OPTIMIZATION OF OPERATION FOR CHP-PLANTS WITH HEAT ACCUMULATORS - USING A MILP-FORMULATION

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Abstract. *The power generation system in Denmark is extensively based on small combined heat and power plants (CHP plants), producing both electricity and district heating. This project deals with smaller plants spread throughout the country. Often a heat accumulator is used to enable electricity production, even when the heat demand is low. This system forms a very complex problem, both for sizing, designing and operation of CHP plants. The objective of the work is the development of a tool for optimisation of the operation of CHP plants, and to even considering the design of the plant. The problem is formulated as a MILP-problem. An actual case is being tested, involving CHP producing units to cover the demand. The results from this project show that it is of major importance to consider the operation of the plant in detail already in the design phase. It is of major importance to consider the optimisation of the plant operation, even at the design stage, as it may cause the contribution margin to rise significantly, if the plant is designed on the basis of a detailed knowledge of the expected operation.*

Keywords: *CHP-plant, Operation, Optimisation*

1. INTRODUCTION

Cogeneration as a facility for district heating can be designed in a various number of ways. As a large number of design options are available, it is often difficult solely by experience to determine the optimal system design for an actual geographical location. Therefore it will, be an advantage to have a computer-based model as a tool for selecting the correct CHP-system configuration. Flow-sheet optimisation for selection of plant configuration of large plants is proposed by (Manninen & Zhu, 1999). Optimal CHP-plant operation for a shorter period of time has been considered in detail by (Spakovsky et. al, 1995). In (Zhao, 1998) the dynamics of the district heating system is taken into account, and a detailed model for plant operation is defined, considering variations during a period of 48 hours. In (Gustafson, 1998) and (Henning, 1997) more overall considerations are discussed, and the operation throughout the year is considered but the design of the heat accumulator is not included. In the present work the objective is to develop a computerised model for finding the optimal configuration of a CHP-plant, including heat accumulator. The objective function of the optimisation is the contribution margin of the plant. In order to reduce the number of plant configurations, that have to be treated, the method described in (Iversen, et. al, 1999) is used. Furthermore, only gas-fired plants are considered, as such plants are very common in Denmark, but the method can easily be applied to other types of plants.

2. POWER GENERATION IN DENMARK

Power generation in Denmark is by tradition based on a number of large coal and gas fired plants, most of them are operated as combined heat and power plants. After natural gas, taken from Danish off-shore resources, has been introduced on the Danish market incentives have been made with the purpose to introduce small combined heat and power plants (CHP plants) as the basis for district heating supply. The plant sizes vary from around 1 MW up to several hundred MW, and may consist of one or more separate units, typically as gas motor or gas turbine systems.

2.1 Electricity price structure

The electricity produced is sold to the national power grid, and one of the incentives introduced is that the price for electricity sold to the grid is higher than what is the basic production price from large power plants. Also the owners of the CHP-plants receive different prices for the electricity, depending on the demand. This is formally defined as a number of tariffs throughout the day, as shown in fig. 1.

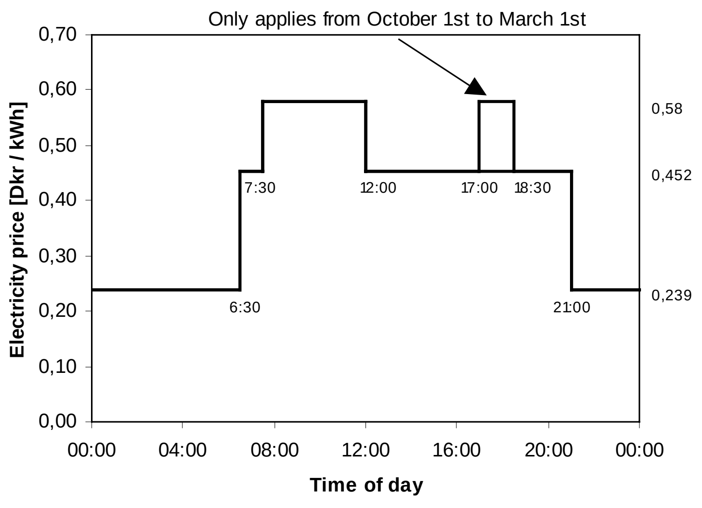


Figure 1 - Tariffs for the electricity price of the Danish power system[1].

[1] 100 US$ is approx. 775 DKR, april 2000

Please note that this only applies for the days of the week, while the entire weekend is defined as low load.

2.1 Heat demand

The demand for district heating varies over the entire year and depends on the weather conditions. The heat demand is calculated based on the Danish Test Reference Year, (TRY). The heat demand is correlated to the TRY-data, it is assumed that 40% of the yearly district heating demand can be considered constant, while the remaining 60% is variable. As mentioned above, only gas-fired plants are considered, and therefore, it is important to be aware of the Danish price structure for natural gas. To support cogeneration for district heating purposes incentives have been introduced; one is that the price for gas used for electricity production is cheaper than gas used for heat production. As it is impossible to convert all the total heat value of the gas into heat or electric power, a certain loss to the surroundings will occur. The price for the part of the gas “lost” is either calculated as defined by the E or the V-formulas, as shown in fig. 2.

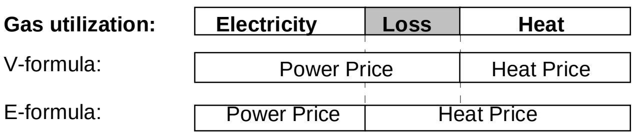


Figure 2 - Principle of the V-formula and E-formula.

Gas, which is used for power production, is purchased at a much lower price (the “power price”) than gas used for heat production. Therefore, it is always more advantageous to use the V-formula. In Denmark it is defined that CHP-plants must use the V-formula the first 5 years of their lifetime, and after that the E-formula.

3. METHODOLOGY

The optimum operation is calculated using the contribution margin of the plant as objective function. It is also preferable to minimise the number of start-ups, as every start-up is associated with additional mechanical wear and typically there is a loss or lower efficiency in the start-up phase. To take the number of start-ups into account a price associated with each start-up is defined. It is thereby possible to value the number of start-ups.

$$\max\ z = f\left(R_{el}, R_{heat}, C_{fuel}, C_{main}, N_{start}\right) \qquad (1)$$

Where z is the contribution margin (DKR), R_{el} is the revenue for sale of electricity (DKR), R_{heat} is the revenue for sale of district-heating (DKR), C_{fuel} is the expense for fuel (DKR), C_{main} is the price for maintenance (DKR) and N_{start} is the number of start-ups.

3.1 Discretization of optimisation problem

In order to solve the optimisation problem it is chosen to divide the time and the load of the plant into a finite number of steps.

Discretization of time

As the heat demand varies throughout the year, it is necessary to perform calculations for the entire year. To simplify the problem calculations are made for 52 weeks (approx. one year), as it is reasoned that the operation during one week only have little or no impact on the operation the next week. Therefore, the week is used as the time domain, and 52 optimisations are made, one for each week with a distinct heat demand. If a division of the time in steps of 30 minutes is used all the variations in the tariffs will be covered, see fig. 1. This provide 48 steps per. day or 336 steps per. week, as this proves to be an optimisation problem of very large scale it is chosen to divide the week into 7 days, and the operation is optimised for each day. Each day is defined as beginning at 6:30 in the morning, where the high load tariff begins, this have proven to be computational attractive, while it is considered only to have minor influence on the result.

Considerations for the heat accumulator

As the optimisation for each day is considered separately, the filling of the heat accumulator is not optimised throughout the week. Instead a simple heuristic rule is used, the weekend is low load, and, therefore, it is preferable to have a full heat accumulator at the beginning of the weekend and an empty heat accumulator Monday morning. Though if the demand is low, it might not be possible to use all the heat in a full heat accumulator and in these situations the loading of the heat accumulator Friday evening must be adjusted to fit the actual demand. This is formulated as

$$
\begin{aligned}
&Q_{store,i,1,1} = Q_{store,\min} \\
&Q_{store,i,5,30} = \min\left(Q_1; Q_2; Q_3\right) \\
&\quad Q_1 = 9.5h \cdot \dot{Q}_{demand,i,5} + 24h \cdot \dot{Q}_{demand,i,6} + 24h \cdot \dot{Q}_{demand,i,7} \\
&\quad Q_2 = \sum_{j=1}^{4} \max\left(24h \cdot \left(\dot{Q}_{prod,\max} - \dot{Q}_{demand,i,j}\right);0\right) + \max\left(14.5h \cdot \left(\dot{Q}_{prod,\max} - \dot{Q}_{demand,i,5}\right);0\right) \\
&\quad Q_3 = Q_{store,\max}
\end{aligned} \tag{2}
$$

Where $Q_{store,i,1,1}$ is the loading of the heat accumulator (MJ) in the *i'th* week at day 1 (Monday) at time-step 1 (beginning at 6:30). $Q_{store,i,5,30}$ is the load of the heat accumulator (MJ) at day 5 (Friday), at time-step 30 (beginning at 21:00). $\dot{Q}_{demand,i,j}$ is the heat demand (MW) at day *j*. $\dot{Q}_{prod,\max}$ is the maximum heat production at the plant, not accounting for auxiliary boilers. $Q_{store,max}$ is the maximum capacity of the heat accumulator (MJ). $Q_{store,min}$ is the minimum capacity of the heat accumulator (MJ). Please note that often minimum and maximum capacities are defined as 5% and 95% of the total possible capacity. This is in order to ensure that the heat accumulator can cover small variations in the heat demand.

Hereby the load of the heat accumulator is calculated for Monday morning 6:30 and Friday evening 21:00. In order to reach the desired level at Friday 21:00 heat must be accumulated through out the week.

$$Q_{store,i,j,1} = \max\left(\dot{Q}_{prod,\max} - \dot{Q}_{demand,i,j};0\right) \cdot 24h \frac{Q_{store,i,5,30} - Q_{store,\min}}{Q_2 - Q_{store,\min}} + Q_{store,i,j-1,1} \quad \forall j \in [2;5] \tag{3}$$

Discretization of the load

In addition, the load of each power production units at the plant is considered to be on / off, which means that during a time step a unit can either be turned off or operate at 100% load.

With these simplifications a power plant consisting of 5 gas engines can have 6 discrete modes of operation at each time step, which is 6^{24} different load combinations, for each day.

3.2 Formulation of the optimisation problem for one day

Using the above discretization of the time and the load the optimisation problem can be formulated. The objective function is the contribution margin, for one day the contribution margin is defined as:

$$\begin{aligned} \max \quad z_{i,j} = &\sum_{k=1}^{48} N_{unit,i,j,k} t \left(\dot{Q}_{elec}\left(R_{elec,i,j,k} - C_{main}\right) + \dot{Q}_{demand,i,j} R_{heat}\left(1 - x_{loss}\right)\right) \\ &- \sum_{k=1}^{48} N_{startup,i,j,k} C_{startup} - \sum_{k=1}^{48} C_{gas,heat} \dot{Q}_{boiler,i,j,k} t \\ &- \sum_{k=1}^{48} N_{units,i,j,k} t \left(\dot{Q}_{heat} C_{gas,heat} + \left(\frac{\dot{Q}_{el} + \dot{Q}_{heat}}{\eta_{total}} - \dot{Q}_{heat}\right) C_{gas,elec}\right) \end{aligned} \tag{4}$$

Where i is the week, j is the day (Monday is day 1), k is the time-step, where the time-step from 6:30 to 7:00 is the first. N_{units} is the number of units in operation. t is the length of each time-step (h). Q_{elec} is the electricity production for one unit (MW). R_{elec} is the revenue for electricity (DKR / MWh). C_{main} is the maintenance cost associated with the electricity production (DKR / MWh). Q_{demand} is the heat demand, R_{heat} is the revenue for heat (DKR / MWh) and x_{loss} is the loss (%) of heat in the distribution system. $N_{startup}$ is the number of start-ups and $C_{startup}$ is the cost associated with each start-up. $C_{gas,heat}$ is the price of gas used for heat production (DKR / MWh), Q_{boiler} is the heat production on auxiliary boilers (MW). Q_{heat} is the heat production for one unit, η_{total} is the overall efficiency for each unit and $C_{gas,elec}$ is the price of gas used for power production (DKR/MWh).

Furthermore the loading of the heat accumulator must be calculated for each time-step.

$$N_{units,i,j,k} \dot{Q}_{heat} t - \dot{Q}_{demand,i,j} t + \dot{Q}_{boiler,i,j,k} t + Q_{store,i,j,k} = Q_{store,i,j,k+1} \quad \forall k \in [1;48] \tag{5}$$

Please note that in this way there are 49 variables for the store each day, as each variable describes the amount of heat in the storage at the beginning of each time-step, and the 49th variable describes the amount of heat in the storage at the end of the 48th time-step.

To count the number of start-ups the following constraint are used:

$$1-\left(N_{units,i,j,k}-N_{units,i,j,k-1}\right)+N_{startup,i,j,k}\geq 1 \quad \forall k\in[2;48] \tag{6}$$

While the number of start-ups at the beginning of the first time-step is determined by:

$$1-\left(N_{units,i,j,1}-N_{unit,init}\right)+N_{startup,i,j,1}\geq 1 \tag{7}$$

Where $N_{units,init}$ is the initial number of units which are running at the beginning of the day. Furthermore the following variable bounds are used:

$$\begin{aligned}
&0\leq N_{units,i,j,k}\leq N_{units,\max} \qquad \forall k\in[1;48]\\
&0\leq \dot{Q}_{boiler,i,j,k}\leq \dot{Q}_{boiler,\max} \qquad \forall k\in[1;48]\\
&Q_{store,\min}\leq Q_{store,i,j,k}\leq Q_{store,\max} \qquad \forall k\in[1;49]\\
&\dot{Q}_{boiler}\in R^{i\cdot j\cdot k}\\
&Q_{store}\in R^{i\cdot j\cdot (k+1)}\\
&N_{units,i,j,k}\in \left(Z^{+}\right)^{i\cdot j\cdot k}
\end{aligned} \tag{8}$$

Equation (3) – (7) defines the optimisation problem. It is a MILP-problem as it is formulated by linear functions, consisting of both continuous and integer variables. The optimisation must be solved for every day through out the year, and hereby the yearly contribution margin can be calculated:

$$z_{year}=\sum_{i=1}^{52}\sum_{j=1}^{7}z_{i,j} \tag{9}$$

3.3 Summary

The methods for optimisation of operation for a week are described above. The problem has been solved using MILP-solvers from LINDO Systems Inc (LINDO).

4 CASE STUDY

The objective of this work is, as described above, to use the optimisation of operation as a basis for the plant design. In the following, it will be illustrated that the optimal design of a plant is highly dependent on the operation, while it is, of course, necessary to determine optimum operation already at the design stage. As an example it will be examined which plant type and which size of heat accumulator is optimum for a CHP-plant with a peak heat demand at 21.5 MJ_{heat}/s, the data is based on an actual design case from a major Danish consultant.

4.1 Definition of case

Referring to "The Danish Energy Agency" a CHP-plant must be designed so it covers at most 60% of the maximum heat load, which in this case means a maximum of 12.9 MJ_{heat}/s. Two different plant configurations are considered, the choice of these plants are based on the method described in (Iversen et. al. 1999), the overall plant characteristics are shown in table 1.

Table 1. Overall data for the two CHP-plants

Plant Concept	3 Gas Engines	4 Gas Engines
Max. heat production [MJ/s]	13.9 (4.63 pr. engine)	14.0 (3.50 pr. engine)
Engine size [MW]	3.7	2.7
Overall efficiency [%]	91.5	91.6
Investment cost [million DKR]	66.1	69.4

Both plants are equipped with auxiliary boilers to cover the peak demand. The investment cost of the plants does not include the price for a heat accumulator.

In order to apply the operational optimisation on the two plants the yearly heat demand is needed. Based on the Danish Test Reference Year (TRY) the duration curve for the heat demand is calculated, see figure 3.

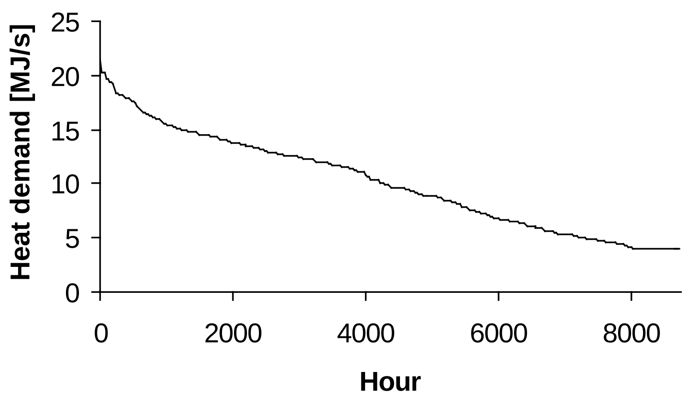


Figure 3 – Duration curve for the heat demand. It is assumed that 40% of the yearly demand is constant load.

In the calculations it is assumed that the heat demand is constant during the day, which e.g. implies that the variation of the heat demand in the morning is not taken into account. To compensate for this assumption it is chosen to define that the heat accumulator can never have a content that is less than 5% of the total and on the other hand it can never be filled to more than 95%. Hereby it is assumed that variation during the day can be included in these margins.

4.2 Optimisation of operation for one week

The method described above is used to optimise the operation, as mentioned above this is done on a daily basis, where the loading of the heat accumulator at the beginning and the end of the day is calculated by the method described in 3.1. An example of the results for optimisation for week is given in fig. 4, where the optimisation of operation for week 22, with the plant configuration with 4 gas-engines is shown.

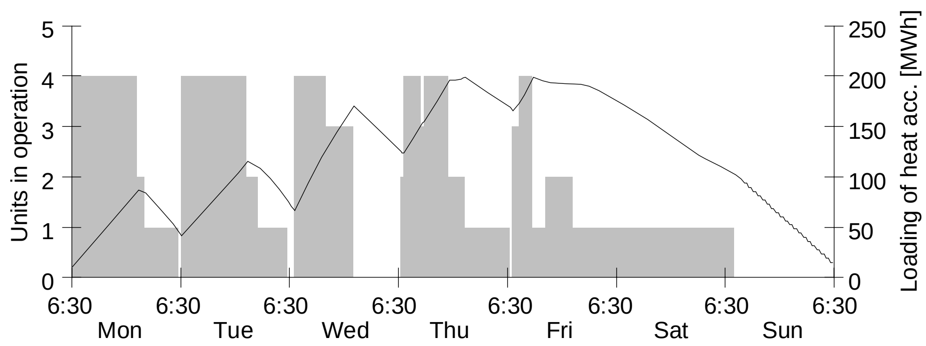


Figure 4 – Optimisation of the operation for week 22. The plant is designed with 4 gas-engines and one auxiliary boiler.

The loading of the heat accumulator is built-up through the week, and from the figure it is apparent that the maximum filling of the storage constraints the operation during Thursday where two engines have to be turned off in the high-load period in the afternoon, while one unit have to operate during the night. If a larger store had been selected the operation would resemble the pattern seen for the first three days of the week, where the operation is not constrained by the loading of the accumulator. It should also be noted that one engine is operating during Saturday, the operation during the weekend could also be decreased if the plant had a larger storage. Finally attention should be given to the operation during Thursday, where one engine is turned off, and after approx. one hour the engine is turned on again. This does not seem reasonable, as the storage does not constraint the operation, and the engine is thereby not forced to stop. Most probably it is because the convergence criterion for the MILP-solver has been set to accept a solution when the duality gap of the MILP-problem is less than 5%. Therefore the solution presented here is not the actual global optimum, but due to the computational effort of closing the duality gap, it has been considered acceptable to have solutions within 5%.

The above result provides the operator of the plant with an operational strategy for the week, or alternatively the actual operation in the past week can be compared to the optimal operation. Finally it should once again be emphasised that the optimal operation is based on a forecast of the heat demand, in reality this forecast might be wrong.

4.3 Optimisation for the entire year, using the optimal operation in design

If the optimisation is applied for the entire year, the yearly contribution margin for each of the plant configurations can be calculated, and it can hereby be determined which one is the better. Furthermore the optimal yearly contribution margin can be calculated for several different sizes of the heat accumulator, to find the optimal size. In fig. 5, the yearly contribution margin as a function of the heat accumulator size is shown.

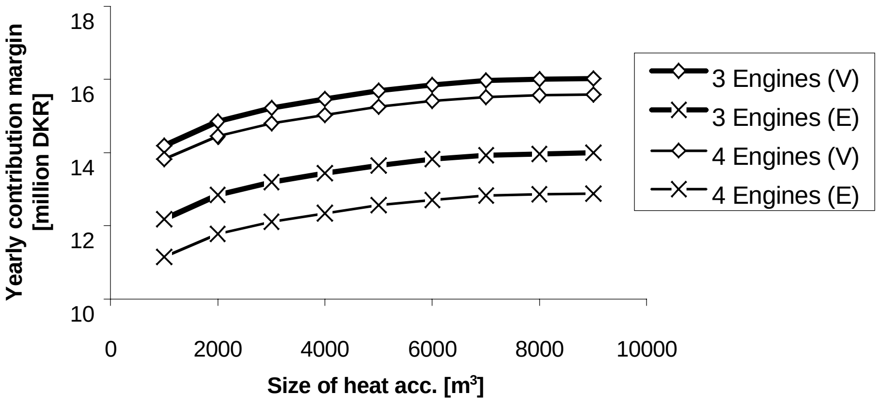
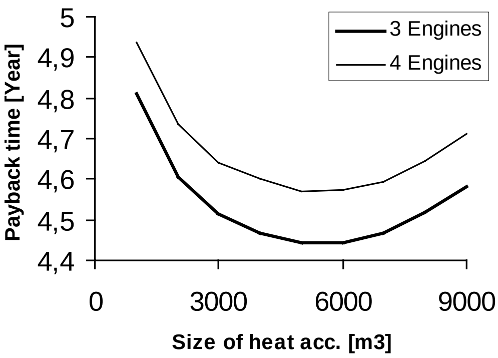


Figure 5 – Yearly contribution margin for both the plant configurations. Furthermore the contribution margin is calculated both with respect to the E-formula and V-formula (ref. section 2.1)

From this it can be seen that the plant with 3 engines have a higher contribution margin, than the plant with 4 engines. To determine the optimal size of the heat accumulator the price must be estimated, this is done by the correlation given in (Iversen et. al., 1999).

$$C_{\text{heatacc}} = 6{,}8 \cdot 10^{-8} V_{\text{acc}}^2 - 2{,}87 \cdot 10^{-5} \cdot V_{\text{acc}} + 2{,}08 \quad (10)$$

Where $C_{heatacc}$ is the investment price for the heat accumulator [million DKR], V_{acc} is the volume of the heat accumulator [m^3]. Using this expression for the price the payback time and the net present value for the plant can be calculated for each of the plant configurations. For the calculation of the net present value a life time of 20 years is used, and an internal rate of return of 10%.



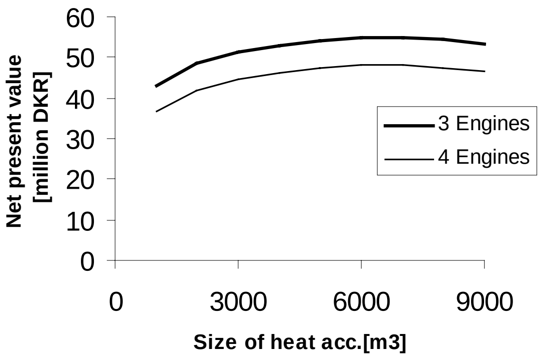


Figure 6 – Payback time and net present value for the plants.

The result is shown in fig. 6, and it can be concluded that a accumulator around 5-6000 m^3, is considered to be the optimum. The method of net present value favours a larger accumulator, this is because the increased contribution margin for all years in the plants living time is taken into account.

Furthermore it is shown that the plant with 3 gas engines is better than the plant with 4 gas engines. This implies that in this particular case it is more feasible to have a plant with fewer and larger units than a plant with more units, even though this reduces the flexibility.

5 CONCLUSION

The method presented here is useful in optimising operation and design of a CHP-plant. It can both be used as a tool for optimising the operation of an existing plant, or, as shown in the case study, used as a tool in the design phase, to determine which plant design is the most feasible. When the method is applied to an existing plant it can determine the optimal operation with respect to the contribution margin, thus predicting the operation of the plant in order to increase the earnings. In the present formulation the method assumes that the plant consists of a number of equal units, this is actually the case in plenty of the Danish CHP-plants, but of course more complex configurations also do exist. In order to handle plants consisting of different units (both in type and size), the method have to be revised.

In the design phase the method is particularly usable when the optimal size of the heat accumulator is to be determined. Though it has only been applied to a single case, where the plant was entirely based on gas engines, it can easily be used for all kind of plants. The method can also be used to compare a number of proposed plant designs, in order to determine which one is the most feasible.

The downside is the high computational times, as hundreds of MILP-problems have to be solved before the optimal size of a heat accumulator can be determined, typically calculation times for the case study was around 8-9 hours on a Pentium Celeron 300. Future developments of the method will both concentrate on reducing the computational time, and introducing the possibility to consider part-load operation with one or more units.

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