OPTIMIZING WELL INTERVENTION ROUTES

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Abstract. This work presents a method for optimizing the itinerary of workover rigs, i.e., the search for the route of minimum total cost, and demonstrates the importance of the dynamics of reservoir behaviour. The total cost of a route includes the rig expenses (transport, assembly and operation), which are functions of time and distances, plus the losses of revenue in wells waiting for the rig, which are also dependent of time. A reservoir simulator is used to evaluate the monetary influence of the well shutdown on the present value of the production curve. Finally, search algorithms are employed to determine the route of minimal cost. The Simulated Annealing algorithm was also successful in optimizing the distribution of a list of wells among different workover rigs. The rational approach presented here is recommended for management teams as a standard procedure to define the priority of wells scheduled for workover.

Keywords: *Well intervention, Reservoir Simulation, Optimization, Simulated Annealing*

1. INTRODUCTION.

1.1 Contents and goals.

This paper presents a study of the methodology for optimizing the itinerary of workover rigs. The task at hand is the search for the route of minimum total cost for a given list of wells waiting for the workover rig. The total cost includes the rig expenses (transport, assembly and operation), which are functions of time and distances, plus the losses of revenue in the wells waiting for the rig, which are dependent of time. Therefore, the total cost depends on the ordering of the wells in the itinerary.

In order to compute the wells' losses, it is necessary to evaluate the monetary influence of each well shutdown on the present value of the production curve. In the present work, this analysis is made through the use of a reservoir simulator. The results are presented in the form of *revenue loss factors*. Failure to account correctly for the revenue losses would mislead the itinerary programmer to select less than optimum routes.

Four algorithms to determine the optimum route are described – *follow the next closest well*, *follow the next best payoff*, *deep search* and *simulated annealing* – and tested comparing their results with a *full search*. The simulated annealing is also tested in optimizing the distribution of a group of wells among different workover rigs.

1.2 Context and motivation

Bottom-well equipments are prone to failure, which results in the reduction of production or shutdown of the well. The return to normal operational conditions requires the intervention of a workover rig. Therefore, when failure is detected, the well is logged in a list of wells waiting for workover, and then, the management team has to decide how to arrange such lists. Usually, the ordering of the list is the subject of discussions during a technical meeting, and the result is based mostly on opinion and superficial calculations. A rational procedure is needed, based on a consistent method. Ideally, it should be a procedure that integrates the existing knowledge of the reservoir behavior with the logistical programming of the production operations.

At the same time, the frequent shutdown of a large number of wells for several days may have a significant impact on the revenue of an oil field, thus justifying the concern to minimize the cost of workover.

With all the aforementioned in mind, a rational approach is recommended as a standard procedure to define the priority of the wells scheduled for workover. The proposed methodology is based on hard data of the wells and reservoirs, and on a clearly defined criteria – minimum cost.

2. LITERATURE REVIEW.

The following review is divided in two parts – quantification *of losses* and *search for the best path*, which are directly related to the main issues of the present study.

2.1 Quantification of losses

The fundamental concepts employed in the present work are detailed in Thuesen & Fabrycky (1984) and Newendorp (1975). Accordingly, the minimum acceptable rate of return will be taken as the smallest internal rate of return among the approved projects, and will be adopted in the computation of the present value of the monetary variables. Also, a delay in revenue will be equated to a loss and will comprise the total cost of the workover.

In the literature, it´s difficult to find specific works about the quantification of losses due to well shutdown, but there are a few works that have dealt with quantification of losses as a side issue.

In Bomar & Callais' work (1993) on drillbit selection, the production delay is simply neglected. The work of Eagle (1996) defines a constant annual loss, denominated "opportunity rate", related to the delay of the drilling of wells, but it doesn't indicate how to determine such rate. In the work of Narvaez & Ferrer (1991), the advanced volume of oil resulting from an improvement in the production rate is directly computed as a monetary gain, while the real gain was actually the increase in the present value of the revenues. This raises the topic of apparent gain and apparent loss, which can also be found in the works of Torres (1991), Costa (1995) and Brown (1980). Alves (1989), on the other hand, accounts for the real monetary loss, and correctly indicates the basic concept of how to determine it, but falls short in detailing it, using some simplifications that do not consider the influence of the reservoir behavior. Still, these simplifications may be accurate enough in some practical cases. It´s also worth mentioning the works of Clegg et al (1993), for recommending the tabulation of the economic consequences when evaluating gas-lift methods, and Kikuchi (1996), for employing the present value of production revenues in the evaluation of projects for oilfield development and for using a reservoir simulator in such studies.

2.2 Search for the best path

The search for the best path for a workover rig, is a problem in the same broad category as the "*travelling salesman problem*" and the "*minimum running distance problem*"; in short terms, it's a combinatorial optimization problem. It consists of establishing sequences of actions or objects aiming at a certain goal, which is translated to obtaining the maximum or minimum of a variable. Such problems are called "NP-difficult" when there aren't algorithms to solve them in a period of time that is a polynomial function of the number of variables (Papadimitriou & Steiglitz, 1982). This is just the case of the workover rig itinerary problem.

There are exact methods of solution and approximate methods of solution. The exact methods may require an enourmous amount of memory and computing time, sometimes exceeding the capacity and power presently available in the whole planet. The approximate methods are not mathematically guaranteed to arrive at the real solution nor to provide a unique solution, but many times, a solution close to the exact solution is good enough. These approximate methods are built upon heuristic approaches (for definition, see Nicholson, 1971), and must be validated through experiment. A sub-class of heuristic methods, denominated meta-heuristic, uses random processes (Elmer & Gillet, 1993). In the present work, the simulated annealing (Press, 1992), one of the meta-heuristic techniques for solving combinatorial optimization problems, is applied to the search for the best workover rig route.

3. REVENUE LOSS DUE TO WELL SHUTDOWN.

3.1 Definitions

In the present work, monetary values are expressed in terms of equivalent cubic meters of petroleum – "ecmp" (or equivalent barrels of petroleum – "ebp"), instead of a particular nation's currency. In addition, the present value of a monetary variable with value v at time t is calculated according to $PV(v@t) = v\mathrm{e}^{-kt}$, where $k = \ln(1+T)/t_\mathrm{u}$, with T being the adopted rate of return with respect to the unit time t_u; alongside, the present value of a cumulative variable X, with a distribution x along a period of time, is computed from $PV(X) = \int x\mathrm{e}^{-kt}\,dt$.

The actual loss L is equated to the decrease of the present value of the production revenues, caused, mainly, by the delay due to the shutdown; therefore

$$L = P - P_\mathrm{d}, \qquad (1)$$

where P and P_d are the present values of the produced oil calculated, respectively, without and with the delay caused by the well shutdown. The loss will depend on the production rate at the time of shutdown – Q_s, on the period that the well is kept shut – t_s, and on the well-reservoir configuration and conditions. It's usefull to express the loss in terms of the "*loss factor*", a non-dimensional quantity defined by

$$F = \frac{L}{L_\mathrm{s}}, \qquad (2)$$

where, $L_s = Q_s t_s$ is an approximate measure of the expected volume of oil that was **not** produced during the shutdown – the *apparent loss*.

3.2 Case studies

Figure 1 shows an example of production rate history for a reservoir with one well: the dashed line represents normal operation without shutdow, while the thick line displays production after the interruption, where two features deserve notice. Firts, there is a temporary production increase due to the evolution of the pressure profile inside the reservoir (pressure build up) during the interruption. Second, after this spurt dies out, the production behavior is restored to what it would have been without the shutdown, except for a time lag, t_d (actual delay). This delay is a function of the shutdown period t_s. Clearly, the two histories have different present values. The delay causes the present value to fall, while that initial spurt of oil provides a small anticipation of revenue, which pushes the present value up a bit, but not enough to offset the effect of the delay. The thin line in Figure 1 represents the history of the "frozen" reservoir, that is, assuming that the pressure profile inside the reservoir doesn't change during the interruption, thus the well resumes operating "as it was" just before shutdown, there is no spurt of oil, only a delay.

Figure 2 illustrates the behavior of the presure profile in the neighborhood of the well (r – distance from well, ΔP – pressure depletion at r). The thick black line represents the state of the reservoir at shutdown, the thick gray line represents the state at re-opening, and the dots represent the state a long period after the interruption. The thin black line depicts the state to which the thick black line would have evolved without the interruption. These last two states are lagging by a period t_d.

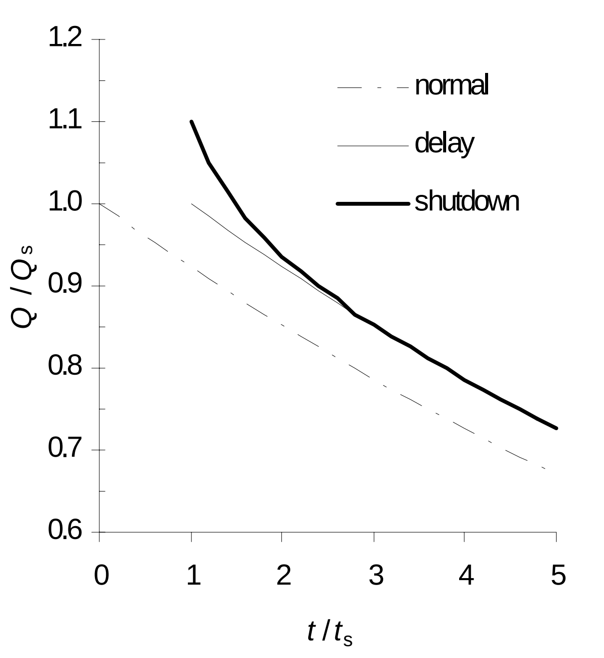

Figure 1 – Production rate history

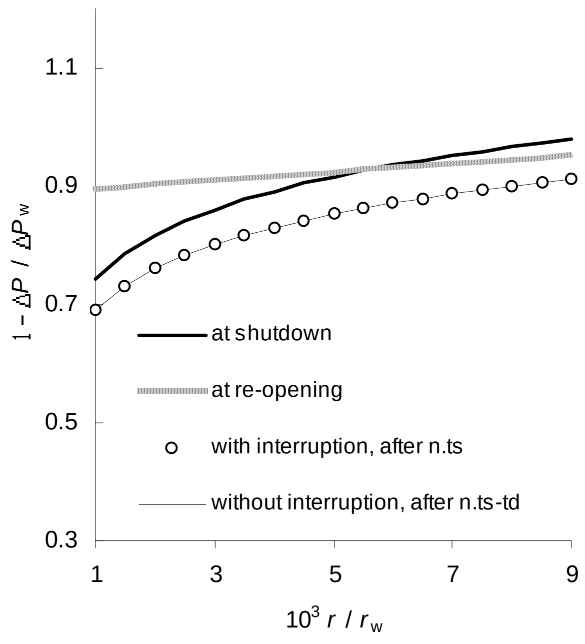

Figure 2 – Pressure profile

The ratio between the delay period t_d and the shutdown period t_s , $I = t_d / t_s$, was found to be fairly close to unit, for the one-well reservoir, and, in general, approximately equal to the ratio of the well production rate, Q_w, to the total reservoir production rate, Q_r, that is,

$$I = \frac{Q_w}{Q_r} . \tag{3}$$

Figures 3 and 4 present examples of the determination of loss factors, employing a reservoir simulator. Figure 3 shows the result for a radial reservoir with a single well at the center; it is noticed that the real loss is about 10% of the apparent loss, in this example. This result confirms that the use of the apparent loss as a criteria for economic evaluation of well shutdown may lead to gross errors, and that, consequently, it is important to consider the correct concept of loss as defined by Equation 1. Figure 4 shows the result for a square reservoir with two wells at opposite corners, where only one well is closed.The continuous line shows the behavior with interference between the wells, while the dashed line represents the case of no interference. When interference is significant, the production of the open well will increase a little during shutdown of the other well; thus compensating in part the revenue loss of the closed well. For the same reason, the final delay time is only a fraction of the shutdown time, as expressed in Equation 3.

Other cases were studied: reservoir with four wells, reservoir with water injection, reservoir with gas coning. The detailed analysis is not the main subject of this paper, and is found in Paiva (1997), but the main lesson is that reservoir simulation is a valuable tool in the evaluation of revenue loss factors due to well shutdown. Therefore, we propose that the reservoir management team performs a periodical analysis of shutdown losses, and registers the loss factors in a database for use by the workover rig programmer.

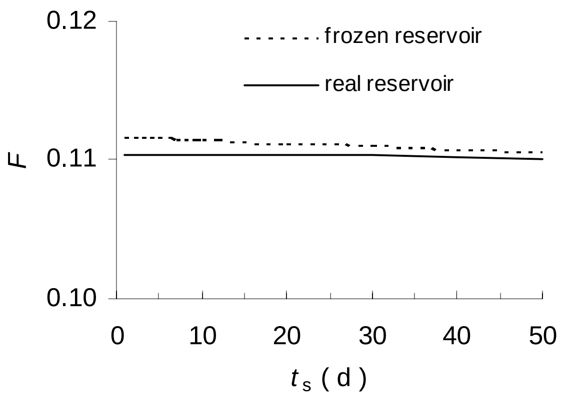

Figure 3 – Case study A: one-well reservoir.

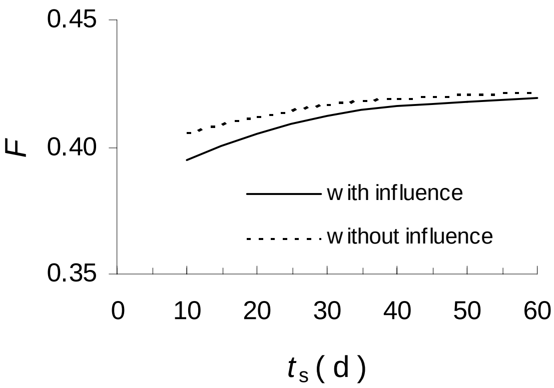

Figure 4 – Case study B: two-well reservoir

4. ITINERARY OF WORKOVER RIGS.

4.1 Cost

The total cost of a route includes the revenue loss of each well - the daily revenue loss of the well, $R_i = F_i Q_{si}$, multiplied by the time the well remains shut awaiting for the workover, t_{si} :

$$L_i = R_i t_{si} \ , \tag{4}$$

where,

$$t_{si} = t_{s(i-1)} + \frac{d_{i-1,i}}{v_{WR}} + t_{wi} \quad . \tag{5}$$

In Equation 5, the first term is the time that the previous well remains shut; the second term is the time it takes for the rig to travel the distance between the two consecutive wells, $d_{i-1,i}$, with average speed v_{WR}; the third term is the time required for workover. The total cost also includes the rig's cost for each well service - a flat fee for "transport, assembly and disassembly", C_{tad}, plus the travelling cost $C_t = c_d.d_{i-1,i}$, where c_d is the distance fee, plus the operation cost $C_{op} = c_t.t_{wi}$, where c_t is the worktime fee:

$$C_i = C_{tad} + c_d d_{i-1,i} + c_t t_{wi} \quad . \tag{6}$$

Finally, the objective function to be optimized (minimum total cost of the itinerary) becomes

$$w = \sum_{i=1}^{m} \ (L_i + C_i) \quad . \tag{7}$$

4.2 Optimization

The *full search* algorithm is the reference for comparison among different algorithms. It provides the exact solution, but requires the calculation of the cost of **all** possible paths, that is **m!** possibilities for *m* wells. The basic concern behind the *next closest well* algorithm is the reduction of the travelling cost, therefore, at each step it selects the well closest to the present position of the rig - smallest distance $d_{WR,i}$. In the *next best payoff* algorithm, a comparison is made at each step to determine which well is the more worthwhile, according to some game rule. In the present work, the rule is to choose the well that offers the greatest value for the parameter G (Paiva, 1997) defined by

$$G_i = \frac{R_i}{t_{wi}} - \left(c_d + \frac{(2 + R_f)}{v_{WR}} \right) \frac{d_{WR,i}}{t_{wi}} \quad , \tag{8}$$

where $R_f = \sum_{j=i+1}^{m} R_j$. Alternatively, other rules may be formulated. The *deep search* algorithm is an extension of the *next best payoff*, where, instead of picking only the well with the greatest G, a group of wells with $G \geq G_{ref}$ is selected, generating a spread of possibilities. The reference value G_{ref} is at a level, between the maximum and minimum values G_{max} and G_{min}, given by

$$G_{ref} = G_{min} + b(G_{max} - G_{min}) \quad , \tag{9}$$

depending on the strictness parameter b, which ranges from 0 to 1; if $b = 1$ the search is the same as *next best payoff*, if $b = 0$ the search is the same as the *full search*. Last, the *simulated annealing* algorithm starts with any initially defined itinerary, then generates random alterations in the itinerary to test their effect on the total cost; allowing modification of the itinerary when the total cost decreases. To prevent the procedure from getting stuck in a local minimum, the algorithm also allows alterations that cause a certain degree of increase in the objective function (a process colorfully referred to as "system shaking").

Generally, the *next best payoff* results in better routes than the *next closest well*, because it takes into account the worth of the wells, however, it may not provide the path of global minimum cost. The *deep search* fares better than the simpler *next best payoff*, because it explores more alternatives, on the other hand it takes more computations. The simulated *annealing* is better when dealing with a large number of wells, but it expends too many unnecessary calculations when rearranging a small number of wells.

Finally, the *simulated annealing* was employed to distribute a group of wells among the itineraries of several workover rigs. This kind of problem is more difficult, and an intuitive approach would hardly provide an optimum solution. That's another advantage of the rational computational methodology recommended in the present work over the lax decision-making management meetings, which are vulnerable to arguments of questionable validity.

4.3 Case studies

In all the following examples the closed wells are a subgroup of the fictitious region depicted in Figure 5, composed of several clusters located in reservoirs designated by two letter codes. A database contains the distances between any two clusters and the distances between wells in the same cluster, plus the revenue loss factors and other optimization parameters (Table 1).

The first example is described by this job-matrix $\begin{Bmatrix} 1 & 2 & 3 & 4 \\ \text{RJ} & \text{SP} & \text{GO} & \text{SC} \\ 36.4 & 36.4 & 72.7 & 36.4 \\ 3.1 & 1.0 & 2.0 & 1.0 \end{Bmatrix}$, where the first line gives the well number, the second line its location, the third line its production rate Q_s, in m^3/d, and the fourth line its workover time t_w, in days.The rig is initially at RJ.

The results of the optimization are shown in Table 2. The best route is {4 2 3 1}, costing 796 ecmp. It's observed that the simulated annealing tested a large number of routes, more than the number of actual possibilities (24), because it tested the same route more than once. Table 3 shows the results that would have been obtained *if the effect of the loss factors had been neglected*, that is, using the apparent loss. This clearly led to the wrong solution {3 2 4 1}, with a real cost of 809 ecmp, 13 ecmp above the best route.

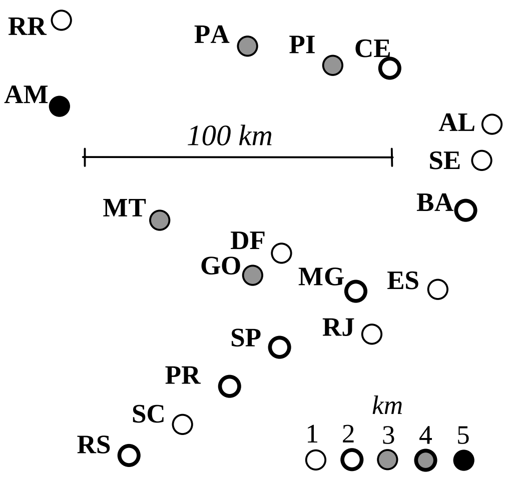

Figure 5 - Map of fictitious production region
(for illustration only, not true scale).

Table 1 - Optimization parameters

C_{tad}	15	ecmp
c_d	0.072	ecmp/km
c_t	2.16	ecmp/h
v_{WR}	30	km/h

F

AL	.24	PA	.42
AM	.75	PI	.36
BA	.63	PR	.48
CE	.50	RJ	.32
DF	.75	RR	.44
ES	.87	RS	.48
GO	.51	SC	.99
MG	.93	SE	.71
MT	.77	SP	.65

Table 2 - Example 1

Search	Itinerary	Cost 10^3ecmp	Routes tested
	1 2 3 4	1.102	
nxt closest	1 2 4 3	1.065	1
nxt best	4 2 3 1	0.796	1
deep	4 2 3 1	0.796	24
sim. ann.	4 2 3 1	0.796	800
full	4 2 3 1	0.796	24

Table 3 - Example 1 ($F_i = 1$)

Search	Itinerary	Cost 10^3ecmp	Routes tested
	1 2 3 4	1.474	
nxt closest	1 2 4 3	1.478	1
nxt best	2 3 4 1	1.174	1
deep	3 2 4 1	1.171	24
sim. ann.	3 2 4 1	1.171	800
full	3 2 4 1	1.171	24

In the second example, the rig is initially at ES and the job-matrix is

$$\begin{Bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ SP & SE & AL & AL & GO & MG & MT & RS & SC \\ 11.3 & 4.5 & 31.9 & 2.4 & 15.6 & 14.9 & 13.9 & 16.6 & 14.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{Bmatrix}.$$

In the third example, the rig is also at ES and the job-matrix is

$$\begin{Bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\ SP & SE & AL & AL & GO & MG & MT & RS & SC & CE & PI & SE & AL & AL & GO & PI & SE & AL & AL & GO \\ 11.3 & 4.5 & 31.9 & 2.4 & 15.6 & 14.9 & 13.9 & 16.6 & 14.0 & 21.4 & 29.7 & 4.5 & 31.9 & 2.4 & 15.6 & 29.7 & 4.5 & 31.9 & 2.4 & 15.6 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 2.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 3.0 & 1.0 & 1.0 & 1.0 & 1.0 & 3.0 \end{Bmatrix}$$

The results are given in tables 4 and 5. *Without the loss factors*, the result for example 2 would have been {3 2 6 8 9 5 7 1 4}, costing 1047 ecmp, 89 ecmp above the best route, while for example 3 one would have arrived at itinerary {3 18 13 11 16 10 5 7 6 1 8 9 15 20 17 12 2 14 19 4}, costing 3226 ecmp, 115 ecmp above the best route. Currently, it's impossible to apply the exact full search to 20 wells, the number of calculations is staggering.

Table 4 - Example 2

Search	Itinerary	Cost 10^3ecmp	Routes tested
	1 2 3 4 5 6 7 8 9	1.142	
nxt closest	6 1 9 8 5 7 2 3 4	0.972	1
nxt best	6 9 8 1 5 7 3 2 4	0.962	1
deep	6 9 8 1 7 5 3 2 4	0.958	354
sim. ann.	6 9 8 7 5 1 3 2 4	0.960	1800
full	6 9 8 1 7 5 3 2 4	0.958	362880

Table 5 - Example 3

Search	Itinerary	Cost 10^3ecmp	Routes tested
	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	3.585	
nxt closest	6 1 9 8 5 15 20 7 16 11 10 4 13 14 18 19 3 2 12 17	3.506	1
nxt best	6 1 9 8 15 5 20 7 16 11 10 13 18 3 12 2 17 4 19 14	3.448	1
deep	6 5 7 9 8 1 20 15 11 16 10 3 18 13 17 2 12 19 4 14	3.378	24192
sim. ann.	6 10 16 11 7 5 9 8 1 3 13 18 17 2 12 15 20 19 4 14	3.111	6000
full			(2.4×10^{18})

The final example deals with the distribution of 20 wells for 3 (or 2) workover rigs (WR), initially at RJ, GO and SC, with the following job-matrix

$$\left\{\begin{array}{ccccccccccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 \\ RJ & GO & SC & RJ & SP & MG & BA & SE & ES & SC & RS & GO & PA & AL & DF & PR & RS & MG & SP & RJ & ES & AM & RR \\ WR & WR & WR & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\ & & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 3 & 1 & 1 & 1 & 1 & 1 & 3 \end{array}\right\}.$$

Only the computer code for the simulated annealing was adapted to deal with this kind of problem. It ran in 50 seconds on a 90 MHz processor, testing 1.05×10^6 routes, resulting in the itineraries {**1:** 6 21 9 7 4 20 14} // {**2:** 12 10 5 19 16 17 11} // {**3:** 18 8 15 22 13 23}, costing 1878 ecmp. Using only the rigs 1 and 2, the calculation took 20 seconds, and the distribution selected was {**1:** 6 21 9 15 5 7 22 8 14 13 23} // {**2:** 10 16 19 4 11 17 20 18 12}, costing 2444 ecmp. Table 6 summarizes the results for the two cases, with the total distance travelled by each rig and their total operating time. As shown, the procedure can determine the optimum distribution of wells among a fixed number of rigs, and is also helpful in the analysis of the optimum number of rigs.

Table 6 - Example 4

Rig	3 rigs		2 rigs	
	km	days	km	days
1	230	8.2	668	15.3
2	137	8.1	274	13.5
3	413	7.3	0	0
sum	780	23.6	942	28.8
cost	*1878* ecmp		*2444* ecmp	

5. CONCLUSION.

Clearly, the behavior of the reservoir has a great impact on the optimization procedure due to the effect of the loss factors. Therefore, knowledge of the reservoir characteristics and its dynamics is of paramount importance. Also, reliable simulation models and codes are needed for an accurate analysis.

This work has established the usefulness of a computational procedure to solve a practical operational problem - the optimization of the itinerary of workover rigs. The procedure provides a consistent criteria to be used in all cases, setting a standard for the programming of operations. Consequently, intuitive, fallible, solutions and trial-and-error approaches can be avoided.

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