Inverse Analysis of Turbulent Boundary Layers

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Abstract. *An inverse problem is solved for the estimation of upstream velocity profiles in an incompressible turbulent boundary layer over a smooth flat plate. The procedure is based on the boundary layer morphology, making use of the law of the wall and the law of the wake to estimate boundary layer parameters from measured velocity histories. The paper also presents a direct problem approach for the solution of the turbulent boundary layer equations. The direct approach resorts to a finite difference method and to the Cebeci-Smith turbulence model. The friction velocity, Von Kármán constant, law of the wall constant, Coles's wake-strength parameter and boundary layer thickness for the initial profile are determined as unknown parameters by the Levenberg-Marquardt algorithm. The effects on solution about the location of the measurement station are examined. The results provided by the direct numerical simulation of the flow are validated by data obtained through the hotwire anemometry technique in a low-speed wind tunnel. The estimated upstream velocity profiles are shown to compare favourably with hotwire anemometry measurements at the same location.*

Keywords: *turbulence, boundary layer, inverse problem, law of the wall.*

1. Introduction

Inverse problems have originated in the heat transfer community in connection with the estimation of surface heat flux histories from measured temperature histories inside a heat-conducting body. In convective environments, early studies were carried out by Keller and Cebeci(1972), Cebeci et al.(1975) and Cebeci(1976) in connection with the determination of the spatial variation of the flow free-stream velocity for a given local wall shear stress. However, as recognized by Moutsoglou(1989), Cebeci and his co-workers failed to capture the ill-posed nature of the problem as the calculated values of the direct problem were used as boundary conditions for the inverse problem. This procedure caused an unnecessary contamination of the inverse problem that made its results difficult to assess.

The purpose of the present work is to propose a new methodology to the solution of an inverse problem for the estimation of upstream velocity profiles for incompressible turbulent boundary layers over smooth flat plates. The solution procedure aims at developing a very robust method which can be used confidently to predict local and global parameters of the flow. As recorded by Cebeci(1976), "a slight error in the experimental skin-friction coefficient will severely affect the computed velocity distribution". Of couse, the same remark is valid if we consider the computed skin-friction coefficient. In fact, the solution sensitivity on the chosen value of the skin-friction is known to be high for turbulent flows and a classical way to overcome this difficulty is to appeal to the asymptotic two-deck structure of the turbulent boundary layer. Here, the unknown upstream velocity profile will be represented by the composite Coles's law of the wall, law of the wake profile; then, the friction velocity, Von Kármán's constant, the law of the wall constant, Coles's wake-strength parameter and the boundary layer thickness for the initial profile will be determined as unknown parameters by the Levenberg-Marquardt algorithm.

The solution procedure will resort to velocity measurements obtained at several different downstream locations in the stream; the measurements were obtained through the hotwire anemometry technique. The direct

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problem is solved by a finite difference method that uses the Cebeci-Smith turbulence model. The effects on solution concerning the location of the measurement station are examined.

2. A Short Review on Inverse Problems

A wide variety of inverse heat conduction problems have been solved in the last two decades for the estimation of initial or boundary conditions, physical properties, geometric parameters, or heat source intensities. Özisik and Orlande(2000) and Su and Silva Neto(2000), among others, present some of the methods developed for the solution of such problems. Despite many potential applications, inverse convection problems have only recently received some attention. Moutsoglou(1989) apparently was the first to address an inverse convection problem that has used a sequential function specification algorithm for the estimation of the asymmetric heat flux in mixed convection in a vertical channel. The same author, Moutsoglou(1990), has also applied the whole domain regularization technique in an inverse analysis to estimate wall heat flux in an elliptic laminar forced convection problem. Raghunath(1993) applied the quasi-Newton conjugate gradient method, which is a special case of the conjugate gradient method, to obtain the temperature profile at the entrance of a thermally developing hydrodynamically developed laminar flow between parallel plates. Huang and Özisik(1992) have applied the regular and modified conjugate gradient methods for the estimation of a steady state wall heat flux in a hydrodynamically developed laminar flow in a parallel plate duct. The same method has been applied by Bokar and Özisik(1995) to estimate the time dependence of inlet temperature in similar flow conditions. Liu and Özisik(1996a) have used the Levenberg-Marquardt algorithm for estimation of the thermal conductivity and thermal capacity of a laminar flow through a circular duct by using transient temperature readings at a single downstream location. Machado and Orlande(1997) have used the conjugate gradient method with an adjoint equation to estimate the timewise and spacewise variation of the wall heat flux in a parallel plate channel. An inverse problem for estimating the heat flux to a power-law non-Newtonian fluid in a parallel plate channel flow was solved by Machado and Orlande(1998) by using the same method. Hsu et al.(1998) applied the linear least-squares method for simultaneous estimation of the inlet temperature and wall heat flux in a laminar circular duct flow. Huang and Chen(2000) have applied the conjugate gradient method in a three-dimensional inverse forced convection problem to estimate a surface heat flux. Li and Yan(1999) applied the conjugate gradient method for the estimation of the space and time dependent wall heat flux for unsteady laminar forced convection between parallel flat plates, similar to that studied by Machado and Orlande(1997). Cho et. al.(1999a) developed an optimization procedure to find the inlet concentration profile for uniform deposition in a cylindrical chemical vapor deposition chamber using local random search technique. In a similar work, Cho et. al.(1999b) solved an optimization problem to find the inlet velocity profile that yields as uniform an epitaxial layer as possible in a vertical metalorganic chemical vapor deposition (MOCVD) reactor.

Few works have been published on inverse problems in turbulent flows despite its obvious technological relevance. Liu and Özisik(1996) applied the conjugate gradient method with an adjoint equation for solving the inverse turbulent convection problem of estimating the timewise varying wall heat flux in parallel plate ducts. Su et al.(2001) applied the Levenberg-Marquardt method to estimate nonuniform wall heat flux in a thermally developing, hydrodynamically developed turbulent flow in a circular pipe based on temperature measurements obtained at several different locations in the stream. Su and Silva Neto(2001) solved an inverse heat convection problem to estimate simultaneously the inlet temperature profile and the wall heat flux distribution in a thermally developing, hydrodynamically developed turbulent flow in a circular pipe based on temperature measurements obtained at several different positions in the stream, using the Levenberg-Marquardt method.

While all above mentioned works are dedicated to internal flow and heat transfer problems, Alekseev(1997) has shown the feasibility of estimation of fream stream parameters in a compressible laminar boundary layer which is governed by the parabolized Navier-Stokes (PNS) equations.

3. Mathematical Formulation of the Direct Problem

The Reynolds averaged governing equations for a steady, incompressible and two-dimensional turbulent boundary layer can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y} - \overline{u'v'}\right). \tag{2}$$

The notation is classical. The eddy viscosity concept is used, what implies that the turbulent stress term is related to the mean rate of strain by

$$-\overline{u'v'} = \nu_t \frac{\partial u}{\partial y}. \qquad (3)$$

In the Cebeci-Smith model, the eddy viscosity model of Boussinesq is involked together with the mixing length concept of Prandtl and the Van Driest damping function for the characteristic length of the flow. Thus, near to the wall

$$\nu_t = l^2 |\frac{\partial u}{\partial y}|, \qquad (4)$$

where

$$l = \kappa y \big[1 - \exp{(-\frac{y^+}{A^+})}\big], \qquad (5)$$

and $y^+ = yu_\tau/\nu$, u_τ is the friction velocity , ν is the kinematic viscosity, and $\kappa = 0.41$ (Von K*á*rm*á*n constant).
Constant A^+ is defined by

$$A^+ = 26\left(1 + yp^+\right)^{-\frac{1}{2}}, \qquad (6)$$

where $p^+ = (dp/dx)/\rho(u_\tau)^2$.
The defect region is described through an eddy viscosity of the type

$$\nu_t = C_1 u_e \delta_1 \gamma, \qquad (7)$$

where δ_1 is the boundary layer displacement thickness, $C_1 = 0.0168$, and γ is the intermittency factor of Klebanoff, given by

$$\gamma = \big[1 + 5.5(\frac{y}{\delta})^6\big]^{-1}, \qquad (8)$$

and δ is the boundary layer thickness.
The partial differential equations have to be solved with appropriate boundary conditions,

$$u = 0, \quad \text{for} \quad y = 0 \qquad (9)$$

$$v = 0, \quad \text{for} \quad y = 0 \qquad (10)$$

$$u = u_e(x), \quad \text{as} \quad y \to \infty \qquad (11)$$

$$u = u_0(y), \quad \text{for} \quad x = x_0 \qquad (12)$$

$$v = v_0(y). \quad \text{for} \quad x = x_0 \qquad (13)$$

If all fluid properties, coefficients of turbulent modelling, and boundary conditions are known, the direct problem given by Eqs. (1) to (13) can be solved to obtain the velocity field of the turbulent boundary layer. In this work, the direct problem defined by equations Eq. (1) to Eq. (13) is solved through an implicit finite difference method.

4. Solution of the Inverse Problem

In the inverse problem considered in this work, we are looking for the unknown upstream velocity profile $u_0(y)$; this must be evaluated from velocity measurements taken at several downstream points in the flow field.
The unknown upstream velocity profile is represented by the composite Coles's law of the wall, law of the wake formulation

$$u_0(y) = u_\tau \left[\frac{1}{\kappa} \ln y^+ + A + \frac{2\tilde{\pi}}{\kappa} \sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right], \qquad (14)$$

where κ (Von Kármán constant), A (law of the wall constant), u_τ (friction velocity), $\tilde{\pi}$ (Cole's wake-strength) and δ (boundary layer thickness) are parameters to be determined.

Upon the parameterization given by Eq. (14), the inverse problem has been formulated as a parameter estimation problem. The solution of this inverse problem for the estimation of the five unknown parameters is based on the minimization of the ordinary least squares norm defined by

$$R(\vec{P}) = \sum_{m=1}^{M} \left[u_m(\vec{P}) - Z_m\right]^2, \tag{15}$$

where $u_m(x_m, y_m)$ are the calculated velocities and $Z_m(x_m, y_m)$ are the measured velocities at points (x_m, y_m), $m = 1, 2, ..., M$, with M being the total number of measurement points.

The vector of unknown parameters is formed by

$$\vec{P}^T = [p_1, p_2, p_3, p_4, p_5] = [\kappa, A, u_\tau, \tilde{\pi}, \delta]. \tag{16}$$

Equation (15) can be written in the following form,

$$R(\vec{P}) = [\vec{u}(\vec{P}) - \vec{Z}]^T[\vec{u}(\vec{P}) - \vec{Z}] = \vec{F}^T\vec{F} \tag{17}$$

with $\vec{F}$ being the difference vector between calculated and measured velocities, $F_m = u_m - Z_m$, $m = 1, 2, ..., M$.

As the inverse problem is solved as an optimization problem, our objective is to minimize the norm $R(\vec{P})$,

$$\frac{\partial R}{\partial p_n} = \frac{\partial}{\partial p_n}(\vec{F}^T\vec{F}) = 0, \quad n = 1, ..., 5. \tag{18}$$

Considering a Taylor expansion,

$$F(\vec{P}^{k+1}) = F(\vec{P}^k + \Delta\vec{P}^k) = F(\vec{P}^k) + \sum_{n=1}^{N_P} \frac{\partial F(\vec{P})}{\partial p_n}\Delta p_n + O(\Delta p_n^2), \tag{19}$$

keeping only the terms up to the first order terms in Eq. (19), and plugging the resulting expression into Eq. (18), we obtain the normal equation,

$$J^T J\Delta\vec{P}^k = -J^T\vec{F}, \tag{20}$$

where the elements of the Jacobian matrix are

$$J_{mn} = \frac{\partial u_m}{\partial p_n}, \quad m = 1, 2, ..., M \quad \text{and} \quad n = 1, .., 5. \tag{21}$$

Summing up with a damping factor λ to improve the convergence behaviour we have the Levenberg-Marquardt method,

$$(J^T J + \lambda D)\Delta\vec{P} = -J^T\vec{F}, \tag{22}$$

where D represents the diagonal matrix.

Equation (22) is then written in a form convenient to be used in an iterative procedure,

$$\Delta P^k = -(J^{kT}J^k + \lambda^k D^k)^{-1}J^{kT}\vec{F}^k, \tag{23}$$

where k is the iteration index.

A new estimation of the parameters, $\vec{P}^{k+1}$, is calculated by

$$\vec{P}^{k+1} = \vec{P}^k + \Delta\vec{P}^k. \tag{24}$$

Please, note that the problem given by Eq. (22) is different from that given by Eq. (20). Nevertheless, the procedure aims at reducing the value of the damping factor with the iterations so that when convergence is achieved, the obtained solution is about the same as that for the original problem. The iterative procedure starts with an initial guess for parameters, $\vec{P}^0$, and new estimates, $\vec{P}^{k+1}$ are sequentially obtained using Eq. (24) with $\Delta\vec{P}^k$ given by Eq. (23) until the convergence criterion

$$|\frac{\Delta p_n^k}{p_n^k}| < \epsilon, \quad n = 1, ..., 5 \tag{25}$$

is satisfied, where ϵ is a small real number, such as 10^{-8}. The elements of the Jacobian matrix as well as the right hand term of Eq. (22) are calculated by using the solution of the direct problem defined by Eq. (1) to Eq. (13), as described in the previous section.

5. The Classical Approach

For the simple case of a turbulent flow over a flat plate at zero incidence, approximate methods based on the momentum integral equation can be easily deducted for the estimation of some flow parameters. In these methods, the boundary layer thickness is approximated by a suitable empirical equation; then, if the velocity distribution is considered to follow a certain form the momentum equation can be integrated to provide a relation between the displacement thickness, momentum thickness and shearing stress at the wall.

The assumption of a 1/7-th-power law of velocity distribution advanced by Prandtl relied on the idea that small differences in the velocity profile are not important since the drag will be evaluated from an integral. Thus, he considered that the velocity distribution in the boundary layer on a plate is identical with that inside a circular pipe. Hence integration of the momentum equation from the initial value $\delta = 0$ at $x = 0$ furnishes

$$\delta_2 = 0.036\, x \left(\frac{U_\infty\, x}{\nu}\right)^{-1/5}, \tag{26}$$

$$\delta = \frac{72}{7}\delta_2, \tag{27}$$

$$c_f = 0.0576\frac{u_e x}{\nu}, \tag{28}$$

$$u_\tau = u_e\sqrt{\frac{c_f}{2}}. \tag{29}$$

The four above equations together with the composite law of the wall/law of the wake can now be used to evaluate the velocity profile at any location from a given velocity profile at any other location. The steps are the following:

- From a given experimental velocity profile calculate δ_2.
- From Eq. 26 calculate the distance of the experimental velocity profile to a virtual plate origin.
- From Eq. 26 calculate δ_2 for the unknown profile.
- From Eq. 27 calculate δ for the unknown profile.
- From Eqs. 28 and 29 calculate u_τ for the unknown profile.
- From Eq. 14 construct the unknown velocity profile.

To implement Eq. 14 in the classical approach one needs to know the values of parameters κ, A and $\tilde{\pi}$. Here, the following values were considered:

$$\kappa = 0.4, \tag{30}$$

$$A = 5.0, \tag{31}$$

$$\tilde{\pi} = -0,05757\ln^2 R_{\delta 2} + 1.062\ln R_{\delta 2} - 4,317; \quad R_{\delta 2} < 5600, \tag{32}$$

$$\tilde{\pi} = 0.55 \geq 5600. \tag{33}$$

6. Experimental Apparatus and Instrumentation

The experiments were carried out in a low-speed wind tunnel located at the Laboratory of Turbulence Mechanics of COPPE/UFRJ. The wind tunnel is of open circuit type and has a 5 m long test section with square cross section of 0.67 m x 0.67 m. Wind speed is continuously variable from 0.5 to 3.5 m/s. The turbulent intensity level in the freestream was about 1.0%. Mean velocity profiles and turbulent intensity levels were measured by using a DANTEC hotwire anemometer series 55M with a standard P11 probe. A Pitot tube, a high precision inclined multi-tube manometer, and a computer controlled traverse gear were also used. Output signals of the hotwire anemometer were transmitted to a PC through a 16-bit data aquisition card.

Six longitudinal velocity profiles were measured at stations 3.20m, 3.25m, 3.30m, 3.35m, 3.40m and 3.45m from the beginning of the test section. All profiles were measured over the central line of the test section. Around 60 mean velocity measurement points were taken for each profile.

The friction velocity (u_τ), Coles's wake-strength parameter ($\tilde{\pi}$), boundary layer thicknes (δ), Von Kármán constant (κ) and the law of the wall constant (A) for each measured velocity profile were obtained through a program specially developed in the Mathematica^{TM} software package environment.

7. Results

The study is to be developed in three parts. The objective of the first part was to validate the numerical solution for the direct problem by comparison with some experimental data. The velocity profile measured at station $3.20m$ was used as the initial condition for the calculation of velocity profiles at the same stations where the measurements were performed. Figure 1 shows a comparison between velocity profiles obtained through the numerical simulation and the experimental profiles at station 3.45m. In Fig. 2, the same results are shown in inner variables.

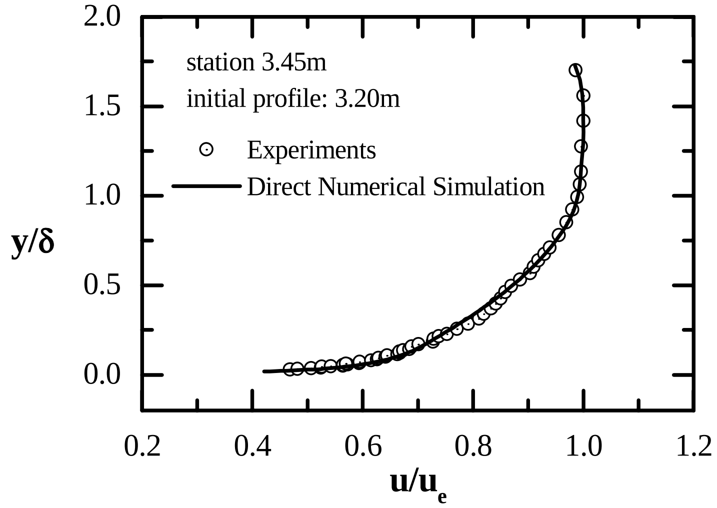

Figure 1: Validation of the numerical solution for the direct problem. Comparison of calculated and measured velocity profiles. Points denote the experimental data.

The second part aimed at estimating the upstream velocity profile, at station $x = 3.20m$, by the inverse method. These profiles were then to be compared with profiles obtained experimentally and by the classical approach. A single experimental profile at a downstream station was used in the inverse analysis. We successfully estimated the upstream velocity profile using measured profiles at stations $x = 3.25m$, $x = 3.35m$ and $x = 3.45m$. Figure 3 shows that the estimated upstream velocity profile agrees well with the measured upstream profile. Figure 4 shows the same comparison in inner variables.

In the third part, we checked the precision of the numerical simulation of the turbulent boundary layer, as a direct problem, if the estimated initial profile was used as the initial condition. We used the estimated values of parameters u_τ, κ, A, $\tilde{\pi}$ and δ to construct the initial condition and compared the results with that obtained by using directly the measured initial profile. Figure 5 shows a comparison between the velocity profiles at station 3.45m when: i) an inverse initial profile is used as an initial condition, ii) the classical approach is used to find the initial condition. Figure 6 shows a comparison of these results in inner variables.

The friction velocity is a flow parameter that is notoriously difficult to determine experimentally. In this work, the friction velocity was determined by means of a non linear regression program developed in the Mathematica software package for treatment of the experimental data.

Table 1, in addition, shows values of friction velocity estimated by the inverse method compared with the measured values of friction velocity at station $x = 3.20m$. As can be seen, the relative errors for u_τ were less than 5%. This is clear indication that inverse analysis can be used successfully to determine the friction velocity from mean velocity measurements in the downstream flow field.

Figures 7 and 8 show the predicted and measured values of C_f and of δ_2. The inverse method shows a clear advantage over the classical approach. In fact, for the present conditions, the classical approach tends to underestimates the values of δ what results in higher predicted values of C_f.

8. Conclusion

An inverse analysis for the estimation of upstream velocity profiles in an incompressible turbulent boundary layer over a smooth flat plate was carried out. The turbulent boundary layer direct problem with an algebraic

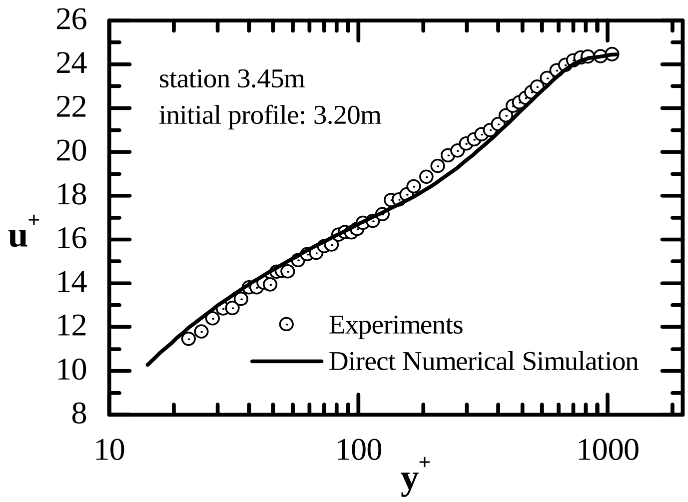

Figure 2: Validation of the numerical solution for the direct problem. Comparison of calculated and measured velocity profiles in inner variable.

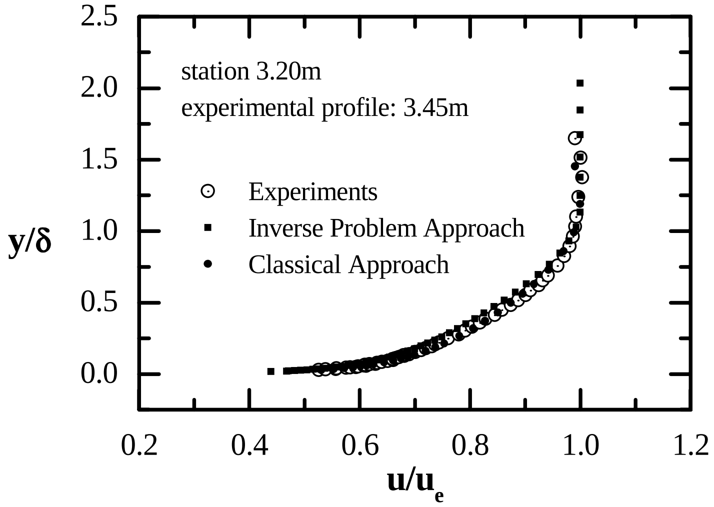

Figure 3: Comparison of estimated initial velocity profile with experimental data using one measured velocity profile at station 3.45m.

Table 1: Comparison of flow parameters at x=3.20m.

Parameter	Experiments	Inverse Problem (x=3.45m)	Classical Approach
$u_\tau(m/s)$	0.160	0.153	0.162
$\delta(m)$	0.0728	0.0637	0.0753
A	5.247	4.964	5.00
κ	0.420	0.414	0.410
$\tilde{\pi}$	0.495	0.600	0.445

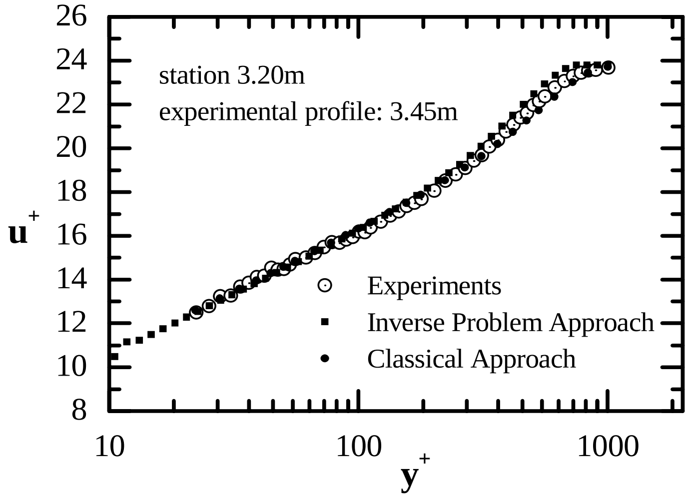

Figure 4: Comparison of estimated initial velocity profile in inner variable with experimental data using one measured at station 3.45m.

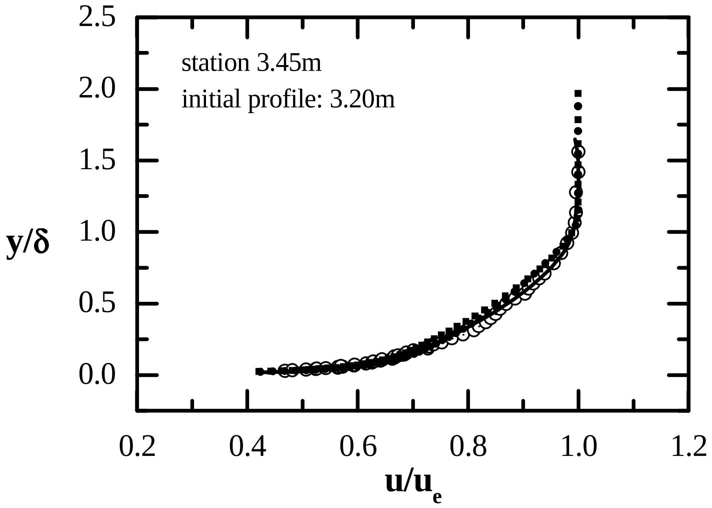

Figure 5: Calculated dowstream velocity profiles using experimental and estimated initial profiles. Points denote experiments; filled squares, initial profile given by inverse method; filled circles, initial profile given by classical approach; line, initial profile given by experiments.

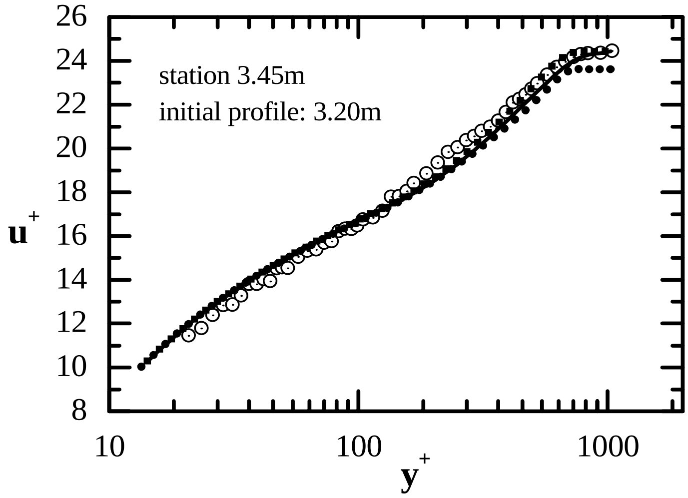

Figure 6: Calculated dowstream velocity profiles using experimental and estimated initial profiles; inner variables. Points denote experiments; filled squares, initial profile given by inverse method; filled circles, initial profile given by classical approach; line, initial profile given by experiments.

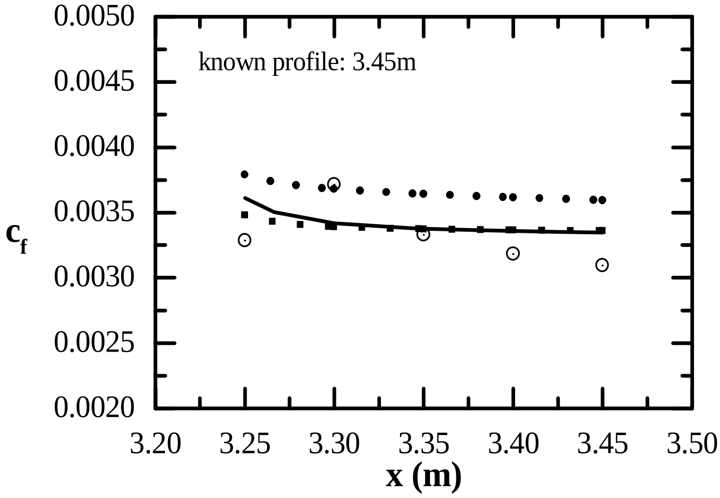

Figure 7: Estimation of friction coefficient using one measured station, 3.45m. Points denote experiments; filled squares, inverse method; filled circles, classical approach; line, direct method.

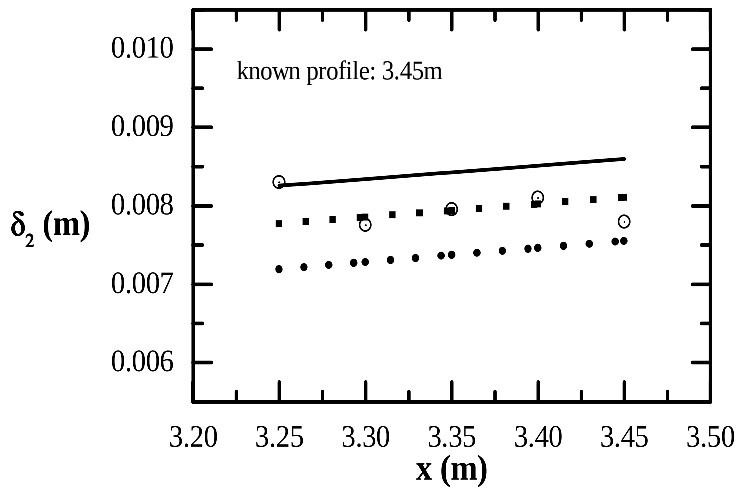

Figure 8: Estimation of momentum thickness using one measured station, 3.45m. Points denote experiments; filled squares, inverse method; filled circles, classical approach; line, direct method.

turbulence model was solved through a finite difference method, which was validated against data obtained in a low-speed wind tunnel. The inverse problem for the estimation of initial velocity profiles was formulated as a parameter estimation problem that searched for the friction velocity, the Von Kármán constant, the law of the wall constant, the Coles's wake-strength parameter and the boundary layer thickness at an upstream station in the turbulent boundary layer. We have shown, through comparison with the measured velocity profile at the same station, that the upstream velocity profile can be accurately estimated if experimental data of velocity measurement within 25 cm from the inlet station is used. The proposed inverse analysis can be used to generate an accurate and smooth initial velocity profile for numerical simulation of turbulent boundary layer and to determine accurately some boundary layer parameters, such as the friction velocity and the boundary layer momentum thickness, that are difficult to measure directly.

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