

# AN UNIFIED APPROACH TO EVALUATE AND COMPENSATE THE TOOL POSITIONING ERROR IN PARALLEL KINEMATIC MACHINES

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***Abstract.** Parallel Kinematic Machines (PKMs) became attractive architectures due to some potential advantages over their traditional serial counterparts. Among them, one can mention: high rigidity, lightness, fast dynamic response, precision and high load capacity. In addition, they can produce workpieces with very complex geometries in a single set-up, which would be rather difficult to obtain from conventional or even CNC machine-tools. Despite of the efforts from academia and industry, there are a great number of problems that remain to be solved, like calibration, motion planning and control. This work proposes an unified approach to evaluate and compensate the undesirable effects caused by possible error sources observed in parallel kinematic machines. The full paper also presents a practical example for a bidimensional PKM, where the proposed approach is applied.*

**Keywords:** calibration, precision, machine-tools, parallel robots

## 1 Introduction

Most of commercially available machine-tools are based on serial kinematic structures, i.e., their actuators and moving links are assembled serially, one after the other, resulting only one open-loop kinematic chain to position and orient the cutting tool. During this last decade, both academic and industrial communities have demonstrated a research interest on using another kind of kinematic structure, known as parallel kinematic machine (PKM), which is characterized by the presence of many independent limbs (kinematic chains), actuating in-parallel or simultaneously on a moving platform. This nonconventional architecture becomes attractive due to some potential advantages over its traditional serial counterpart. Among them, one can mention: high rigidity, lightness, fast dynamic response, precision and high load capacity. In addition, they can produce workpieces with very complex geometries in a single set-up, which would be rather difficult to obtain from conventional or even CNC machine-tools.

Despite of the efforts from academia and industry, there are a great number of problems that remain to be solved, like motion planning (singularity avoidance), control and calibration. In fact, calibration of PKMs is needed to keep accuracy and repeatability in acceptable levels during all phases of machining operation. Though, calibration still remains as one of the main obstacles for practical utilization of PKMs in factories [1].

Pott and Hiller [2] proposed a method to describe the displacements of the tool center point in terms of error amplifications for uncertainties in all kinematic parameters. This was done by using the differential geometric properties of kinetostatic transmission. Nakamura and Murai [3] discussed the design issue of closed kinematic chains under the influence of machining errors. Their analysis considered two classes of machining errors: one refers to errors than can be absorbed by clearances in unactuated joints, while the other deals with unabsorbed errors that can cause elastic deformation of the structure. Oiwa [4] introduced a compensation system consisting of some

measuring devices for elastic deformation and thermal expansion of the links and frame. Renaud et al. [5] proposed an algorithm to achieve kinematic calibration of parallel mechanisms by using vision sensors to perform measurements on the legs of the mechanism. Sato et al. [6] presented two methods to increase the robustness of calibration calculation. One of them employs nonsymmetrical links, while the other considers *a priori* knowledge in a non-linear squares method.

This work proposes an unified approach to evaluate and compensate the undesirable effects caused by possible error sources observed in parallel kinematic machines. The proposed approach has four steps. First, identification of possible error sources. Second, develop mathematical models to predict the positioning error of the tool inside the feasible workspace caused by each error source independently. Third, compare these errors in order to evaluate which are preponderant and those are irrelevant. Forth, propose alternative forms for compensating the tool positioning error in control algorithms in such way to eliminate or even at least minimize those errors. The idea is to constrain such errors in a reasonable band, in accordance with the desired quality for the workpiece. The full paper also presents a practical example for a bidimensional PKM, where the proposed approach is applied.

## 2 Parallel kinematic machines and the analysed system

Parallel kinematic machines (fig.1) are composed by a moving platform that carries the tool connected to a fixed base by at least two independent limbs or legs. According to the employed limbs, such unconventional machines can be classified in two main categories: the first corresponds to architectures composed by fixed-length struts and the second is represented by structures with variable-length struts. In both cases, the translational motion of prismatic joints is achieved by the action of rotary actuators coupled to ball-screw devices. For PKMs with fixed-length struts, the actuators are attached to the base, while in architectures with variable-length struts, the actuators perform small rotation with respect to the center of universal or spherical joints.



**Fig. 1. Parallel kinematic machine (PKM)**

In order to illustrate the approach to be presented in the following sections, we focus our analysis on a bidimensional (planar) PKM with fixed-length struts. This two-degree-of-freedom system (fig.2) is responsible for positioning a tool (point P) by the action of two actuated sliding blocks. So, the input motions are described by the variables  $d_1$  and  $d_2$ , while the output motions are represented by the  $x$ - and  $y$ -coordinates of point P. The parameter  $L$  corresponds to the length of each strut.

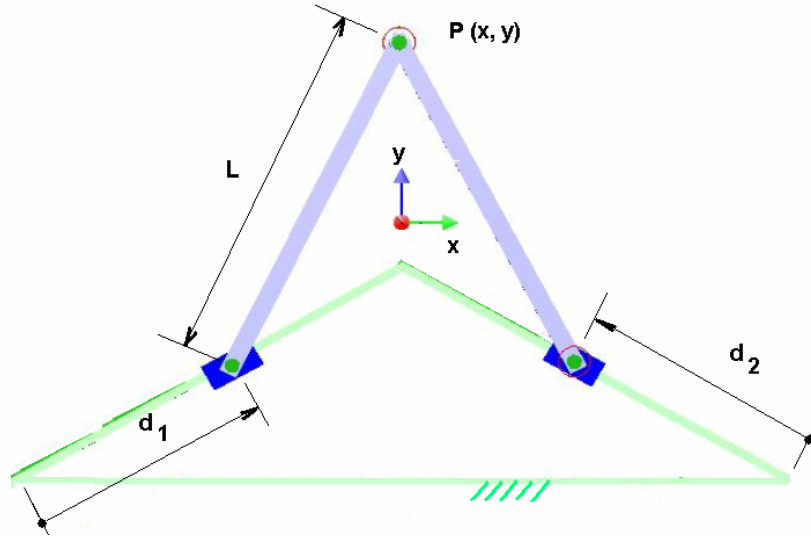


Fig.2 . The analysed system: a bidimensional PKM

### 3 Identification of error sources and their mathematical modelling

The actual position of the cutting tool, when it moves along the available workspace, deviates from the desired one because of many different error sources. Typically, such error sources are associated to dimensional and geometric tolerances, elastic and thermal deformation of the structure, backlash in joints, sensor resolution and actuator precision. As a consequence, they cause a strong influence on the overall precision of a machine.

In this work, we consider as possible error sources the dimensional tolerances due to the manufacturing process of each strut. In addition, we take into account the elastic deformation of each strut by the action of machining forces. Moreover, the thermal expansion of the struts, expected during the machining process, is also considered.

Regarding the dimensional tolerances for the struts, the correspondent minimum and maximum deviation values, respectively,  $\Delta L_{min\_toler}$  and  $\Delta L_{max\_toler}$ , are directly associated to the length of the struts, in accordance with the standard DIN 7168.

The length deviation  $\Delta L_{th\_exp}$  due to thermal expansion is calculated from a unidimensional model for each strut

$$\Delta L_{th\_exp} = L \times \alpha \times (T_f - T_i) \quad (1)$$

where  $L$ ,  $\alpha$ ,  $T_f$ ,  $T_i$  are, respectively, the length of the strut, the thermal expansion coefficient, final and initial temperatures.

Considering that both struts behave as trusses, the length deviation for strut  $i$   $\Delta L_{sta\_def}^{(i)}$ , associated to static deformation, is

$$\Delta L_{sta\_def}^{(i)} = \frac{F^{(i)} \times L}{A \times E} \quad i = 1, 2 \quad (2)$$

where  $F^{(i)}$ ,  $A$ ,  $E$  are, respectively, the axial force on the strut  $i$ , the section area and modulus of elasticity of the strut material. Consequently, the minimum and maximum overall deviation in the length of strut  $i$  are

$$\tilde{\Delta L}_{ov\_min}^{(i)} = \Delta L_{min\_toler} + \Delta L_{th\_exp} + \Delta L_{sta\_def}^{(i)} \quad i = 1, 2 \quad (3)$$

$$\tilde{\Delta L}_{ov\_max}^{(i)} = \Delta L_{max\_toler} + \Delta L_{th\_exp} + \Delta L_{sta\_def}^{(i)} \quad i = 1, 2 \quad (4)$$

Hence, the corrected length of strut  $i$   $\tilde{L}^{(i)}$  is bounded by the lower and upper limits,  $\tilde{L}_{min}^{(i)}$  and  $\tilde{L}_{max}^{(i)}$ , respectively, and constitutes a modified PKM parameter according to previously mentioned effects.

$$\tilde{L}_{min}^{(i)} = L + \Delta\tilde{L}_{ov\_min}^{(i)} \quad i = 1, 2 \quad (5)$$

$$\tilde{L}_{max}^{(i)} = L + \Delta\tilde{L}_{ov\_max}^{(i)} \quad i = 1, 2 \quad (6)$$

For the analysed bidimensional PKM, the starting point is the definition of desired x- and y-coordinates of the tool (point  $P$ ). Then, considering the uncorrected parameter  $L$  and performing inverse position kinematics [7], we can calculate the actuator displacements  $d_1$  and  $d_2$ . From these two theoretical values and now assuming that the PKM is subjected to the considered error sources, we can determine the corrected position for point  $P$ , called  $\tilde{P}$ , by performing direct position kinematics. The correspondent equations for both inverse and direct position kinematics are fully described in [8]. Consequently, the overall error  $\Delta\tilde{P}$  represents a vector with x- and y-components.

$$\Delta\tilde{P} = P - \tilde{P} = [\Delta\tilde{P}_x, \Delta\tilde{P}_y]^T \quad (7)$$

**Table 1. Adopted parameters and process variables**

$\alpha$ [ $^{\circ}\text{C}^{-1}$ ]	$T_i$ [ $^{\circ}\text{C}$ ]	$T_f$ [ $^{\circ}\text{C}$ ]	$E$ [GPa]	$F_H$ [kN]	$F_V$ [kN]
$1,1 \times 10^{-5}$	20	75	210	10	5

In order to estimate the overall error, we present in table 1 some adopted parameters and process variables. The forces  $F_H$  and  $F_V$  represent, respectively, the horizontal and vertical components of the machining. These values are obtained assuming that the average chip thickness  $h_m$  is 0,2 mm, specific cutting force  $K_m$  equals 3000 N/mm<sup>2</sup>, cutting depth  $a_a$  is 2 mm and machined width  $a_r$  is equal to 2,5 mm. In addition, table 2, 3 and 4 shows 25 selected locations for the cutting tool that corresponds to the desired coordinates for point P. These locations constitute a region, inside the feasible workspace, where the error prediction analysis is conducted.

**Table 2. Selected locations for the cutting tool (from 1 to 11)**

P	1	2	3	4	5	6	7	8	9	10	11
x[mm]	0	100	200	300	450	-100	0	100	220	380	-200
y[mm]	0	70	120	160	180	70	140	180	240	260	120

**Table 3. Selected locations for the cutting tool (from 12 to 22)**

P	12	13	14	15	16	17	18	19	20	21	22
x[mm]	-100	0	120	300	-320	-240	-120	0	180	-450	-380
y[mm]	180	250	300	340	160	240	300	360	400	180	260

**Table 4. Selected locations for the cutting tool (from 23 to 25)**

P	23	24	25
x[mm]	-300	-180	0
y[mm]	340	420	470

## 4 Results

From the parameters and assumptions mentioned in previous section, we present here the estimated errors due to dimensional tolerances, elastic deformation and thermal expansion of the struts. Figures 3 and 4 show, respectively, the positioning error of the tool due to each error source with respect to X- and Y-axes and the overall positioning error of the tool with respect to the same axes.

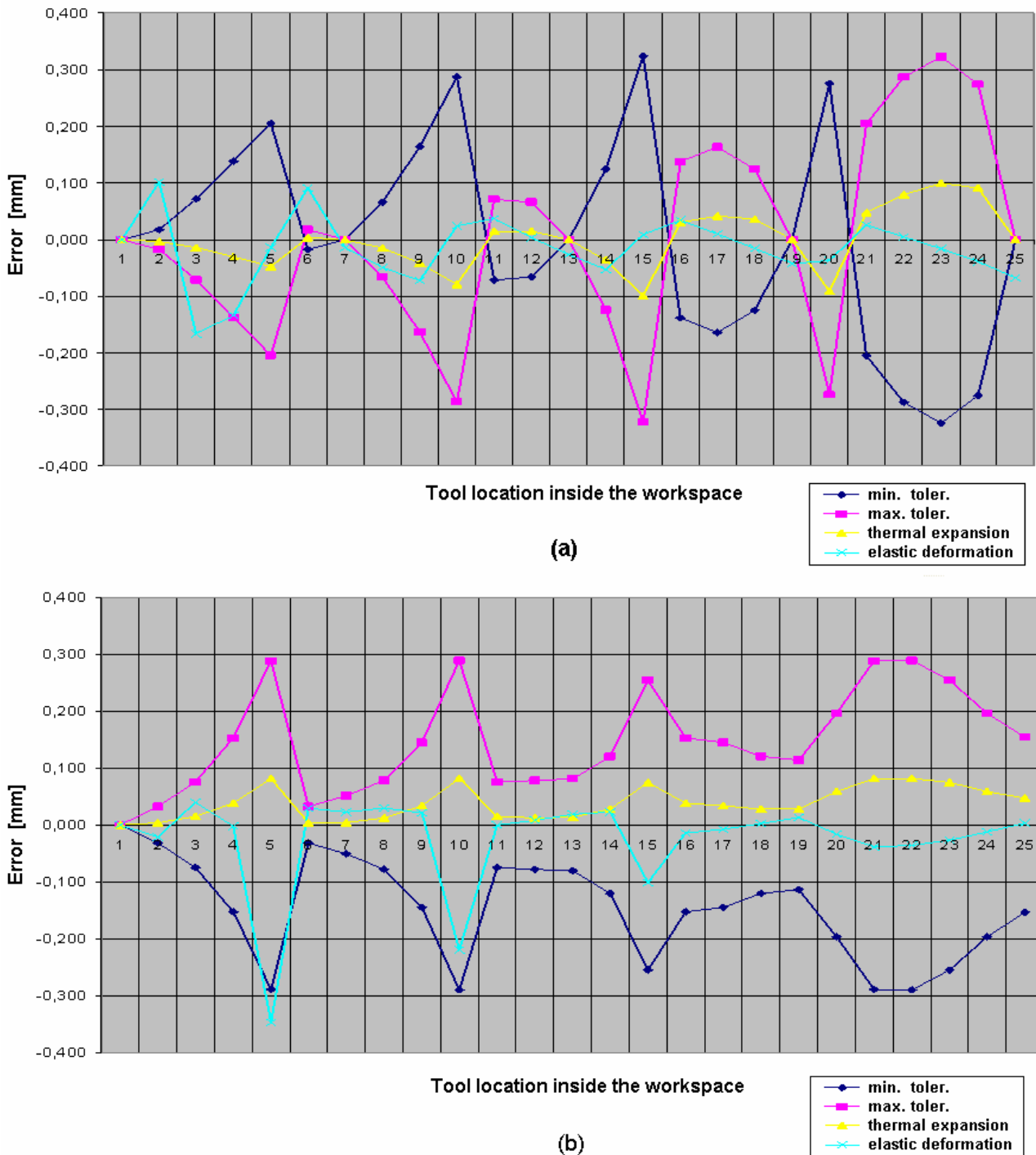


Fig. 3. Positioning error of the tool due to each error source with respect to (a) X- and (b) Y-axes.

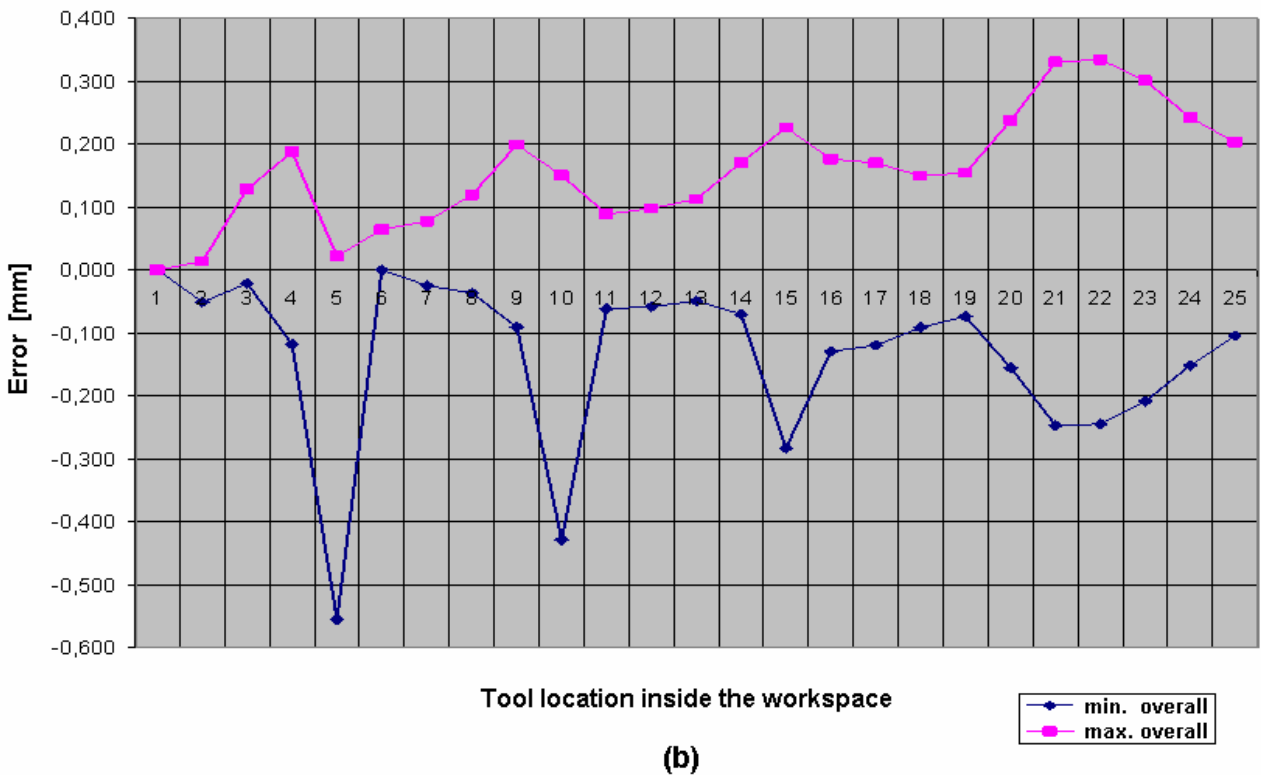
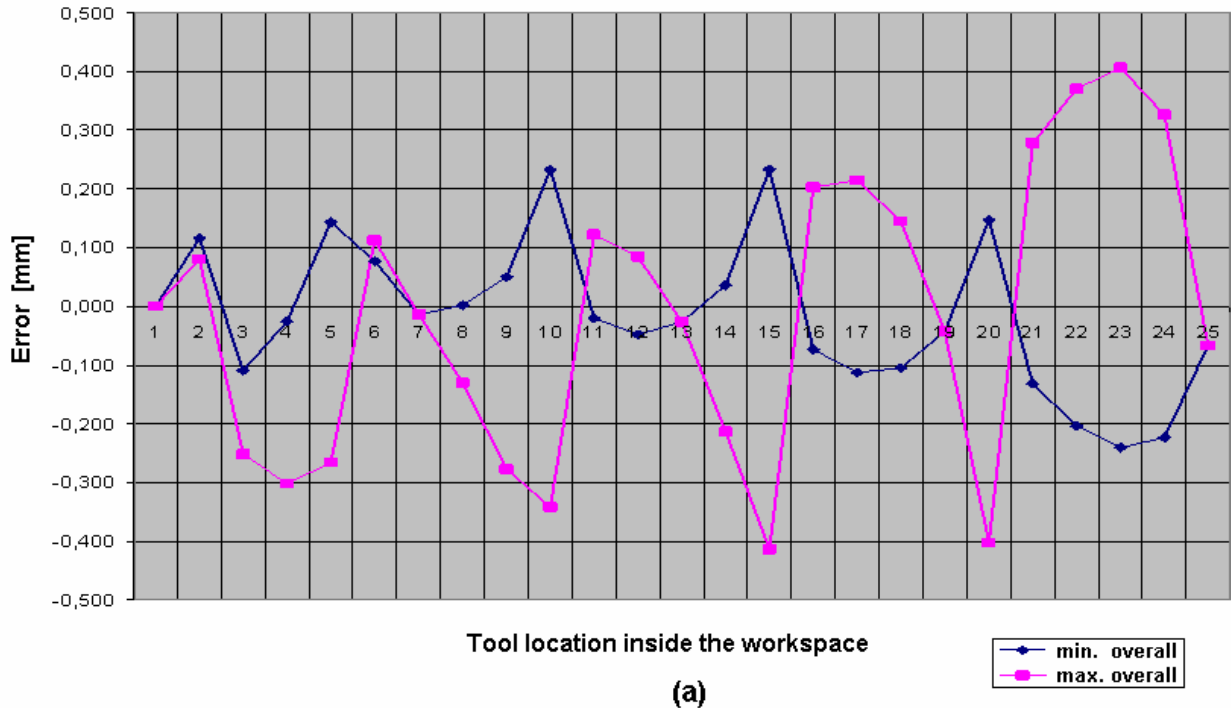


Fig. 4. Overall positioning error of the tool with respect to (a) X- and (b) Y-axes.

## 5 Comparative analysis of error sources

By analysing the positioning error of the tool in fig. 3 and 4, one can notice that the estimated values are strongly dependent of the location of the tool itself inside the feasible workspace. In addition, we observed that the dimensional tolerances and elastic deformation are more preponderant than thermal expansion. Eventhough, in more precise applications, the thermal expansion should also be considered. Tables 5 and 6 also confirm these remarks.

**Table 5. Highest estimated errors (absolute values)**

	Axis X [mm]	Axis Y [mm]
Tolerances (min. or max.)	0,324	0,290
Elastic deformation	0,166	0,347
Thermal expansion	0,100	0,081

**Table 6. Highest estimated errors (percentage values)**

	Axis X [%]	Axis Y [%]
Tolerances	0,157	0,159
Elastic deformation	0,166	0,191
Thermal expansion	0,109	0,045
Overall (max. toler.)	0,544	0,838
Overall (min. toler.)	0,326	1,009

The values presented in table 6 correspond to those calculated by eqs. (8) and (9).

$$\Delta\tilde{P}_{x,y}^{source} = \frac{(P_{x,y} - \tilde{P}_{x,y}^{source})}{P_{x,y}} \times 100 \text{ [%]} \quad (8)$$

$$\Delta\tilde{P}_{x,y} = \frac{(P_{x,y} - \tilde{P}_{x,y})}{P_{x,y}} \times 100 \text{ [%]} \quad (9)$$

## 6 Compensation of the tool positioning error

To compensate the tool positioning errors estimated in section 4, we propose to consider the corrected mean value for the strut parameters  $\tilde{L}^{(i)}$  ( $i = 1, 2$ ), calculated for each tool location inside the workspace, before performing the inverse kinematics. Hence, we can calculate the corrected actuator displacements  $\tilde{d}_1$  and  $\tilde{d}_2$ . By applying the direct kinematics from these two values, we will obtain the corrected coordinates of point P, which will be really close to the desired tool position location. So, the remaining error with this procedure will be half of the dimensional tolerance range at most.

## 7 Conclusion

This work proposed an unified approach to evaluate and compensate the undesirable effects caused by possible error sources observed in parallel kinematic machines. We considered here the effects of dimensional tolerances, elastic deformation and thermal expansion as possible error sources in machining operations.

By analysing the error plots, one can notice that the estimated values are strongly dependent of the location of the tool itself inside the feasible workspace. In addition, we observed that the dimensional tolerances and elastic deformation are more preponderant than thermal expansion.

In future works, the authors will analyse the influence of other error sources such as joint backlash, joint stiffness, sensor and actuator accuracy and extend this approach to three-dimensional PKMs.

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