modes can be grouped in two sets: longitudinal and lateral-directional.

Two modes belong to the longitudinal set, the “short period” and the “phugoid”. Both are second order dynamics with their associated damping and natural frequency. While the first has a typical period from 1 to 4 seconds and damping from 0.3 to 1.0, the second responds from 30 seconds to 2 minutes with a 0.05 to 0.1 damping ratio.

The lateral-directional set contains the three other modes: “roll”, “spiral” and “dutch roll”. The “roll” mode is a first order dynamics whose time constant measures roll command efficiency. The “spiral” mode is a tendency of the aircraft to roll away from level flight, and, finally, the “dutch roll” is as coupled rolling-yawing second order motion.

In flying qualities evaluation, the main goal is to evaluate how the pilot interacts with the airplane, his workload and also aircraft behavior. Reduced models are typically applied in a LOES pattern, and some model simplifications are done. Although high order aircraft dynamics are common, the LOES should adequately characterize the motion with minimum order.

### 3.1. Longitudinal Model

The longitudinal dynamics can be encapsulated only by "short period" model as follows (Tischler, 1994)

$$G_{lon}(s)=\frac{\dot{\theta}(s)}{\delta(s)}=\frac{K_{\dot{\theta}}\left(s+1/T_{\theta_2}\right)e^{-\tau_e s}}{s^2+2\xi_{sp}w_{sp}s+w_{sp}^2} \quad (11)$$

where the output $\dot{\theta}$ is the pitch rate and the input $\delta$ is the elevator deflection.

A typical flight test data is shown in Fig. 2.

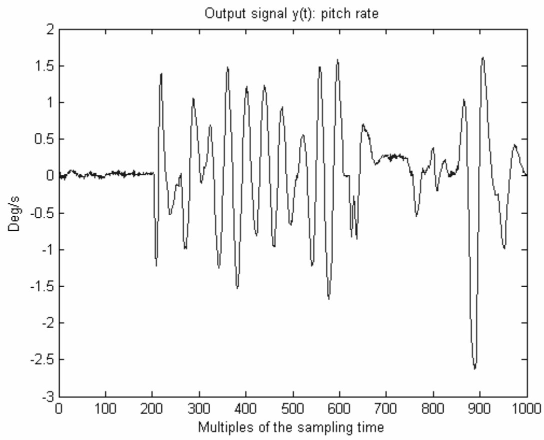


Figure 2. Flight test data for longitudinal LOES identification (the input signal is elevator deflection)

By using the first step in Fig. 1, namely the SVD identification, the time delay estimate and the mean square prediction error were

$$\hat{\tau}=0 \quad \text{and} \quad J=0.114 \quad (12)$$

and the corresponding parameter estimates

$$\xi_{sp}=0.96, w_{sp}=2.01, K_{\dot{\theta}}=1.26, T_{\theta_2}=0.36 \quad (13)$$

By feeding the second step in Fig. 1 with the initial parameters (13), the mean square prediction error was reduced to $J=0.013$ and the improved parameter estimates were

$$\xi_{sp}=0.49, w_{sp}=1.11, K_{\dot{\theta}}=1.09, T_{\theta_2}=5.65 \quad (14)$$

The prediction capability of the identified model, i.e., with parameter estimates (14) and time delay estimate $\hat{\tau}=0$, is shown in Fig. 3.

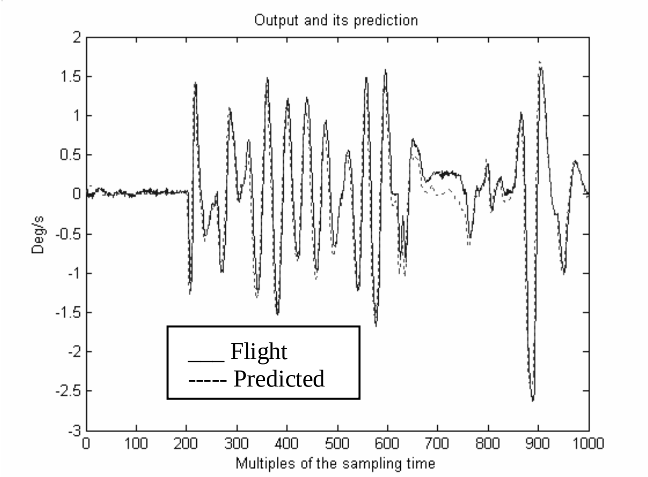


Figure 3. Measured output and its prediction by the model with $\hat{\tau} = 0$ and $\xi_{sp} = 0.49, w_{sp} = 1.11, K_{\dot{\theta}} = 1.09, T_{\theta_2} = 5.65$

Besides the flight data shown in Fig. 2, 5 other flight tests were performed, with aircraft in the same configuration and flight conditions, and the results for these 6 independent flight data are summarized in Table 1.

Table 1. Estimated parameters.

| File | $n_r$ | $\xi_{sp}$ | $w_{sp}$ | $K_{\dot{\theta}}$ | $T_{\theta_2}$ | $\tau$ |
|---|---|---|---|---|---|---|
| sp_1 | *1000* | *0.49* | *1.11* | *1.08* | *5.57* | *0* |
| sp_2 | *550* | *0.46* | *1.14* | *1.04* | *3.82* | *0* |
| sp_3 | *386* | *0.49* | *1.01* | *1.74* | *3.74* | *0* |
| sp_4 | *978* | *0.49* | *1.00* | *1.59* | *2.72* | *0* |
| sp_5 | *694* | *0.48* | *1.12* | *1.60* | *2.12* | *0* |
| sp_6 | *706* | *0.44* | *1.08* | *1.58* | *3.04* | *0* |

From Table 1 we conclude that the parameters estimates $\xi_{sp}$ and $w_{sp}$ are quite precise, i.e., by using the $1\sigma$ band we conclude that the true values are likely in the interval $(0.47 \pm 0.02)$ and $(1.07 \pm 0.06)$, respectively. The parameters estimate with largest variance are $T_{\theta_2}$ and $K_{\dot{\theta}}$. Both are related to input frequency contents, and a correlation is expected when only low frequency data is applied. A frequency sweep or a sharp multi-step input would be more effective to reduce correlation.

### 3.2. Lateral-Directional Model

The lateral-directional model can be represented by the "dutch roll" and the "roll" modes, as below (Tischler, 1994),

$$G_{latd}(s) = \frac{p(s)}{\delta_A(s)} = \frac{K_{\phi_A} \cdot \left(s^2 + 2\xi_\phi w_{n_\phi} s + w_{n_\phi}^2\right) e^{-\tau_{\phi_A} s}}{\left(s + \frac{1}{T_r}\right)\left(s^2 + 2\xi_D w_{n_D} s + w_{n_D}^2\right)} \tag{15}$$

where $p$ stands for the aircraft roll angle derivative and $\delta_A$ is the aileron deflection.

A typical flight test data is shown in Fig. 4.

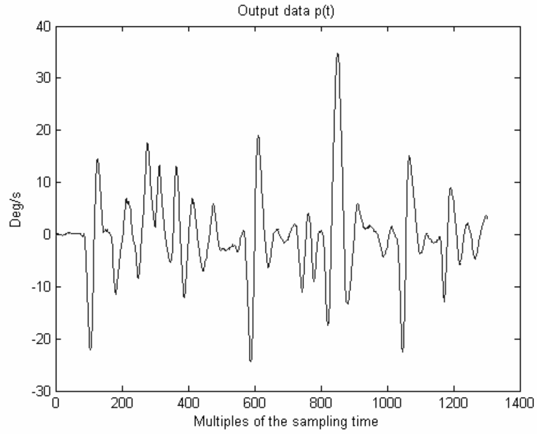


Figure 4. Flight test data for lateral-directional LOES identification (the input signal is aileron deflection).

By using the procedure indicated in Fig. 1, the best time delay was $\hat{\tau}=0$ and the SVD produced estimated parameters which correspond to a discrete third order system with a negative real pole: the damping is explained by this pole, and not by a pair of complex conjugated poles. Therefore, there is no correspondent third order continuous system. This happens because the dynamics in Fig. 4 can not be approximated by the model (15) with small prediction error. Indeed, the mean squared prediction error for the model obtained with the SVD was *9.892*. If the parameters of the model obtained by the SVD were transferred directly as initial estimates for the Nelder-Mead algorithm, it would converge to a discrete model which decreases the prediction error, but which still possesses a negative real pole. The natural solution to this problem would be to replace the Nelder-Mead method by some global optimization algorithm, like the genetic one. However, this would increase the computational burden considerably. Therefore, since a reasonable estimate for the parameters in equation (15) is available, a better solution seems to give this initial estimate, which possesses physical meaning, to the Nelder-Mead algorithm. By so doing, the following parameter estimates were obtained,

$$\begin{aligned} &\xi_D = 0.244 \;,\; w_{n_D} = 1.230 \;,\; 1/T_R = 4.284 \\ &\xi_\phi = 0.644 \;,\; w_{n_\phi} = 1.303 \;,\; K_{\phi_A} = 2.115 \end{aligned} \tag{16}$$

The prediction capability of the identified model, i.e., with parameter estimates (16) and time delay estimate $\hat{\tau}=0$, is shown in Fig. 5.

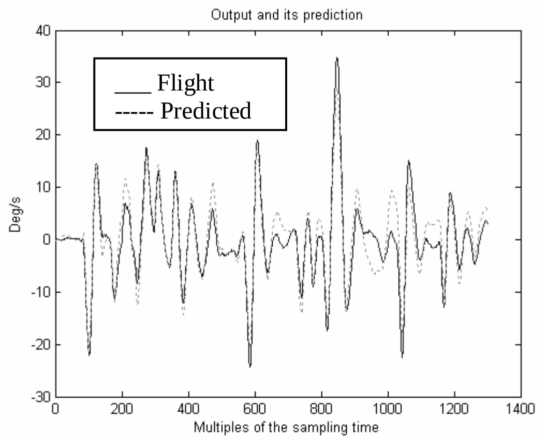


Figure 5. Measured output and its prediction by the model with $\hat{\tau}=0$ and parameters given by (16)

The mean squared prediction error corresponding to Fig. 5 is *8.703*, which is still high and then further strengthen the conclusion about the inadequate SVD parameter estimate aforementioned: the flight data in Fig. 4 represents a considerably more complex dynamics than that allowed for by the model (15). This explains why a good initial estimate is necessary for the Nelder-Mead method. This is also the case for the frequency approach used in section 4.

**4. Comparison with the Frequency Domain Approach**

In order to compare the performance of the approach for LOES identification proposed in Fig. 1 with a well established technique, the procedure described in (Tischler, 1994), developed in the frequency domain, is now applied to the flight data shown in Fig. 4.

Briefly, the method consists of estimating experimental frequency responses curves and fit magnitude and phase of a parametric LOES model using a minimization technique. Although initial condition is not an issue for frequency domain methods, the choice of a proper window could be critical and influence the results substantially. Those different sources of error make the comparison between both methods more effective.

The FRF structure was forced to have no time delay, two pair of conjugated poles and two pairs of conjugated zeros, otherwise a real negative pole would show up, as happened in the time domain SVD procedure.

The following parameters were estimated for the lateral-directional case,

$$\begin{aligned} &\xi_D = 0.318 \;\;,\; w_{n_D} = 1.299 \;\;,\; 1/T_R = 3.069 \\ &\xi_\phi = 0.440 \;\;,\; w_{n_\phi} = 1.268 \;\;,\; K_{\phi_A} = 2.153 \end{aligned} \tag{17}$$

All estimated parameters are quite similar to those obtained in (16). FRF magnitude curve fit is depicted in Fig. 6.

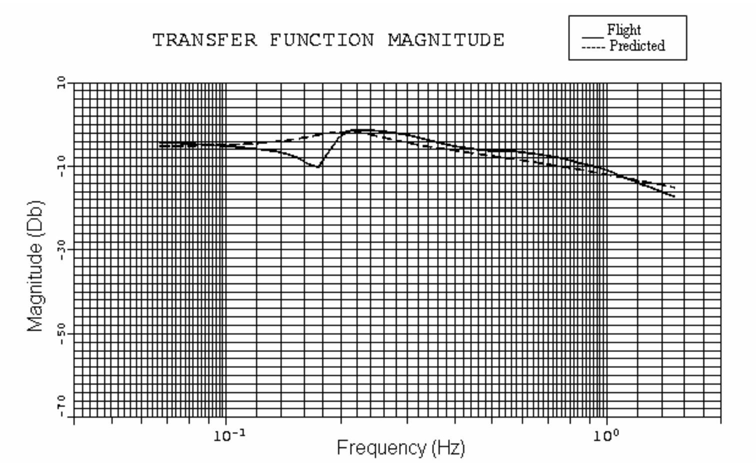


Figure 6. Experimental FRF and its estimated model with $\hat{\tau} = 0$ and parameters given by (17)

It is evident some small disturbance in the experimental FRF near *0.18 Hz*. This effect disappeared for some others window size, but this one proved to be the more convenient for other aspects beyond the scope of this paper.

For comparing the predictions capabilities of the models (16) and (17), the output predictions for both models are displayed in Fig. 7.

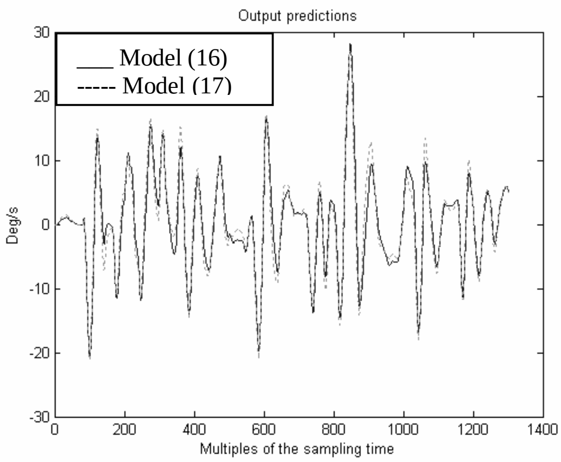


Figure 7. Comparison of output predictions for models (16) and (17).

The output predictions in Fig. 7 are quite similar. However, since model (16) was obtained from the output prediction error minimization, it should provide smaller prediction error: indeed, the mean squared prediction error for the two graphics in Fig. 7, for the models (16) and (17), are *8.703* and *11.080* , respectively.

**5. Conclusions**

A procedure for identifying LOES model in the time domain was proposed in this paper. It envolves discretization by using an approximation which preserves linearity in the discrete parameters, which enables the use of the efficient SVD technique for solving the ensuing LS problem. Models with different time delays are fit via SVD, and the one with smallest prediction error is selected as starting point for an improvement phase, based on the Nelder-Mead optimization method, which has the prediction error as cost function. This 2 steps procedure enables us to better tackle real data, since in the LOES case we are actively searching for a model which is simpler that the true system generating the data.

Experimental results for both longitudinal and lateral-directional flight data were presented, showing good results. Comparison with a frequency domain LOES identification procedure was also carried out and discussed. As expected, the proposed approach produces models with smaller prediction error.

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