NUMERICAL OPTIMIZATION AND PERFORMANCE COMPARISON OF STAGGERED CIRCULAR AND ELLIPTIC TUBES IN FORCED CONVECTION

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Abstract. *In this study, a two-dimensional (2-D) heat transfer analysis was performed in circular and elliptic tubes heat exchangers. The numerical results for the equilateral triangle staggering configuration, obtained with the finite element method were then validated qualitatively by means of direct comparison to previously published experimental results for circular tubes heat exchangers (Stanescu et al., 1996). Next, a numerical geometric optimization was conducted to maximize the total heat transfer rate between the given volume and the given external flow both for circular and elliptic arrangements, for general staggering configurations. The results are reported for air in the laminar regime, in the range* $300 \le \mathrm{Re}_L \le 800$*, where L is the swept length of the fixed volume. Circular and elliptical arrangements with the same flow obstruction cross sectional area were compared on the basis of maximum total heat transfer. The effect of ellipses eccentricity was also investigated. A relative heat transfer gain of up to 13 % is observed in the optimal elliptical arrangement, as compared to the optimal circular one. The heat transfer gain, combined with the relative pressure drop reduction of up to 25 % observed in previous studies (Brauer, 1964; Bordalo and Saboya, 1995) show the elliptical arrangement has the potential for a considerably better overall performance than the traditional circular one.*

Keywords: *Finite element method, Heat exchangers, Geometric optimization*

1. INTRODUCTION

Economic and environmental considerations have brought the need for performance improvement on all engineering applications, aiming to rationalize the use of available energy and reduction of lost work. Many industrial applications require the use of tubes heat exchangers, that have to be sized according to space availability. Therefore, the volume to be occupied by the array of tubes is fixed. The volume constrained optimization problem consists on finding the optimal spacing between tubes (or cylinders), of a known geometry, such that maximum overall heat transfer (or thermal conductance) between the array and the surrounding fluid is achieved. A typical application of such fundamental optimization results is on the development of cooling techniques for electronic packages. Considerable effort has been put on finding optimal spacings for many different types of geometries, both for natural and forced convection (Bar-Cohen and Rohsenow, 1984; Kim et al., 1991; Knight et al., 1991; Anand et al., 1992; Knight et al., 1992; Bejan and Sciubba, 1992; Bejan and Morega, 1993; Bejan, 1995; Bejan et al., 1995; Stanescu et al., 1996; Ledezma et al., 1996; Fowler et al., 1997).

Stanescu et al. (1996) reported the optimal spacing of circular cylinders in free-stream cross-flow forced convection, which followed the study presented by Bejan et al. (1995) on the optimization of arrays of circular cylinders in natural convection. Both studies considered only equilateral triangle staggering configurations. The tube geometry was not investigated in those studies as an additional degree of freedom. The elliptic tube geometry is expected to perform better, aerodynamically, than the circular one, i.e., combining reduction in total drag force and increase in total heat transfer, as it was reported by Rocha et al. (1997), when comparing elliptic and circular sections in the specific cases of one and two-row tubes and plate fin heat exchangers. The results showed a heat transfer gain of up to 18 % when comparing elliptic to circular arrangements in the studied cases.

The present study focuses on the geometric optimization (optimal spacing) of staggered circular and elliptic tubes in a fixed volume. The problem is treated in a fundamental (geometric) sense, without specific reference to an application (electronics cooling, compact heat exchangers, etc). The optimizations are conducted numerically, by using the finite element method to solve the conservation equations (mass, momentum and energy), to obtain the velocity and temperature fields inside the arrays, thereafter computing the overall heat transfer rate between the tubes and the fluid. First, the numerical results obtained with the finite element code are validated by direct comparison to previously published experimental results for circular tubes heat exchangers with equilateral triangle staggering configurations (Stanescu et al., 1996). Next, the equilateral triangle staggering configuration is relaxed and numerical optimization results are obtained for circular and elliptic arrangements, for general staggering configurations. Circular and elliptical arrangements with the same flow obstruction cross sectional area are then compared on the basis of maximum total heat transfer. Appropriate nondimensional groups are defined and the optimization results reported in dimensionless charts.

2. THEORY

Figure 1 is a general simple sketch of the problem configuration. It was shown by Fowler and Bejan (1994) that in the laminar regime, the flow through a large bank of cylinders can be simulated accurately by calculating the flow through a single channel, such as that illustrated by the unit cell seen in Fig. 1. Because of the geometric symmetries, there is no fluid exchange and no heat transfer between adjacent channels. In Figure 1, L and H are the length and height of the array, and not shown is the width of the array (tube length), W.

The governing equations are the mass, momentum and energy equations which were simplified in accordance with the assumptions of two-dimensional incompressible steady-state flow with constant properties, for a Newtonian fluid:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re_L}\left[\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right] \qquad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re_L}\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right] \qquad (3)$$

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{1}{Pe_L}\left[\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right] \tag{4}$$

where dimensionless variables have been defined as follows:

$$(X,Y)=\frac{(x,y)}{L}\ ;\ P=\frac{p}{\rho U_\infty^2} \tag{5}$$

$$(U,V)=\frac{(u,v)}{U_\infty}\ ;\ \theta=\frac{T-T_\infty}{T_w-T_\infty}\ ;\ Re_L=\frac{U_\infty L}{\nu}\ \text{and}\ Pe_L=\frac{U_\infty L}{\alpha} \tag{6}$$

where (x, y) – Cartesian coordinates, m; p – pressure, N/m^2; ρ - fluid density, kg/m^3; U_∞ - free stream velocity, m/s; (u, v) – fluid velocities, m/s; T – fluid temperature, K; T_∞ - free stream temperature, K; T_w - tubes surface temperature, K; L – length of the array in the flow direction, m; ν - fluid kinematic viscosity, m^2/s and α - fluid thermal diffusivity, m^2/s.

To complete the problem formulation, the following boundary conditions are specified for the unit cell of Fig. 1:

$$\text{A) } U=1;\frac{\partial V}{\partial Y}=0;\theta=0 \tag{7}$$

$$\text{B) } \frac{\partial U}{\partial X}=0;V=0;\frac{\partial\theta}{\partial X}=0 \tag{8}$$

$$\text{C) } U=V=0;\ \theta=1 \tag{9}$$

$$\text{D) } \frac{\partial U}{\partial Y}=\frac{\partial V}{\partial Y}=0;\frac{\partial\theta}{\partial Y}=0 \tag{10}$$

Once the geometry of the computation domain defined by the unit cell of Fig. 1 is specified, Eqs. (1) - (10) deliver the resulting velocities, pressure and temperature fields in the domain.

The optimization objective is to find the optimal spacing between rows of tubes, S, such that the volumetric heat transfer density is maximized, subject to a volume constraint. The engineering design problem starts by recognizing the finite availability of space, i.e., an available space $L\times H\times W$ as a given volume that is to be filled with a heat exchanger. To maximize the volumetric heat transfer density means that the overall heat transfer rate between the fluid inside the tubes and the fluid outside the tubes will be maximized.

In order to perform the comparison between the elliptic and circular arrangements, a criterion was adopted to preserve similar flow characteristics in the unit cell, i.e., the flow obstruction cross sectional areas of the arrangements under comparison were made equal. The same criterion was adopted by Rocha et al. (1997) for tubes and plate fin heat exchangers. Hence, in all cases, the diameter of the circles, D, was equal to the smaller axis of the ellipses, 2b. The laminar regime for air crossing a bundle of tubes is observed when $Re_D\le 200$, which is the Reynolds number based on the tube diameter (Stanescu et al., 1996).

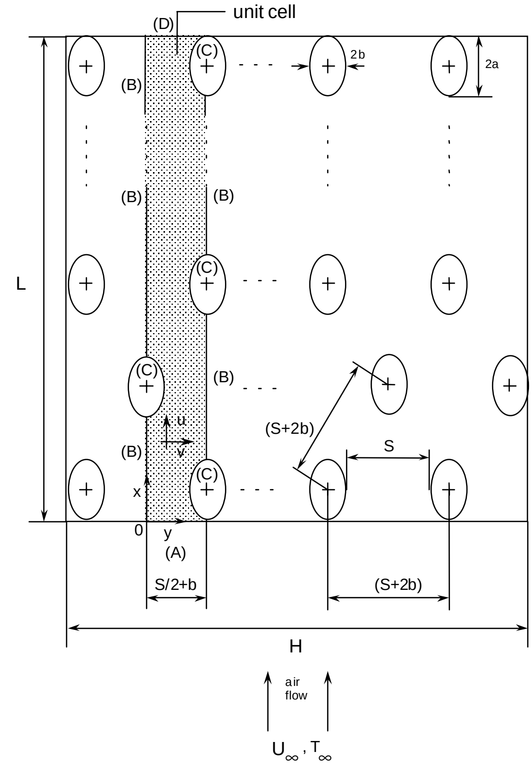

Figure 1 - Problem sketch and computational domain.

3. RESULTS AND DISCUSSION

The finite element method was used to discretize the fluid flow and heat transfer governing equations (1) – (10) and a 2-D isoparametric, four-noded, linear element was implemented for the finite element analysis program, FEAP (Zienkiewicz and Taylor, 1989). This way, the velocities and temperature fields in the unit cell of Fig. 1 were determined.

Initially, the dimensionless overall thermal conductance $\tilde{q}$, or volumetric heat transfer density for the circular arrangements was defined as follows, for the sake of comparison with the results of Stanescu et al. (1996):

$$\tilde{q} = \frac{q/(T_w - T_\infty)}{kLHW/(2b)^2} \tag{11}$$

where the overall heat transfer rate between the tubes and the free stream, q, has been divided by the constrained volume, LHW; k – fluid thermal conductivity, W/(m.K), and 2b = D – ellipse smaller axis or tube diameter.

The calculation of $\tilde{q}$ is conducted numerically, re-arranging equation (11) as follows:

$$\tilde{q} = \frac{N_{ec}}{2}\left[\frac{2b}{L}\right]^2 \frac{2b}{H}\sum_{i=1}^{N}\left[\frac{\overline{q}_i^{\,"} L\pi}{k(T_w - T_\infty)}\right] \tag{12}$$

where $\overline{q}_i^{\,"}$ - average normal heat flux at the i-th tube surface, W/m^2; N_{ec} - number of elemental channels (or unit cells) and N – number of tubes in one elemental channel.

In equation (12), $\overline{q}_i^{''}$ is calculated from the local normal heat flux at the tube surface, as

$$q_i^{''} = -k\left(\frac{\partial T}{\partial n}\right)_i \quad , \; i = 1, \ldots, N \tag{13}$$

where n – normal direction.

The computation of the heat fluxes for equation (13) is done by post-processing the temperature results obtained from the finite element solution.

For the comparison between the elliptic and circular arrangements, the dimensionless overall thermal conductance is computed alternatively from a balance of energy in one elemental channel, noting that:

$$q = N_{ec}\dot{m}_{ec}c_p\left(\overline{T}_{out} - T_\infty\right) \tag{14}$$

where $\dot{m}_{ec} = \rho U_\infty\left(\frac{S+2b}{2}\right)W$, which is the fluid mass flow rate entering one elemental channel, kg/s; c_p - fluid specific heat at constant pressure, J/(kg.K), and $\overline{T}_{out}$ - average fluid temperature at the elemental channel outlet.

The dimensionless overall thermal conductance computed by equation (11), using equation (14) is therefore renamed as $\tilde{q}_*$. The calculation of $\tilde{q}_*$ is conducted numerically, re-arranging equation (11) as follows:

$$\tilde{q}_* = \frac{N_{ec}}{2}\Pr Re_L\left[\frac{2b}{L}\right]^2\frac{2b}{H}\left(\frac{S}{2b}+1\right)\overline{\theta}_{out} \tag{15}$$

The results obtained from equation (15) are therefore expected to be more accurate than the results obtained with equation (12). The reason is that the former are obtained directly from the finite element temperature solution, whereas the latter are obtained from post-processing the finite element solution. It is well known that the numerical error in the derivative of the solution is larger than the numerical error in the solution itself.

To obtain accurate numerical results, several mesh refinement tests were conducted. The monitored quantity was the dimensionless overall thermal conductance, computed either with equation (12) or with equation (15), according to the following criterion:

$$\varepsilon = \left|\tilde{q}_{*,finemesh} - \tilde{q}_{*,coarsemesh}\right| / \left|\tilde{q}_{*,finemesh}\right| \leq 0.01 \tag{16}$$

The criterion defined by equation (16) was used to test the extension of the computational domain defined in the unit cell of Fig. 1. An extra-length L/2 had to be added to the computational domain, upstream and downstream of the unit cell to represent the actual flow, and satisfied equation (16), when compared to an extra-length L. For all cases, the mesh was established with 5460 nodes and 5180 elements, which satisfied equation (16) when compared to a mesh with 5670 nodes and 5380 elements. All meshes were more refined in the regions close to the tubes where the highest gradients in the solution were expected.

The numerical results obtained with the finite element code are validated by direct comparison to previously published experimental results for circular tubes heat exchangers with equilateral triangle staggering configurations obtained with L = 39.2 mm, H = 35.2 mm, W = 134 mm and D = 6.35 mm (Stanescu et al., 1996).

All the arrangements in this study (elliptic and circular) had $N_{ec} = 6$ and N = 4.

The numerical results shown in Fig. 2 were obtained with equation (12). Figure 2 also shows the experimental results obtained by Stanescu et al. (1996) for $Re_D = \frac{U_\infty D}{\nu} = 50 \text{ and } 100$. The experimentally determined $\tilde{q}$ agrees qualitatively with the numerical results, mainly with respect to the identification of $(S/D)_{opt}$. The agreement is remarkable if we think that the tested array was not a large bank of cylinders and, in the experiments, with uniform heat flux, while in the numerical simulations it was infinitely wider and with isothermal cylinders.

Next, numerical optimization results are obtained for the circular and elliptic arrangements, for general staggering configurations. The dimensionless thermal conductance is hereafter computed with equation (15), in the form of $\tilde{q}_*$.

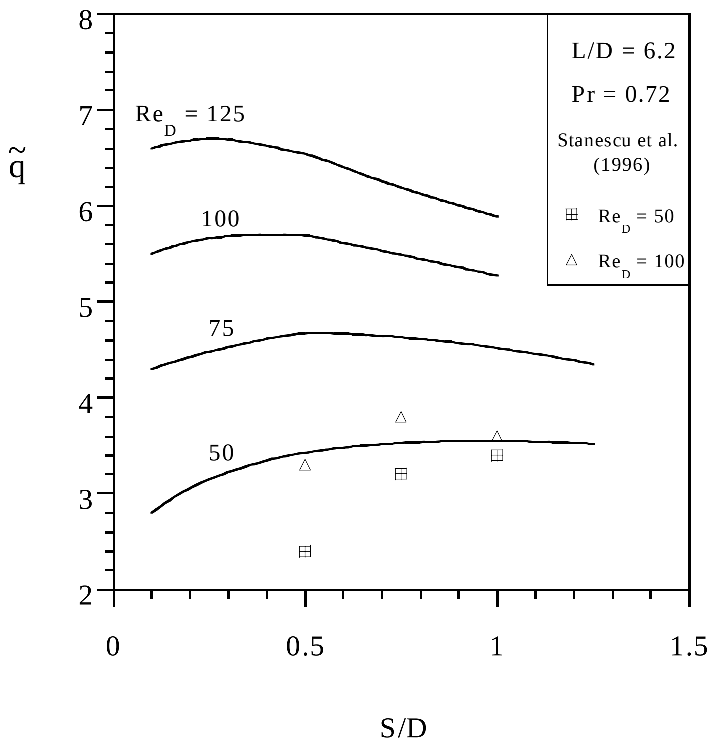

Figure 2 - Numerical and experimental results for circular tubes heat exchangers with equilateral triangle staggering configurations.

Figures 3 and 4 show maxima for $\tilde{q}_*$ with respect to (S/2b), for two different values of ellipses eccentricity, i.e., e = 0.8 and 0.65:

$$e = \frac{b}{a} \tag{19}$$

where b – smaller ellipse semi-axis and a – larger ellipse semi-axis.

The influence of the variation of Re_L is also investigated in Figs. 3 and 4. As Re_L increases $\tilde{q}_*$ increases. The maximum is less pronounced for lower values of Re_L.

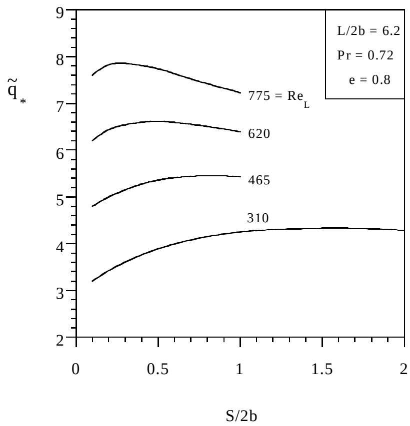

Figure 3 - Numerical results for elliptic tubes (e = 0.8) heat exchangers with general staggering configurations.

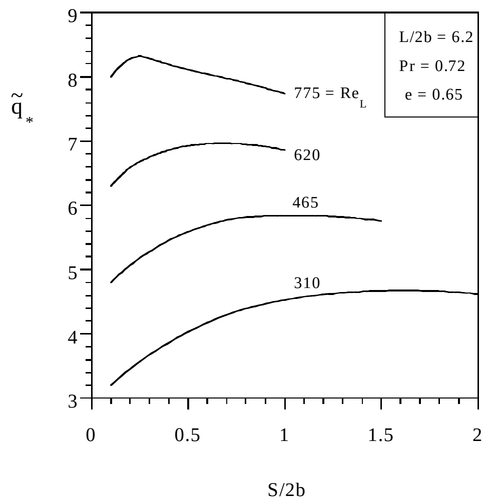

Figure 4 - Numerical results for elliptic tubes (e = 0.65) heat exchangers with general staggering configurations.

Figures 5 and 6 show the effect of ellipses eccentricity on $\tilde{q}_*$, for $Re_L = 465$ and 620, respectively. As the eccentricity decreases, $\tilde{q}_*$ increases, therefore the elliptic geometry improves the overall heat transfer rate between the tubes and the free stream.

The results reported in Figs. 3 – 6 are summarized in Figs 7 and 8. The effect of ellipses eccentricity on $\tilde{q}_{*,max}$ is depicted in Fig. 7, where, for all Re_L, $\tilde{q}_{*,max}$ increases as the eccentricity decreases, i.e., the flatter the ellipses are the higher the overall heat transfer will be. In a quantitative analysis, it is important to stress that a 13 % maximum relative heat transfer gain, in comparison with the traditional circular arrangement, was observed for the elliptical arrangement with e = 0.65, in the numerical simulations. Figure 8 shows that the optimal spacing decreases as the free stream velocity (or Re_L) increases.

There was no loss of generality of the results by fixing $N_{ec} = 6$ in the present study, as it is deduced through equations (12) and (15). The effect of varying the number of tubes in one elemental channel, N, is still to be investigated, but it should be noted that $N = L/a$ represents the limit where the ellipses ends (edges) touch. However, it is not difficult to verify that

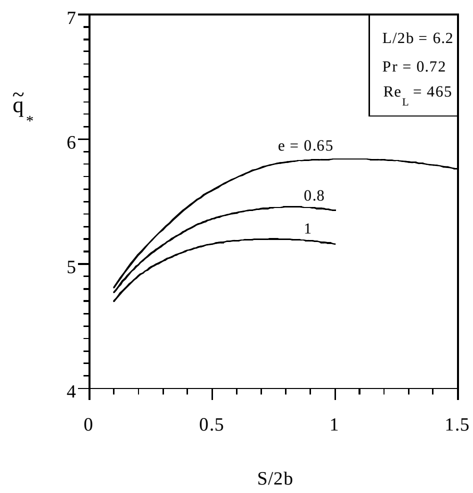

Figure 5 - The effect of ellipses eccentricity on $\tilde{q}_* \left(Re_L = 465\right)$.

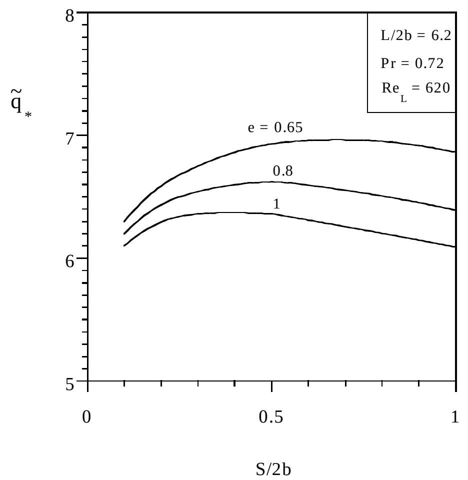

Figure 6 - The effect of ellipses eccentricity on $\tilde{q}_* \left(Re_L = 620\right)$.

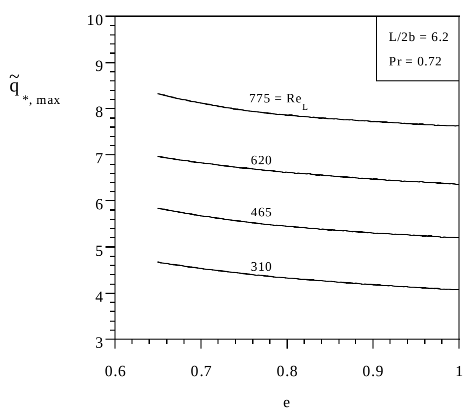

Figure 7 - The effect of ellipses eccentricity on the maximum overall thermal conductance.

the figure of merit given by equation (11) is an analogue of the average Nusselt number for the whole arrangement, $\tilde{q} = \overline{Nu} = \bar{h}(2b)/k$, noting that $\bar{h} = \dfrac{q(2b)}{(T_w - T_\infty)LHW}$, where $\bar{h}$ -

equivalent average heat transfer coefficient, $W/(m^2.K)$. Therefore, for a larger number of rows, $\tilde{q}_{max}$ $(\text{or } \tilde{q}_{*,max})$ computed for N = 4 is a fairly good approximation. This is explained by the fact that, with a large number of rows, the flow will be fully developed, therefore with no significant changes in the average Nusselt number for a particular geometry, either circular or elliptic. This behavior was observed experimentally comparing three-row circular results reported by Saboya and Sparrow (1976), with two-row circular results reported by Rosman et al. (1984), both for finned heat exchangers. The same phenomenon was also observed numerically in a recent study by Fowler et al. (1997), for staggered plates in forced convection, where it was reported that the effect of N on $\tilde{q}_{max}$ is almost nonexistent for $2 \le N \le 65$.

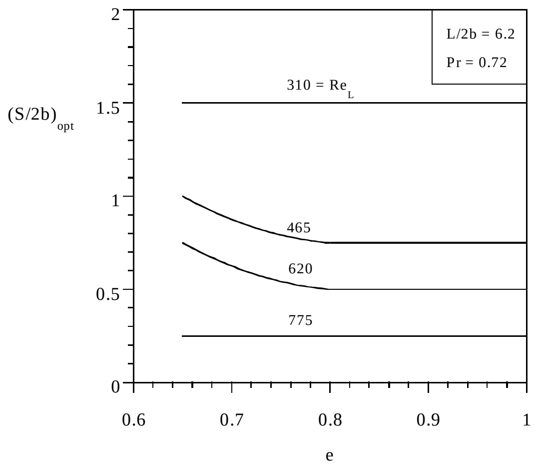

Figure 8 - The effect of ellipses eccentricity on the optimal spacing for maximum overall thermal conductance.

4. CONCLUDING REMARKS

This study demonstrates that the geometric arrangement of staggered circular or elliptic tubes can be optimized for maximum heat transfer, when the optimization is subjected to an overall volume constraint. The existence of optimal spacings between rows of tubes was demonstrated through numerical results obtained from two alternative ways, i.e., equations (12) and (15). The approach was to formulate the problem fundamentally as a volume-constrained geometric optimization study, where appropriate nondimensional groups were identified and generalized results presented in dimensionless charts. From the point of view of practical application of the results herein presented, it is important to stress that they will apply depending on how similar the actual design under consideration is to the configuration presented in Fig. 1, such that the approximate optimal geometry can be predicted. However, from the fundamental point of view, the results show that there will always be an optimal spacing between rows of tubes in circular and elliptic tubes heat exchangers, which is important to be found.

From the heat transfer point of view, it was shown that the elliptic configuration performs better than the circular one. Among the studied cases, a maximum relative heat transfer gain was of 13 %, for e = 0.65, with $Re_L = 465$. The heat transfer gain, combined with the relative pressure drop reduction of up to 25 % observed in previous studies (Brauer, 1964; Bordalo and Saboya, 1995) show the elliptical arrangement has the potential for a considerably better overall performance than the traditional circular one.

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