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ON THE THERMO-PLASTIC-PHASE TRANSFORMATION COUPLING IN SHAPE MEMORY ALLOYS

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Abstract*. Shape memory, pseudoelasticity and thermal expansion are phenomena related to the behavior of shape memory alloys (SMAs). Constitutive models consider phenomenological aspects of these phenomena. The present contribution discusses the thermo-plastic-phase transformation coupling in the modeling of SMA behavior. Numerical results show that this coupling is important in order to describe the two-way shape memory effect.*

Keywords: *Shape memory alloys, Plasticity, Constitutive model, Numerical simulation.*

1. INTRODUCTION

Metallurgical studies have revealed the microstructural aspects of the behavior of SMAs (Otsuka & Ren, 1999; Shaw & Kyriakides, 1995). Basically, there are two possible phases on SMAs: austenite and martensite. In martensitic phase, there are plates that may be internally twin-related. Hence, different deformation orientations of crystallographic plates constitute what is known by martensitic variants. On SMAs there are 24 possible martensitic variants which are arranged in 6 plate groups with 4 plate variants per group (Zhang *et al.*, 1991). Schroeder & Wayman (1977) have shown that when a specimen is deformed bellow a temperature where only martensitic phase is stable, with increasing stress, only one of the 4 variants in a given plate group will begin to grow. This variant is the one that has the largest partial shear stress. On the other hand, because the crystal structure of martensite is less symmetric than the austenite, only a single variant is created on the reverse transformation (Zhang *et al.*, 1991). For one-dimensional cases, it is possible to consider only three variants of martensite on SMAs: the twinned martensite (M), which is stable in the absence of a stress field, and two other martensitic phases ($M+$, $M-$), which are induced by positive and negative stress fields, respectively.

The thermomechanical behavior of shape memory alloys may be modeled either by microscopic or macroscopic point of view. There are many different works dedicated to the constitutive description of the thermomechanical behavior of shape memory alloys, however, this is not a well established topic (James, 2000; Birman, 1997; Bertram, 1982; Souza *et al.*, 1998; Auricchio & Lubliner, 1997; Auricchio & Sacco, 1997; Auricchio *et al.*, 1997; Tanaka & Nagaki, 1982; Liang &

Rogers, 1990; Brinson, 1993; Boyd & Lagoudas, 1994; Ivshin & Pence, 1994; Fremond, 1987, 1996; Abeyaratne *et al.*, 1994).

Plastic strains are concerned in different articles in order to evaluate either effects of these strains in phase transformations or the description of the two-way shape memory effect (Bo & Lagoudas, 1999; Dobovsek, 2000; Govindjee & Hall, 2000; Zhang & McCormick, 2000a,b; Lexcellent *et al*., 2000; Miller & Lagoudas, 2000). The loss of actuation through repeated cycling due to plastic strain development is one of the import aspects related to the effect of plastic strains in SMAs.

This article discusses the thermo-plastic-phase transformation coupling which is incorporated into a model allowing a correct description of the thermomechanical behavior of SMAs. The proposed model is based on the Fremond's theory and includes four phases in the formulation: three variants of martensite and an austenitic phase. Hardening effect is represented by a combination of kinematic and isotropic behaviors. An iterative numerical procedure based on the operator split technique (Ortiz *et al.*, 1983), the orthogonal projection algorithm (Savi & Braga, 1993b) and the return mapping algorithm (Simo & Taylor, 1986; Simo & Hughes, 1998) is developed. Numerical results show that the thermo-plastic-phase transformation is important to describe the thermomechanical behavior of shape memory alloys.

2. CONSTITUTIVE MODEL

Fremond (1987, 1996) has proposed a three-dimensional model for the thermomechanical response of SMA where martensitic transformations are described with the aid of two internal variables. These variables represent volumetric fractions of two variants of martensite ($M+$ and $M-$), and must satisfy constraints regarding the coexistence of three distinct phases, the third being the parent austenitic phase (A). It has been noted that Fremond's original model present some limitations in the description three-dimensional problems (Savi & Braga, 1993a), however, one-dimensional results are qualitatively good. Here, an alternative one-dimensional model is considered introducing a fourth variant of martensitic phase: twinned martensite.

Modeling of SMA behavior can be done within the scope of the standard generalized material (Lemaitre & Chaboche, 1990). With this assumption, the thermomechanical behavior can be described by the Helmholtz free energy, ψ, and the pseudo-potential of dissipation, ϕ. The thermodynamic state is completely defined by a finite number of state variables: deformation, ε, temperature, T, the volumetric fractions of martensitic variants, β_1 and β_2, which are associated with detwinned martensites ($M+$ and $M-$, respectively) and austenite (A), β_3. The fourth phase is associated with twinned martensite (M) and its volumetric fraction is β_4. The plastic phenomenon is described with the aid of plastic strain, ε^p, and the hardening effect is represented by a combination of kinematic and isotropic behaviors, described by variables μ and γ, respectively. Additive decomposition is assumed and the total strain, ε, may be split into a phase transformation part, ε^{SMA}, usually considered on SMA description, and a plastic part, ε^p.

$$\varepsilon = \varepsilon^{SMA} + \varepsilon^p \tag{1}$$

With these assumptions, each phase have a free energy function as follows,

$$M+: \quad \rho\psi_1 = \frac{1}{2}E_M(\varepsilon-\varepsilon^p)^2 - \alpha(\varepsilon-\varepsilon^p) - \Omega_M(T-T_0)(\varepsilon-\varepsilon^p) + \frac{1}{2}K_M\gamma^2 + \frac{1}{2H_M}\mu^2 \tag{2}$$

$$M-: \quad \rho\psi_2 = \frac{1}{2}E_M(\varepsilon-\varepsilon^p)^2 + \alpha(\varepsilon-\varepsilon^p) - \Omega_M(T-T_0)(\varepsilon-\varepsilon^p) + \frac{1}{2}K_M\gamma^2 + \frac{1}{2H_M}\mu^2 \tag{3}$$

$$A: \quad \rho\psi_3 = \frac{1}{2}E_A(\varepsilon-\varepsilon^p)^2 - \frac{L_A}{T_M}(T-T_M) - \Omega_A(T-T_0)(\varepsilon-\varepsilon^p) + \frac{1}{2}K_A\gamma^2 + \frac{1}{2H_A}\mu^2 \tag{4}$$

$$M: \quad \rho\psi_4=\frac{1}{2}E_M(\varepsilon-\varepsilon^p)^2+\frac{L_M}{T_M}(T-T_M)-\Omega_M(T-T_0)(\varepsilon-\varepsilon^p)+\frac{1}{2}K_M\gamma^2+\frac{1}{2H_M}\mu^2 \tag{5}$$

where α, $L_M=L_M(T)$ and $L_A=L_A(T)$ are material parameters that describe martensitic transformation, E_M and E_A represents the elastic moduli for martesitic and austenitic phases, respectively; Ω_M and Ω_A represents the thermal expansion coefficient for martensitic and austenitic phases, respectively; K_M and K_A are the plastic moduli for martensitic and austenitic phases while H_M and H_A are the kinematic hardening moduli for martensitic and austenitic phases; T_M is a temperature below which the martensitic phase becomes stable in the absence of stress while T_0 is a reference temperature; ρ is the density. A free energy for the mixture can be written as follows,

$$\rho\hat{\psi}(\varepsilon,T,\beta_i,\varepsilon^p,\gamma,\mu)=\rho\sum_{i=1}^{4}\beta_i\psi_i(\varepsilon,T,\varepsilon^p,\gamma,\mu)+\hat{\mathbf{J}}(\beta_i) \tag{6}$$

where the volumetric fraction of the phases must satisfy constraints regarding the coexistence of four distinct phases:

$$0\le\beta_i\le 1 \ (i=1,2,3,4); \quad \beta_1+\beta_2+\beta_3+\beta_4=1 \tag{7}$$

In the absence of stress, detwinned martensites, $M+$ and $M-$, do not exist. In order to include this physical aspect, an additional constraint must be written,

$$\beta_1=\beta_2=0 \ \text{ if } \sigma=0 \text{ and } \ \beta_1^S=\beta_2^S=0 \tag{8}$$

where β_1^S and β_2^S are the values of β_1 and β_2, respectively, when the phase transformation begins to take place. With these considerations, $\hat{\mathbf{J}}$ is the indicator function of the convex τ (Rockafellar, 1970):

$$\tau=\left\{\beta_i\in\Re \;\middle|\; \begin{array}{l} 0\le\beta_i\le 1\,(i=1,2,3,4);\ \beta_1+\beta_2+\beta_3+\beta_4=1; \\ \beta_1=\beta_2=0 \ \text{ if } \sigma=0 \text{ and } \ \beta_1^S=\beta_2^S=0 \end{array}\right\} \tag{9}$$

Using constraints (7), β_4 can be eliminated and the free energy can be rewritten as:

$$\rho\psi(\varepsilon,T,\beta_1,\beta_2,\beta_3,\varepsilon^p,\gamma,\mu)=\rho\tilde{\psi}(\varepsilon,T,\beta_1,\beta_2,\beta_3,\varepsilon^p,\gamma,\mu)+\mathbf{J}(\beta_1,\beta_2,\beta_3) \tag{10}$$

where,

$$\begin{aligned}
\rho\tilde{\psi}=&\ \beta_1\left[-\alpha(\varepsilon-\varepsilon^p)-\frac{L_M}{T_M}(T-T_M)\right]+\beta_2\left[\alpha(\varepsilon-\varepsilon^p)-\frac{L_M}{T_M}(T-T_M)\right]+\\
&+\beta_3\left[\frac{1}{2}(E_A-E_M)(\varepsilon-\varepsilon^p)^2-\frac{(L_A+L_M)}{T_M}(T-T_M)-(\Omega_A-\Omega_M)(T-T_0)(\varepsilon-\varepsilon^p)+\right.\\
&\left.+\frac{1}{2}(K_A-K_M)\gamma^2+\left(\frac{1}{2H_A}-\frac{1}{2H_M}\right)\mu^2\right]+\\
&+\frac{1}{2}E_M(\varepsilon-\varepsilon^p)^2+\frac{L_M}{T_M}(T-T_M)-\Omega_M(T-T_0)(\varepsilon-\varepsilon^p)+\frac{1}{2}K_M\gamma^2+\frac{1}{2H_M}\mu^2
\end{aligned} \tag{11}$$

Now, **J** represents the indicator function of the tetrahedron π of the set (Figure 1),

$$\pi = \left\{ \beta_i \in \Re \left| \begin{array}{l} 0 \le \beta_i \le 1 \left(i = 1, 2, 3 \right); \beta_1 + \beta_2 + \beta_3 \le 1; \\ \beta_1 = \beta_2 = 0 \text{ if } \sigma = 0 \text{ and } \beta_1^S = \beta_2^S = 0 \end{array} \right. \right\} \tag{12}$$

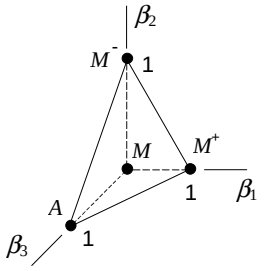


Figure 1 - Tetrahedron of the constraints π.

State equations can be obtained from the Helmholtz free energy as follows:

$$\sigma = \rho \frac{\partial \psi}{\partial \varepsilon} = E(\varepsilon - \varepsilon^p) + \alpha(\beta_2 - \beta_1) - \Omega(T - T_0) \tag{13}$$

$$B_1 \in -\rho \frac{\partial \psi}{\partial \beta_1} = \alpha(\varepsilon - \varepsilon^p) + \frac{L_M}{T_M}(T - T_M) - \partial_1 J \tag{14}$$

$$B_2 \in -\rho \frac{\partial \psi}{\partial \beta_2} = -\alpha(\varepsilon - \varepsilon^p) + \frac{L_M}{T_M}(T - T_M) - \partial_2 J \tag{15}$$

$$\begin{aligned} B_3 \in -\rho \frac{\partial \psi}{\partial \beta_3} = & -\frac{1}{2}(E_A - E_M)(\varepsilon - \varepsilon^p)^2 + \frac{L_M + L_A}{T_M}(T - T_M) + \\ & + (\Omega_A - \Omega_M)(T - T_0)(\varepsilon - \varepsilon^p) - \frac{1}{2}(K_A - K_M)\gamma^2 - \left(\frac{1}{2H_A} - \frac{1}{2H_M} \right) \mu^2 - \partial_3 J \end{aligned} \tag{16}$$

$$X = -\rho \frac{\partial \psi}{\partial \varepsilon^p} = E(\varepsilon - \varepsilon^p) + \alpha(\beta_2 - \beta_1) - \Omega(T - T_0) = \sigma \tag{17}$$

$$Y = -\rho \frac{\partial \psi}{\partial \gamma} = -K\gamma \tag{18}$$

$$Z = -\rho \frac{\partial \psi}{\partial \mu} = -\frac{1}{H}\mu \tag{19}$$

where B_i are thermodynamic forces and σ represents the uniaxial stress; ∂_i is the *sub-differential* with respect to β_i (Rockafellar, 1970). Lagrange multipliers offer a good alternative to represent sub-differentials of the indicator function (Savi & Braga, 1993b). Furthermore, the following definitions are considered:

$$E = E_M - \beta_3(E_M - E_A) \tag{20}$$

$$\Omega = \Omega_M - \beta_3(\Omega_M - \Omega_A) \tag{21}$$

$$K = K_M - \beta_3(K_M - K_A) \tag{22}$$

$$\frac{1}{H} = \frac{1}{H_M} - \beta_3 \left(\frac{1}{H_M} - \frac{1}{H_A} \right) \tag{23}$$

In order to describe the dissipation processes, it is necessary to introduce a pseudo-potential of dissipation. This pseudo-potential can be written through its dual ϕ^*. Considering the following type,

$$\phi^* = \frac{1}{2\eta}\left[(B_1 + \eta_{ci}Y + \eta_{ck}Z)^2 + (B_2 + \eta_{ci}Y + \eta_{ck}Z)^2 + (B_3 - \eta_{ci}Y - \eta_{ck}Z)^2\right] + I_f \tag{24}$$

where I_f is the indicator function related to the yield surface defined as follows,

$$f = \left|X + HZ\right| - (\sigma_Y - Y) \tag{25}$$

The parameter η is associated with the internal dissipation of the material while η_{ci} and η_{ck} are related to plastic-phase transformation coupling. The parameter η_{ci} is associated with isotropic hardening coupling while η_{ck} is associated with kinematic hardening. At this point, it is possible to write the following complementary equations:

$$\dot{\beta}_1 \in \partial_{B_1}\phi^* = \frac{B_1}{\eta} + \frac{\eta_{ci}}{\eta}Y + \frac{\eta_{ck}}{\eta}Z = \frac{B_1}{\eta} - \frac{\eta_{ci}}{\eta}K\gamma - \frac{\eta_{ck}}{\eta}\frac{\mu}{H} \tag{26}$$

$$\dot{\beta}_2 \in \partial_{B_2}\phi^* = \frac{B_2}{\eta} + \frac{\eta_{ci}}{\eta}Y + \frac{\eta_{ck}}{\eta}Z = \frac{B_2}{\eta} - \frac{\eta_{ci}}{\eta}K\gamma - \frac{\eta_{ck}}{\eta}\frac{\mu}{H} \tag{27}$$

$$\dot{\beta}_3 \in \partial_{B_3}\phi^* = \frac{B_3}{\eta} - \frac{\eta_{ci}}{\eta}Y - \frac{\eta_{ck}}{\eta}Z = \frac{B_3}{\eta} + \frac{\eta_{ci}}{\eta}K\gamma + \frac{\eta_{ck}}{\eta}\frac{\mu}{H} \tag{28}$$

$$\dot{\varepsilon}^p \in \partial_X \phi^* = \lambda\,\mathrm{sign}\,(X + HZ) = \lambda\,\mathrm{sign}\,(\sigma - \mu) \tag{29}$$

$$\dot{\gamma} \in \partial_Y \phi^* = \lambda + \eta_{ci}(\dot{\beta}_1 + \dot{\beta}_2 - \dot{\beta}_3) = \left|\dot{\varepsilon}^p\right| + \eta_{ci}(\dot{\beta}_1 + \dot{\beta}_2 - \dot{\beta}_3) \tag{30}$$

$$\dot{\mu} \in \partial_Z \phi^* = \lambda H\,\mathrm{sign}\,(X + H\,Z) + \eta_{ck}(\dot{\beta}_1 + \dot{\beta}_2 - \dot{\beta}_3) = H\dot{\varepsilon}^p + \eta_{ck}(\dot{\beta}_1 + \dot{\beta}_2 - \dot{\beta}_3) \tag{31}$$

where λ is the plastic multiplier. The irreversible nature of plastic flow is represented by means of the *Kuhn-Tucker conditions*. Another constraint must be satisfied when $f(\sigma,\gamma,\mu)=0$. It is referred as *consistency condition* and corresponds to the physical requirement that a stress point on the yield surface must persist on it. These conditions are presented as follows (Simo & Hughes, 1998):

$$\lambda \geq 0\,;\ f(\sigma,\gamma,\mu) \leq 0\,;\ \lambda\, f(\sigma,\gamma,\mu) = 0\ ;\ \lambda\, \dot{f}(\sigma,\gamma,\mu) = 0 \ \text{ if } \ f(\sigma,\gamma,\mu) = 0 \tag{32}$$

These equations form a complete set of constitutive equations. Since the pseudo-potential of dissipation is convex, positive and vanishes at the origin, the Clausius-Duhen inequality (Eringen, 1967), is automatically satisfied if the entropy is defined as $s = -\partial\psi/\partial T$.

Furthermore, it is important to consider the definition of the parameters L_M=L_M(T) and L_A=L_A(T), which is obtained assuming $\dot{\beta}_1 = 0$ and $\varepsilon = \varepsilon_R$ in a critical temperature, T_C, below which there is no residual strain. With this aim, it is necessary to define,

$$\varepsilon_R = \frac{L_M\left[\alpha + \Omega_M(T_M - T_0)\right]}{L_M E_M + \alpha\Omega_M T_M} \qquad T_C = T_M\left[\frac{L_M E_M + \alpha(\Omega_M T_0 - \alpha)}{L_M E_M + \alpha\Omega_M T_M}\right] \tag{33}$$

Hence, using these conditions in Equation (26), the following expressions are obtained,

$$L_M(T)=\begin{cases} L_M = L\,, & \text{if} \quad T \geq T_C \\ L_M = L\,\dfrac{(T_C - T_M)}{(T - T_M)}\,, & \text{if} \quad T < T_C \end{cases} \tag{34}$$

$$L_A(T)=\begin{cases} L_A = L\,, & \text{if} \quad T \geq T_C \\ L_A = 2L - \left[L\,\dfrac{(T_C - T_M)}{(T - T_M)} \right], & \text{if} \quad T < T_C \end{cases} \tag{35}$$

The parameter L may be evaluated as a function of T_A, a temperature above which austenitic phase starts its formation in the absence of stress, as follows:

$$L = \frac{(E_A + E_M)\varepsilon_R^2}{2\left(\dfrac{T_A}{T_M} - 1\right)} \tag{36}$$

The operator split technique (Ortiz *et al.*, 1983) associated with an iterative numerical procedure is developed in order to deal with the nonlinearities in the formulation. The procedure isolates the sub-differentials and uses the implicit Euler method combined with an orthogonal projection algorithm (Savi & Braga, 1993b) to evaluate evolution equations. Orthogonal projections assure that volumetric fractions of the martensitic variants will obey the imposed constraints. In order to satisfy constraints expressed in (12), values of volumetric fractions must stay inside or on the boundary of π, the tetrahedron shown in Figure 1. The elasto-plastic behavior is simulated with the aid of the return mapping algorithm proposed by Simo & Taylor (1986).

3. NUMERICAL SIMULATIONS

In order to evaluate the response predicted by the proposed model, a SMA specimen with typical properties of a Ni-Ti alloy (Table 1), is subjected to different thermomechanical loadings. Stress-driving or temperature-driving simulations are carried out.

Table 1. Thermomechanical properties.

E_A (GPa)	E_M (GPa)	α (MPa)	η (MPa/K)
67	26.30	89.42	0.07

T_M (K)	T_A (K)	T_0 (K)	Ω_A (MPa/K)	Ω_M (MPa/K)	σ_Y^M (MPa)
291.40	307.50	298	0.74	0.17	200
$\sigma_Y^{A,i}$ (MPa)	$\sigma_Y^{A,f}$ (MPa)	K_A (GPa)	K_M (GPa)	H_A (GPa)	H_M (GPa)
690	257.72	1.40	0.40	0.40	0.11

The yield limit σ_Y has a linear variation with T, evaluated with the following expression:

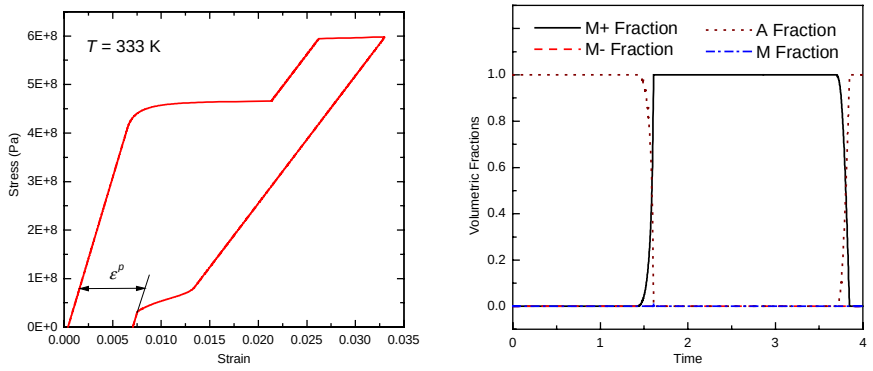
$$T \leq T_M \;\Rightarrow\; \sigma_Y = \sigma_Y^M \tag{37}$$

$$T_M < T \leq T_A \;\Rightarrow\; \sigma_Y = \frac{\sigma_Y^M (T_A - T) + \sigma_Y^{A,i}(T - T_M)}{T_A - T_M} \tag{38}$$

$$T_A < T \leq T_F \;\Rightarrow\; \sigma_Y = \frac{\sigma_Y^{A,i}(T_F - T) + \sigma_Y^{A,f}(T - T_A)}{T_F - T_A} \tag{39}$$

where T_F is used to determine the angular coefficient of the linear interpolation. Moreover, the following parameters are calculated with the aid of Equations (33, 36): ε_R = 0.0033, T_C = 282.04 K and L = 9.18 MPa/K.

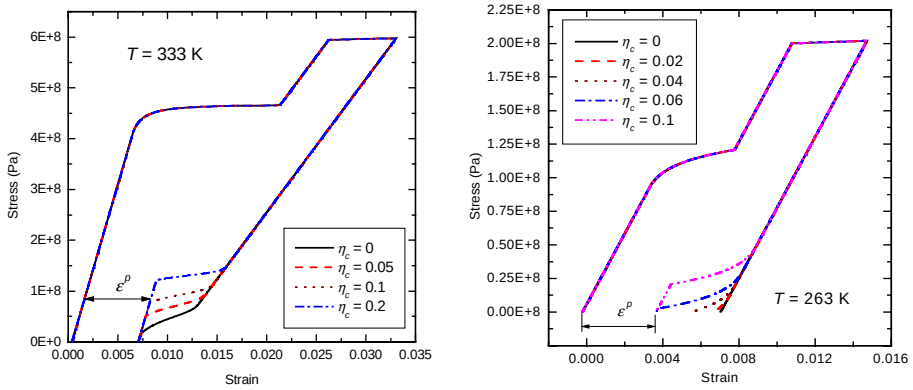
The forthcoming analysis concerns with the effect of plastic strains in the thermomechanical behavior of shape memory alloys. The thermo-plastic parameter is $\eta_c = \eta_{ci} = \eta_{ck} = 0.02$. At first, an isothermal thermomechanical load is considered at T = 333K ($T > T_A$) (Figure 2). Figure 2a shows stress-strain curve related to this load-unloading process. During the loading process, after phase transformation ($A \rightarrow M+$), the yield limit is reached producing plastic strains. Upon unloading, reverse transformation ($M+ \rightarrow A$) is completed and SMA experiences a linear unload. When the unloading process is finished, there are irreversible residual strains associated with plastification. The evolution of volumetric fractions shows the phase transformation related to these processes (Figure 2b).



(a) (b)

Figure 2 - Pseudoelasticity with plastic strain. (a) Stress-strain curve; (b) Volumetric fraction.

A discussion of the plastic-phase transformation coupling is now focused. With this aim, simulations with different values of the parameter η_c are performed. The plastic-phase transformation coupling is now focused considering pseudoelastic and shape memory effects (Figure 3). The alteration of plastic-phase transformation coupling parameter, η_c, shows how this effect tends to anticipate phase transformation. It should be pointed out that this behavior could promote the loss of actuation of the SMA, since the anticipation of phase transformation reduces the amount of phase transformation deformation that can be recovered by either a thermal or a mechanical loading. Figure 3a shows pseudoelastic behavior for different values of coupling parameter. This variation can decrease the internal dissipation of SMA passive actuators, for example. On the other hand, Figure 3b shows a shape memory test, with constant temperature, where this behavior is illustrated. In this situation, the variation of coupling parameter can reduce either the deformation recovery or the force generated by a thermal actuation.



(a) (b)

Figure 3 - Effect of plastic-phase transformation coupling: Stress-strain curves.
(a) Pseudoelasticity; (b) Shape Memory.

A nine-cycle thermomechanical load process, depicted in Figure 4, is now considered. This process is a mechanical loading of the austenitic phase into the plastic region of strain followed by a temperature loading that promote phase transformation. Figure 5a presents stress-strain curves related to this load process while Figure 5c shows the strain-temperature response. The details in Figure 5c show the hysteretic characteristics of phase transformation driven by thermal expansion, indicating that the model is capable to describe the coupling between shape memory effects and thermal expansion. Figure 5b presents evolution of volumetric fractions. These results are in agreement with experimental data presented by Miller & Lagoudas (2000). Notice that the growth of plastic strains tend to enlarge hysteresis loops. This behavior is related to the two-way shape memory effect.

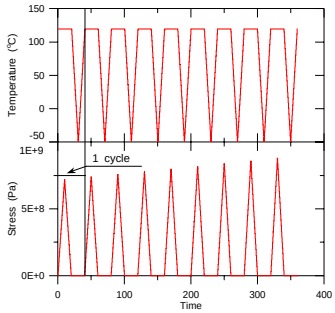
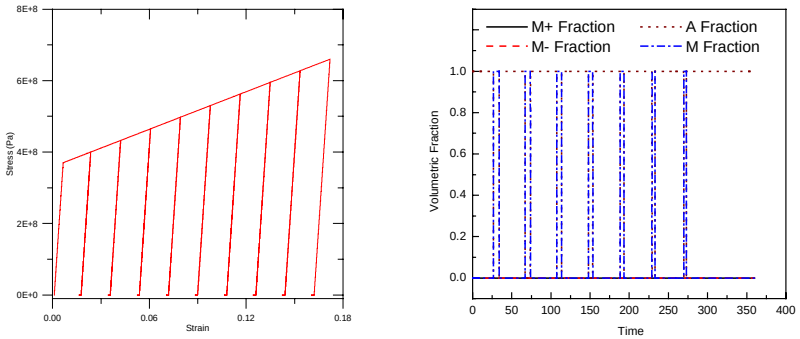
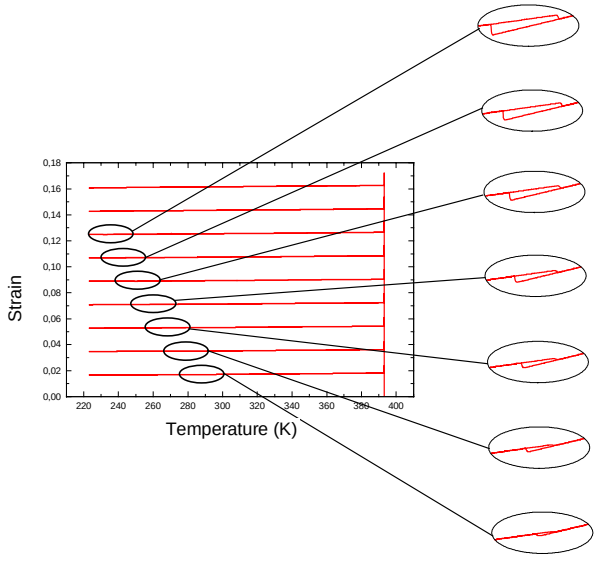


Figure 4 - Nine-cycle thermomechanical load process.



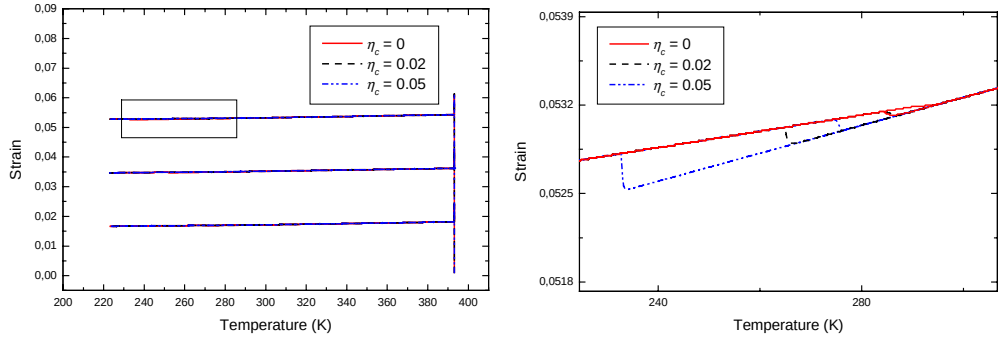
(a) (b)



(c)

Figure 5 - Nine-cycle test.
(a) Stress-strain curve; (b) Volumetric fractions; (b) Strain-temperature curve.

Figure 6 presents strain-temperature curves associated with the three first cycles of the previous test. Notice that plastic-phase transformation coupling parameter tends to change the phase transformation temperature, moving the hysteresis loop. This behavior is related to the growth of hysteresis loops.



(a) (b)

Figure 6 - Effect of plastic-phase transformation coupling: Strain-temperature curves. (a) Three cycles; (b) Enlargement.

4. CONCLUSIONS

Plastic strain description is a goal of the proposed model. Hardening effect is represented by a combination of linear kinematic and isotropic behaviors. A thermo-plastic-phase transformation coupling is incorporated into the model allowing a correct description of the thermomechanical behavior of SMAs. This coupling allows the description of the two-way shape memory effect and, furthermore, tends to anticipate phase transformation, also changing its temperature, after plastic strains occur. All these effects are in close agreement with experimental results.

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