II CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
II NATIONAL CONGRESS OF MECHANICAL ENGINEERING
12 a 16 de Agosto de 2002 - João Pessoa – PB

INVESTIGATION ON THE NON-LINEAR DYNAMIC BEHAVIOR USING TIME-FREQUENCY DISTRIBUTIONS

Demian Gomes da Silva
demian@sc.usp.br

Paulo Sérgio Varoto
varoto@sc.usp.br

Dynamics Laboratory
Department of Mechanical Engineering
School of Engineering of São Carlos – USP
Av. Trabalhador São-carlense 400, CEP 13566-590, São Carlos, S.P, Brasil.

Abstract. *Cohen's based energy distributions constitute a technique, which transforms the time domain vibration response that is represented in the 1-D space in a 2-D representation in the time-frequency domain. The major advantages of this transformation are: (i) enable the analysis of the modifications that occur in the frequency components of a given signal along the time; (ii) identify what is the time duration of such modifications. The present paper is focused on the investigation of the non-linear dynamic behavior in the time-frequency domain by using two types of transformations: the spectrogram and the Wigner-Ville distribution. The results obtained show that: (a) the presence of high-harmonics can be identified and isolated; (b) the amplitude-frequency dependence of the vibration response can be detected and analyzed. The analysis methodology proposed in this paper is illustrated by performing numerical simulations on two non-linear systems that are modeled by the Duffing equation having one and five-degrees-of-freedom.*

Keywords: *Non-Linear behavior, Spectrogram, Wigner-Ville distribution.*

1. INTRODUCTION

Success in product development depends on reaching the market with innovative products that are, for example, smaller, cheaper, and more ergonomic, and yet more reliable. One crucial factor in order to get reliable products is their dynamic behaviour. Undesired vibrational behavior could, for example, lead to fatigue or a high noise levels and in some cases to catastrophic accidents. Thus, the knowledge of the system's dynamic characteristics is crucial in order to achieve a good design.

In almost all design situations, the product to be designed and built is composed of flexible components in such a way that it transforms a time varying input signal into an output signal that presents its own characteristics. In addition to the representation of the system's response, the output signals can also represent the system, for example, the impulse response describes its properties, and can be use to dynamically characterize the system.

When dealing with vibration signals, the system is usually subjected to input signals that for instance can be represented by forces while the system's output responses can be given in terms of displacements, velocities or accelerations. In some practical cases it is difficult to obtain information about the input/output signals in the time domain. To overcome this difficulty many signal processing techniques were developed. Theses techniques are based on three domains: the amplitude, frequency and time-frequency domains. The appropriate domain to assess the system's and input/output characteristics depends on several factors, including availability of measurements, and more important non linear effects presented by the system under study. When the system can be modeled as linear, frequency domain techniques based on the concept of Fourier Series is widely employed in the analysis. However, no real system is linear. Mechanical systems may contain many elements with nonlinear properties, such as, Coulomb sliding friction, nonviscous damping, plastic strain, large deformations, contact deformations, impacts, and geometric nonlinearities (Rao,1992).

For practical purposes, nonlinear systems are in many cases regarded as linear systems since the degree of nonlinearity is insignificant, what allows the system to be considered linear. However, due to the increased requirements in terms of speed, power and reliability of mechanical systems, nonlinearities may become progressively more significant. In theses cases, the assumption of linearity is not appropriate for studying the dynamics of such systems.

For the cases where nonlinear effects is significant, the traditional technique to characterization and analysis based on Fourier methods is uncapable to show important properties about the system. To illustrate this, Fig.(1) shows the impulse response of a SDOF system in two situations: linear and nonlinear (due the stiffness force). Figs. (1a), (1b) and (1c) show respectively the input time history response, the spectral density of the impulse response and the frequency response function (FRF) considering the linear system, and Figs. (1d), (1e) and (1f) show in the same order, but consider the system with a nonlinear stiffness force.

From this figure, in the linear case of Fig.(1a) to Fig(1c) the impulse response consists of a single harmonic mode with natural frequency of 4 Hz represented by the high amplitude in the spectrum to this frequency. The same information can be found from the bode plot representation of the FRF. However, for the nonlinear case showed in Fig.(1d) to Fig.(1f) it is difficult to find with accuracy what is the natural frequency or how the natural frequency changes with the time. This simple example shows that when the nonlinearity is significant, the traditional method to characterize and analyse based on the frequency domain is not adequate.

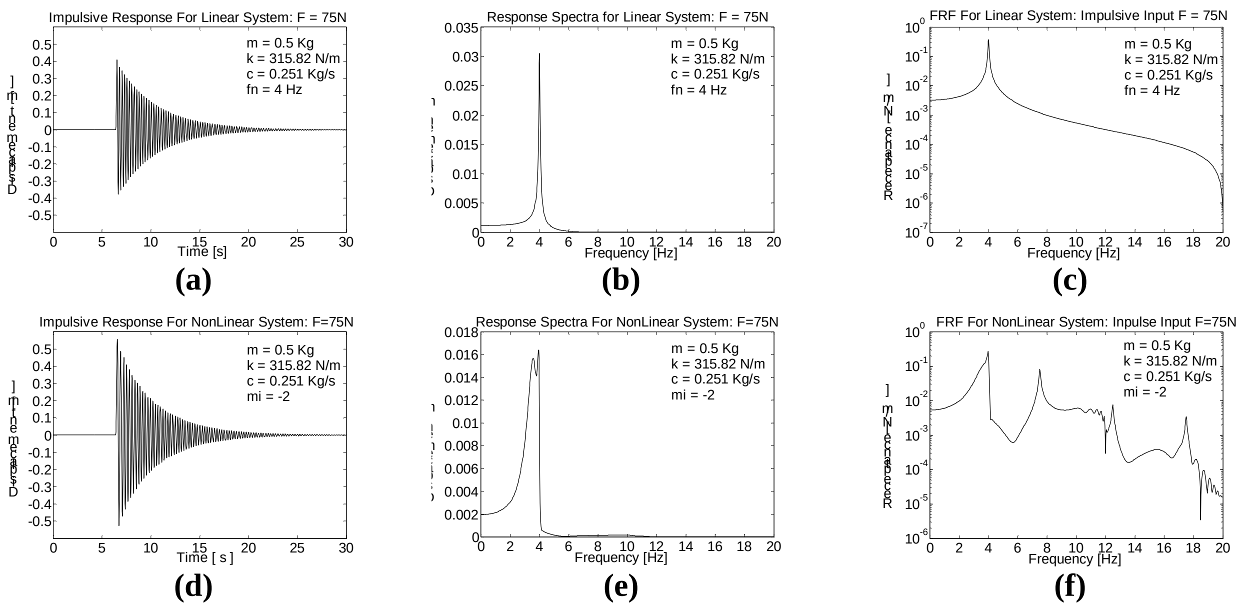


Figure 1. Input response analysis for a SDOF system on two situations: linear e nonlinear (a) time history for the input response of the linear system; (b) spectra to linear input response; (c) FRF to linear system; (d) time history for the input response of the nonlinear system; (e) spectra to nonlinear input response; (f) FRF to nonlinear system.

In order to make the investigation of the dynamic behavior of the nonlinear systems possible, many classical methods were developed in the last decades, such as analytical methods that include pertubation methods, describing function, harmonic balance method, among others (Szemplińska-Stupnicka, 1990; Rao,1992) and experimental methods that include, Hilbert transform (Lin, 1990;

Feldman, 1997), plane-phase (Lin, 1990), Poincarè map (Thompson & Stewart, 1986), High-Order FRF (Chartterjee, 2001), among others. Theses methods provide theoretical and experimental basis for understanding the dynamic behaviour of nonlinear systems.

Currently, a large amount of research results focusing on the new directions of the nonlinear theory can be found, and this shows that the dynamic behaviour of the nonlinear systems is still a topic for further research. Imregun & Chong (2000) studied the coupling between nonlinear sub-systems using variable modal parameters; Woren & Manson (1999) studied the random vibration of the MDOF system using Volterra series; Vakakis (1997) studied nonlinear normal modes and their applications on the vibration theory and Feldman (1997) identify nonlinear behaviour using the Hilbert transform.

The major goal of this article is to employ time-frequency distributions in order to study the dynamic behavior of nonlinear systems. Two nonlinear systems, a SDOF and a MDOF with five degree of freedom are used in numerical simulations with Matlab and Simulink in order to assess the nonlinear dynamics of these systems. Two distributions of the Cohen class (Cohen, 1995) were used, the spectrogram that is based on the short time Fourier transform and the Wigner-Ville distribution. Basic theories related to the spectrogram and the Wigner-Ville distributions are reviewed in order to explain the main differences and advantages between theses quadratic time-frequency representations. The main characteristic of the time-frequency analysis, its major advantages and disadvantages are discussed, based on several numerical simulations under many dynamic conditions.

2. TIME-FREQUENCY ANALYSIS

The analysis of a signal is usually performed in the time or frequency domain. Different properties and parameters can be obtained when the time and frequency analysis are performed. The energy $| x(t) |^2$ of the signal x(t) per unit of time at the particular time t, can be obtained when time domain analysis is used. At the same time, the energy density of the signal per unit frequency at the particular frequency f is given by $| X(f) |^2$ in the frequency domain, where X(f) is the Fourier transform of x(t).

When the spectral content of the signal is changing in time, neither the time nor the frequency domain is sufficient to describe the signal properties accurately. Clearly, the energy density $| X(f) |^2$ becomes a function of time. The analysis of a signal using the joint time-frequency approach is called time-frequency analysis. It gives a new signal representation F(t,f), being a function of time and frequency. Since, in a physical interpretation, the spectrum of the signal is a power frequency distribution, by analogy the function F(t,f) is also called a distribution, and in fact, it represents the signal energy at a time t for different values of frequency f (Boualem, 1992)

There are many different time-frequency distributions with various properties. In this paper two types were used: the spectrogram, which is the squared modulus of short-time Fourier transform (STFT) and the Wigner-Ville distribution (WVD). Both of them, the spectrogram and WVD were briefly showed below (Boualem, 1992; Cohen, 1995).

2.1. Short Time Fourier Transform (STFT)

The short-time Fourier transform is the most widely used method for studying non-stationary signals. The concept behind this mathematical tool is simple and powerful. The STFT is essentially an application of the Fourier transform with a short moving time window to a long time series to obtain its time-frequency representation (Boualem, 1992; Cohen, 1995).

To study the properties of the signal at time t, the signal is multiplied by a window function, h(t), centered at t, to produce a modified signal, that is give as

$$s_t(\tau) = s(\tau)h(\tau - t) \tag{1}$$

The modified signal is a function of two time variables, the fixed time we are interested in, t, and the running time, τ. The window function is chosen to keep the signal as unaltered as possible around the time t, but suppress the signal for instants far away from the time of interest.

Since the modified signal emphasizes the signal around the time t, the Fourier transform will reflect the distribution of frequency around that time, mathematically we can write the STFT like (Cohen, 1995):

$$s_t(\tau) = \frac{1}{\sqrt{2\pi}} \int e^{-jw\tau} s(\tau) h(\tau - t) d\tau \qquad (2)$$

There are many advantages and limitations in the use of the STFT. Perhaps, the major limitation of this technique is to obtain simultaneously good resolutions in both, the time and frequency domains. To represent the signal with more details in the time domain, the only option in the STFT is to make the time window shorter. If the sample rate of the signal remains the same, choosing a shorter time window makes the frequency resolution worse.

The short-time Fourier transform method is ideal in many aspects. It is well defined, based on reasonable physical principles, and for many signals and situations it gives an excellent time-frequency representation. However, for some situations it may not be the best method available in the sense that is does not always give the clearest possible representation of the system dynamics. Thus other methods have been developed, and one of them will be present below.

2.2. Wigner-Ville Distribution Function (WVD)

The Wigner distribution (WD) has been the most widely studied time-frequency distribution. It was developed in quantum mechanics by Wigner and implemented in signal processing by Ville. Many other developments have been made since then. The distribution was rediscovered again in the beginning of the 1990s in a series of papers by Claasen and Mecklenbrauker who also introduced the first discrete approach to the WVD. The distribution has found many successful applications in different areas such as acoustics, speech processing, detection and estimation problems, biosignal processing, seismology and engineering.

The Wigner distribution can be derived by generalising the relationship between the power spectrum and the autocorrelation function for nonstationary time-variant processes. The physical interpretation of the generalised power spectrum F(t,f) is that it represents the instantaneous power density spectrum. This approach leads to the WD defined as show the Eq.(3), where the superscript * denotes the complex conjugate (Boualem, 1992; Cohen, 1995)..

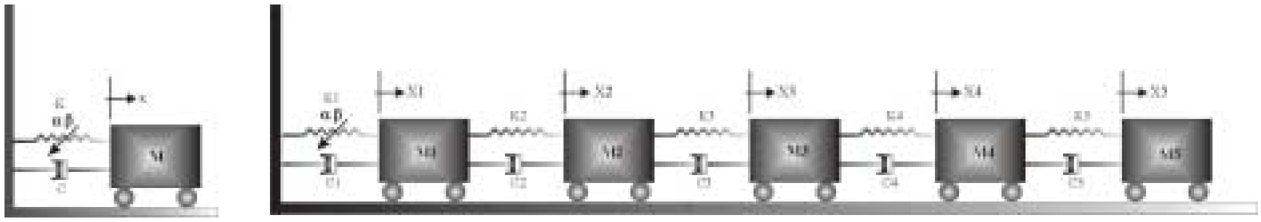
$$W(t,\omega) = \frac{1}{2\pi} \int s^*\left(t - \frac{1}{2}\tau\right) s\left(t + \frac{1}{2}\tau\right) e^{-jw\tau} d\tau \qquad (3)$$

The Wigner distribution (WD) of the analytic signal is called the Wigner-Ville distribution (WVD). The analytic signal is defined as the Hilbert transform of the original signal. It is well known that the spectrum of the analytic signal does not have any negative frequencies, thus the WVD is very often used in practical applications since it avoids interference between positive and negative frequencies.

Signal analysis techniques based on the Wigner-Ville distribution (WVD) can achieve high resolutions in both the frequency and time domains. The original WVD satisfies some desirable distribution properties, such as the reality, shift in time and in frequency, finite support in time and frequency, marginal in time and frequency, instantaneous frequency, and group delay properties. Mathematical definitions of these can found in Cohen (1995). One undesirable property of WVD is that it is not always positive due to the cross or interference terms (ITs).

3. ANALYSIS PROCEDURE

The behavior of the two just presented time-frequency analysis was investigated based on the dynamic behavior of two discrete systems. The first is a SDOF nonlinear system, showed in Fig.(2a). The second is a MDOF nonlinear system with five degrees of freedom, showed in Fig.(2b). In both, the nonlinear effect is due to the nonlinear stiffness force, which is proportional to the cube of displacement.



(a) **(b)**

Figure 2. Nonlinear system studied: (a) SDOF system and (b) MDOF system.

It is assumed that the governing differential equation of SDOF Duffing's nonlinear system as showed in Fig.(2a) can be written as showed in Eq.(4). For the MDOF Duffing's nonlinear system, showed in Fig.(2b), it is assumed that the governing system of differential equation can be written as showed in Eq.(5). Other important parameter is the Duffing's nonlinear coefficient μ, which is described by Eq.(6).

$$m\ddot{x} + c\dot{x} + \alpha x + \beta x^3 = f(t) \tag{4}$$

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [\alpha]\{x\} + [\beta]\{x^3\} = \{f\} \tag{5}$$

$$\mu = \frac{\beta}{\alpha} \tag{6}$$

To obtain the solution of Eq.(4) and Eq.(5), two dynamic models were built using Matworks Simulink®. A Simulink® model consists of three types of elements: sources, the system being modeled, and sinks. In this paper, the sources and the sinks were developed using Matworks Matlab®. In both systems, the SDOF and the MDOF, two input conditions were considered: impulsive and periodic senoidal. A pulse which duration is compatible with interest range approximates the impulse input.

3. RESULTS AND DISCUSSIONS

3.1. SDOF Damped System for Impulse Excitation

To obtain the characteristics of the SDOF, a pulse with duration of the 0.2s was used, which has got enough energy in the range of interest. The SDOF system in analysis, showed in Fig.(2a), has got a natural frequency (linear case) equal to 4 Hz, damping ration of 1%, mass of 0.5Kg, stiffness of 315,82 N/m.

Case 1: Linear with variable force: F=25N; 50N and 75N.

In this case it three amplitude to the impulse excitation were used, 25N, 50N and 75N, in order to evaluate the characteristics of the linear system to the variable force in the time frequency plane, showed by Fig.(3). Figure (3a) to the Fig.(3c) represent the results obtained by spectrogram. By the analysis of those figures, it is possible to see that the energy of the signals is concentrated in a constant frequency of 4 Hz and a range between five to fifteen seconds approximately.

It is also possible to see that the color scale changes with the amplitude of the impulse, from one, the red, when the amplitude is 25 N, to nine, when the amplitude is 75 N, describing thus the changes of the response amplitude.

The same results can be seen in Fig.(3d) to Fig.(3f), that represent the results obtained by Wigner-Ville distribution, with two main differences:(i) There are some distortions in the beginning and (ii) there is a better precision in time and frequency, when compared to the spectrogram.

Case 2: Nonlinear with constant force (75N) and μ variable: μ=-2; -1 and 2.

The main point is to analyze the behavior of the system using time-frequency distributions when the Duffing coefficient (μ) changes. For this analysis three values for μ: -2, -1 and 2 were used. They represent two phenomenon the “softening stiffness” and “hardening stiffness” defined in the classical nonlinear theory.

For the softening spring (μ= -2), in the spectrogram (Fig.(4a)) one can see that most of the energy is concentrated in a frequency range between 3 Hz and 4 Hz and in a time range between 5s and 10s approximately, showing the relation between amplitude-frequency of the response, but one cannot see what this relation is. In Fig. (4d) which represents the Wigner-Ville distribution, one can see accurately that the oscillation frequency varies during the decay in a exponential form. The physical interpretation of this behavior is easily obtained by observing that the stiffness reduction for large amplitude excursions will be accompanied by a slower oscillation. When the duffing coefficient reduces from –2 to –1, the nonlinear effects are strongly reduced and there is a soft decrease of the natural frequency, better observed in WVD.

For the hardening spring, one can better see in the WVD Fig. (4f) than the spectrogram Fig. (4c), that the stiffness increase for large amplitude excursions will be accompanied by a faster oscillation.

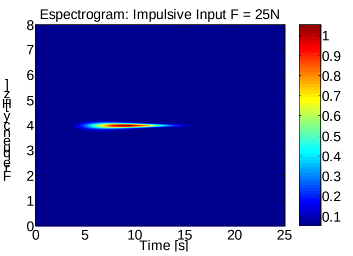
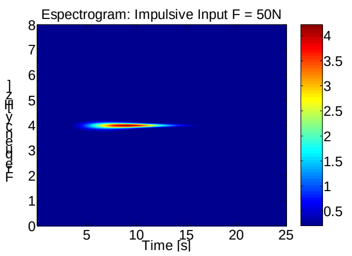
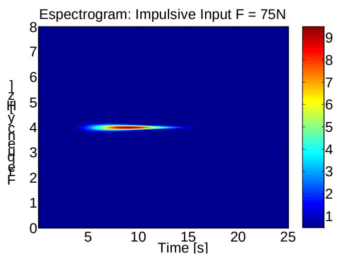
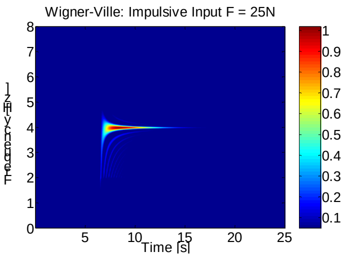
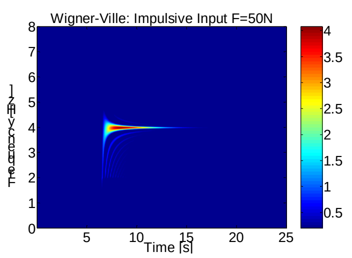
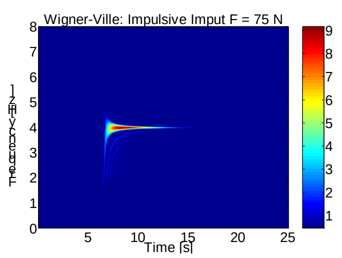
Distribution	Force: F = 25N	Force: F = 50N	Force: F = 75N
Spectrogram	 **(a)**	 **(b)**	 **(c)**
Wigner-Ville	 **(d)**	 **(e)**	 **(f)**

Figure 3. Time-frequency representations, linear system with variable force, 25N; 50N and 75N.

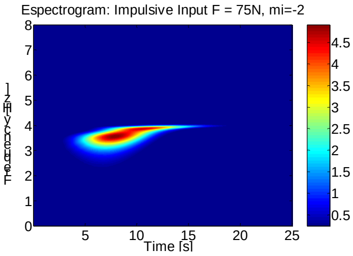
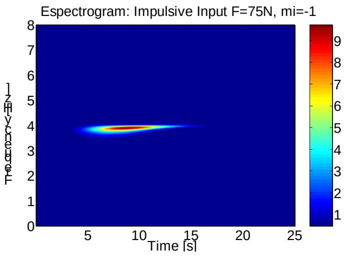
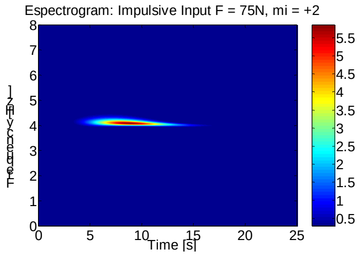
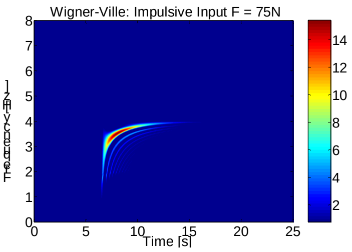
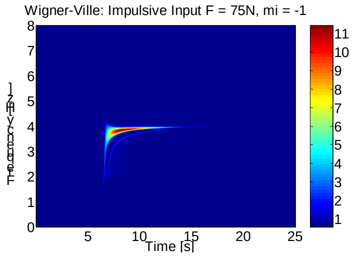
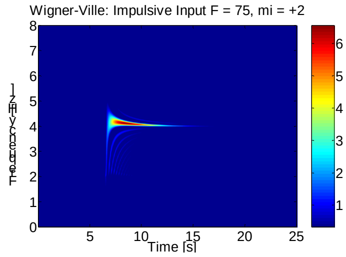
Distribution	μ=-2	μ=-1	μ=2
Espectrogram	 **(a)**	 **(b)**	 **(c)**
Wigner-Ville	 **(d)**	 **(e)**	 **(f)**

Figure 4. Time-frequency representations to nonlinear with constant force (75N) and μ variable: μ=-2; -1 and 2

3.2. SDOF Damped System For Periodic Excitation

Case 1: Nonlinear with μ and frequency constant (μ=3 and f=2Hz) and amplitude variable: F_a=100N and 200N.

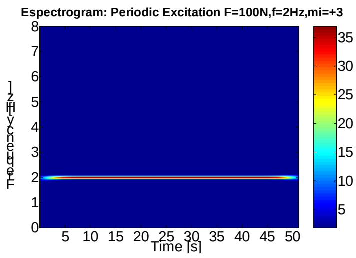
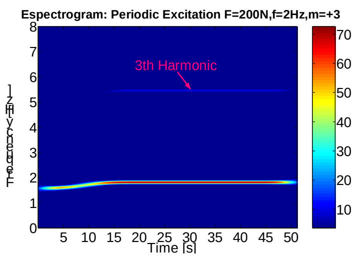
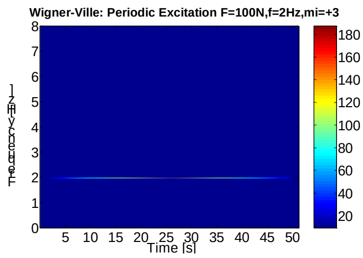
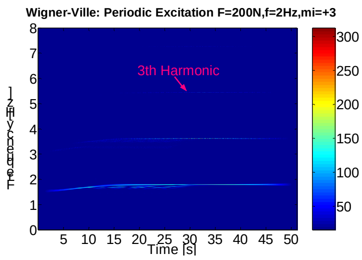
Distribution	Force: F = 100N	Force: F = 200N
Espectrogram	 **(a)**	 **(b)**
Wigner-Ville	 **(c)**	 **(d)**

Figure 5. Time-frequency representations to nonlinear with μ and frequency constant (μ=3 and f=2Hz) and amplitude variable: Fa=100N and 200N.

In this case, the main point is to analyze the response behavior of the SDOF nonlinear system with μ=3 under variable amplitude of excitation. When an amplitude of excitation of 100N is used, in both the spectrogram and WVD, one can see just a single component and such frequency is equal to 2 Hz during all the time without changes, Fig.(5a) and Fig(5c). When the amplitude of the harmonic input becomes larger (200N), the higher-harmonic in the response can be observed in both representations, Fig.(5b) and Fig.(5d).

3.3. MDOF Undamped System For Impulse Excitation

To obtain the characteristics of the multi-degree-of-freedom, a pulse with duration of the 0,025s, smaller than the SDOF system were used, which has gotten enough energy in the range of interest, from 0 to 7 Hz. The MDOF system in analysis was showed in the fig (6), and it has got, for linear case, natural frequencies equal to f_1=0.8, f_2= 2.5, f_3= 4.2, f_4= 5.9 and f_5= 6.8 Hz; masses equal to m_1= 0.4, m_2=1, m_3=0.6 Kg, m_4=1 and m_5= 1Kg, stiffness to all elements equal to 315,89 N/m.

Case 1: Linear for force: F=500N.

It the system as linear was considered. One can find in the spectrogram (Fig.(6a)) all the natural frequencies of the system, and how the first and the fourth natural frequencies have gotten higher amplitudes and the third has gotten the smaller amplitude. The same cannot be found clearly in the WVD representation (Fig.(6b)), due to the large quantity of the cross-terms.

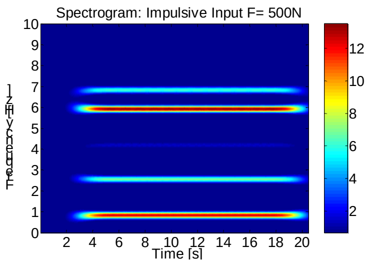
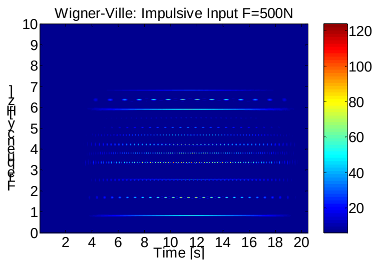
<table>
<tr><th>Spectrogram</th><th>Wigner-Ville</th></tr>
<tr><td>
(a)</td><td>
(b)</td></tr>
</table>

Figure 6. Time-frequency representations of the MDOF linear system.

Case 2: Nonlinear with μ constant (μ=3) and variable force: F=500N; 1000N and 1500N.

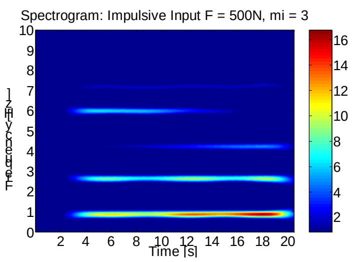
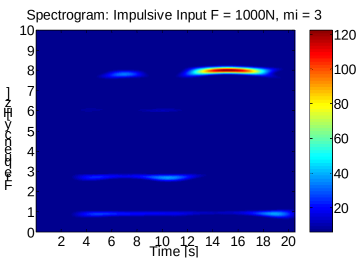
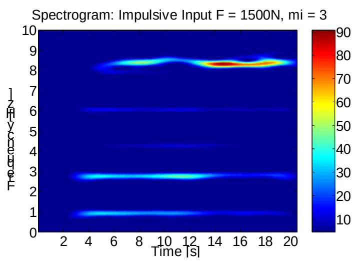
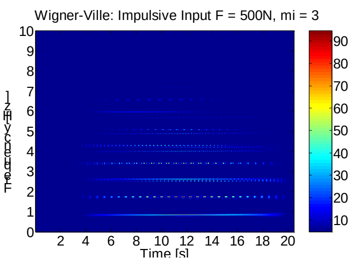
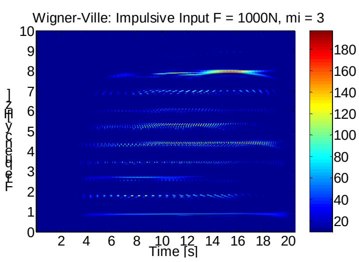
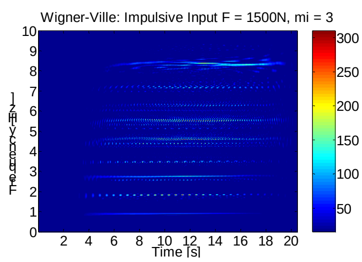
<table>
<tr><th>Distribution</th><th>Force: F = 500N</th><th>Force: F = 1000N</th><th>Force: F = 1500N</th></tr>
<tr><td>Spectrogram</td><td>
(a)</td><td>
(b)</td><td>
(c)</td></tr>
<tr><td>Wigner-Ville</td><td>
(d)</td><td>
(e)</td><td>
(f)</td></tr>
</table>

Figure 7. Time-frequency representations of the MDOF nonlinear system with μ constant (μ=3) and variable force: F=500N; 1000N and 1500N.

Three amplitudes to the excitation signal were considered, 500N, 1000N and 1500N, to evaluate the characteristic of the MDOF system with a nonlinear spring positioned between the ground and the first mass. The impulse excitation was applied on the third mass and the response was obtained in the first mass.One can see in the Fig.(7a) some changes when compared to Fig.(6a): (i) the energy of the signal is not distributed in a uniform way on the natural frequencies anymore; (ii) even undamped the amplitudes of the natural frequencies change with the time.

When the amplitude of the input increases three times, the spectrogram of the Fig.(7c) shows specially that the behavior of the fith natural frequency changes in amplitude and frequency with the time. Once again, the WVD representation was difficult to find information on the dynamic behavior of the MDOF system, due the cross-terms.

3.4. MDOF Damped System For Periodic Excitation

In this item, the dynamic behaviour of the response of the MDOF system, showed in the Fig.(2b), under a single harmonic excitation was analyzed. The excitation was applied on the thrith mass and the response it was obtained on the first mass.

Case 1: Nonlinear Case with μ and frequency constant (μ=3 and f=3.5Hz) and amplitude variable: Fa=500N and 1500N.

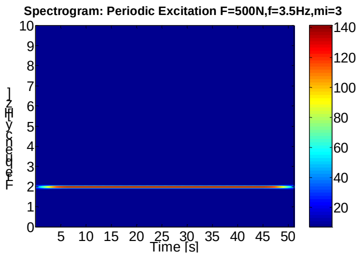
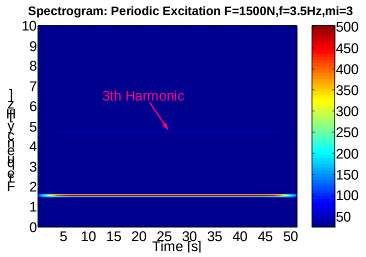
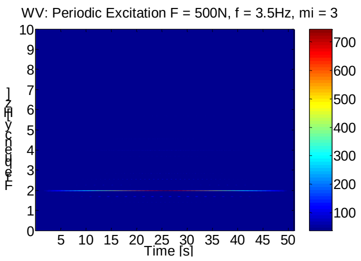
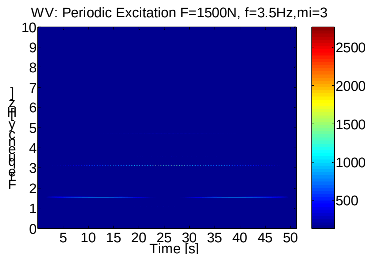
Distribution	Force: F = 500N	Force: F = 1500N
Espectrogram	 **(a)**	 **(b)**
Wigner-Ville	 **(c)**	 **(d)**

Figure 8. Time-frequency representations of the MDOF nonlinear with μ and frequency constant (μ=3 and f=3.5Hz) and amplitude variable: Fa=500N and 1500N.

The main point is to analyze the response behavior of the MDOF nonlinear system with μ=3, under variable amplitude excitation. One can find in the spectrogram, showed in the Fig.(8a), that the system responds with a different frequency of the input. In this case, the excitation frequency is 3.5Hz and the response frequency is 2Hz. Other important characteristic is that the frequency response decreases with the increase of the amplitude of the excitation.

3. CONCLUSIONS

In this paper, we have addressed to analysis of the dynamic behaviors of the Duffing´s nonlinearly structural systems by the time-frequency variations of some response signals. For this, two types of time-frequency techniques were used: the spectrogram and the Wigner-Ville distribution (WVD). The behavior of the two proposed time-frequency analysis was investigated based on the dynamic behavior of two discrete systems. The first is a SDOF nonlinear system, showed in Fig.(2a). The second is a MDOF nonlinear system with five degrees of freedom, showed in Fig.(2b). In both, the nonlinear effect is due to the nonlinear stiffness force, which is proportional to the cube of displacement. The main characteristic of the time-frequency analysis, its main advantage and disadvantage are showed, based on many numeric simulations under many dynamic conditions. The results obtained show that: (a) the presence of high-harmonics can be identified and isolated; (b) the amplitude-frequency dependence of the vibration response can be detected and analyzed; (c) the behaviour of the nonlinear systems are amplitude dependent of the input force; (d) for the softening spring (μ= -2), in the spectrogram (Fig.(4a)) one can see the relation between amplitude-frequency of the response, but one cannot see what this relation is, due to the low accuracy of the spectrogram; (e) in Fig.(4d) which represents the Wigner-Ville distribution, one can see accurately that the oscillation frequency varies during the decay in a exponential form; (f) the spectrogram can analyze the MDOF system satisfactory, but the same not occur with the WVD, due to the cross-terms.

3. ACKNOWLEDGEMENTS

The financial support from CNPq is gratefully acknowledged.

3. REFERENCES

Boualem, B., 1992. “Time–Frequency Signal Analysis: Methods and Application”, Wiley Halsted Press, Melbourne, 546p.

Chong, Y. H., Imregun, M., 2000, “Coupling of Nonlinear Substructures Using Variable Modal Parameters”, Mechanical Systems and Signal Processing, Vol.14, pp.731-46.

Wornen, K., Manson, G., 1999, “Random Vibrations of Multi-Degree-of-Freedom Nonlinear System Using the Volterra Series”, Journal of Sound and Vibration, Vol.226, pp.397-405.

Vakakis, A. F., 1997, “Nonlinear Normal Modes (NNMs) and their Applications in Vibration Theory: an Overview”, Mechanical Systems and Signal Processing, Vol.11, pp.3-22.

Feldman, M., 1997, “Non-linear Free Vibration Identification via the Hilbert Transform”, Journal of Sound and Vibration, Vol.208, pp.475-89.

Lin, R., 1990, “Identification of the Dynamics Characteristics of Nonlinear Structures”,London, 280p.

Szemplińska-Stupnicka, V., 1990, “The Behavior of Nonlinear Vibration Systems”, Kluwer Academic Publishers, London , 251p.

Rao, J. S., 1992, “Advanced Theory of Vibration”, J. Wiley & Sons,,New York, 431p.

Cohen, L., 1995, “Time-frequency Analysis”, Prentice Hall, New Jersey, 300p.

Chatterjee, A., Vyas, N.S., 2001, “Stiffness Non-linearity Classification Through Structured Response Component Analysis Using Volterra Series”, Mechanical Systems and Signal Processing, Vol.15, pp.323-36.

Thompson, J. M. T., Stewart, H. B., 1986, “Nonlinear Dynamics and Chaos”, J. Wiley & Sons,,New York, 376p.

5. AUTHORAL RIGHTS